Non-unitary CFT duals to de Sitter quantum gravity

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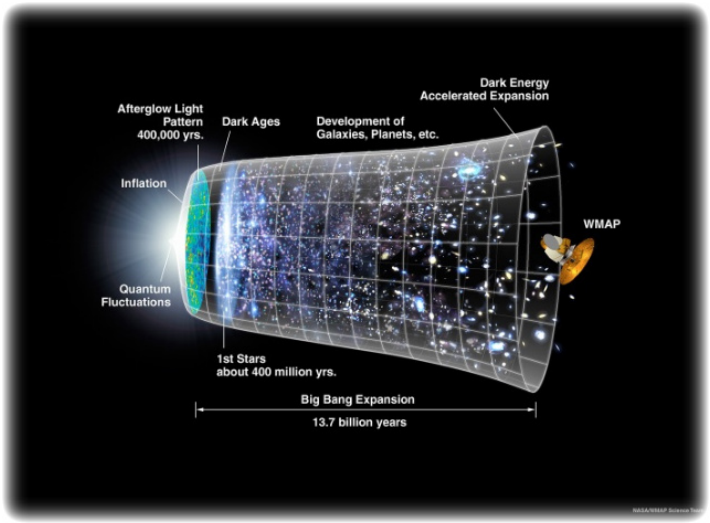
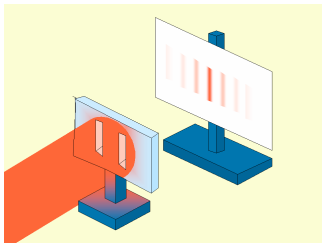
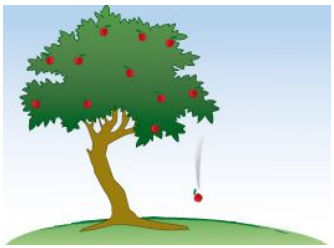
Expanding universe







Accelerated expansion applies both to

- the inflationary era
- our current/late-time universe ("dark energy" domination)

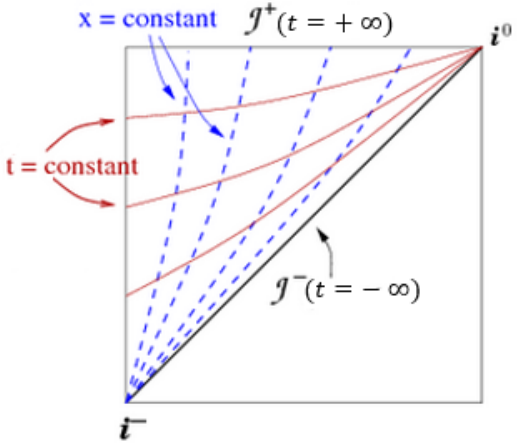
Quantum mechanics + Gravity
+ Accelerated expansion = ?

de Sitter spacetime

Both accelerating eras are well-approximated by the de Sitter geometry:

$$ds^2 = -dt^2 + e^{2Ht} dx_i^2 \, .$$

This represents an *exponentially* expanding spacetime.



Observer cosmological horizons obey thermodynamical laws, with $S = A/4$.

Fundamental questions: how do we understand quantum gravity "in" such a spacetime? What are the observables? What is the meaning of the horizon entropy? Proposal: Construct a holographic dual theory.

Holography: dS/CFT

Holography states that quantum gravity in $d+1$ dimensions is equivalent to a QFT without gravity in d dimensions. There exists a(n incomplete) dictionary between the two in certain cases.

dS/CFT proposal [Maldacena; Strominger; Witten]:

$$\boxed{\Psi_{HH}[\sigma, \dots] = Z_{CFT}[\sigma, \dots]}$$

Renormalization group flow of QFT reconstructs emergent time dimension.

Renormalization group flows are often between two fixed points; maybe in our universe these fixed points are the inflationary era and dark-energy domination! [Strominger]

$Sp(N)$ theory

Certain higher-spin theory, containing all even spins nonlinearly interacting, dual to free $Sp(N)$ theory[1] of *anticommuting* scalars (non-unitary!) [Anninos, Hartman, Strominger]:

$$S = \frac{1}{2}\int d^3x\, \Omega_{ab}\, \partial_i \chi^a \partial_i \chi^b, \qquad \Omega_{ab} = \begin{pmatrix} 0 & 1_{N/2\times N/2} \\ -1_{N/2\times N/2} & 0 \end{pmatrix}.$$

Need to restrict the theory to singlet sector, i.e. only $Sp(N)$-invariant operators in spectrum. Also studied in condensed matter physics [LeClair, Neubert].

Critical theory dual to a different flavor of higher-spin theory:

$$S = \frac{1}{2}\int d^3x\, \left(\partial_i \chi \cdot \partial_i \chi + m\, \chi\cdot\chi + \lambda(\chi\cdot\chi)^2\right).$$

[1] Also exists flavor of duality involving $U(N)$ symmetry.

What to do? Compute wavefunctionals!

Calculate $Z[g_{ij}, m]$ on planar topology:

$$S = \frac{1}{2}\int d^3x\sqrt{g}\,\Omega_{ab}\left(\partial_i\chi^a\partial_j\chi^b g^{ij} + \frac{R[g]}{8}\chi^a\chi^b + m(x^i)\chi^a\chi^b\right).$$

Preserve $SO(3)$ symmetry: consider mass profile $m(r)$ and metric deformation $ds^2 = dr^2 + f(r)^2\, r^2\, d\Omega_2^2$.

Gaussian theory: zeta-regularized partition function computed with Dunne-Kirsten formula on $\mathbb{R}^3$

$$\log\left(\frac{\det\left[-\nabla^2+\mu^2+m(r)\right]}{\det\left[-\nabla^2+\mu^2\right]}\right) = \sum_{l=0}^{\infty}(2l+1)\left(\underbrace{\log T^{(l)}(\infty)}_{\text{Gelfand-Yaglom}} - \underbrace{\frac{\int_0^\infty dr\; r\; m(r)}{2l+1}}_{\text{regularizer}}\right).$$

Constant mass on peanuts: divergence!

Consider peanut geometry and constant mass $m(x_i) = m_0$:

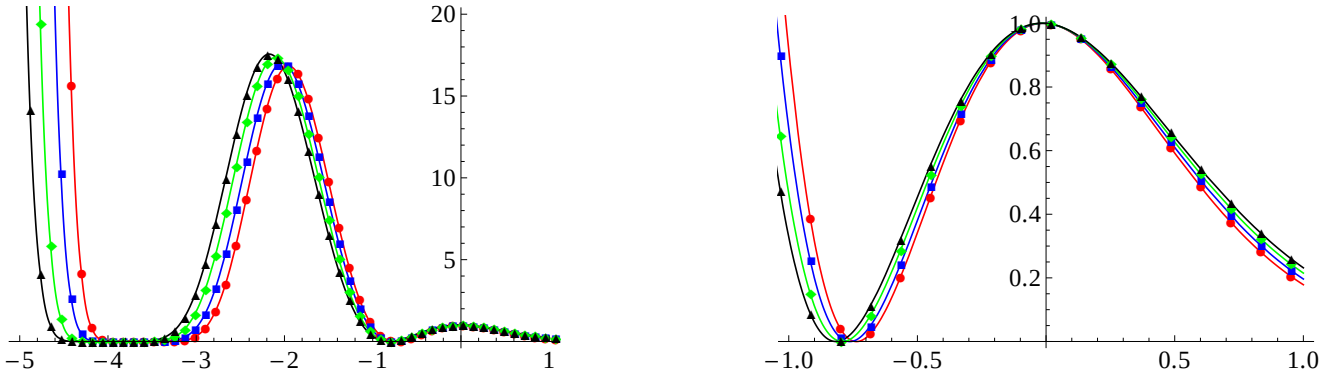


Figure : Left: $|\Psi_{HH}(\zeta, m)|^2$ for for $N = (\ell_{dS}/\ell_P)^2 = 2$ as a function of m_0 for peanut geometries ($l_{max} = 45$). Right: Zoomed in to de Sitter minimum.

$$\phi = \eta\,\nu(x^i) + \eta^2\sqrt{N}m(x^i)$$

Spherical harmonics: killing the divergence

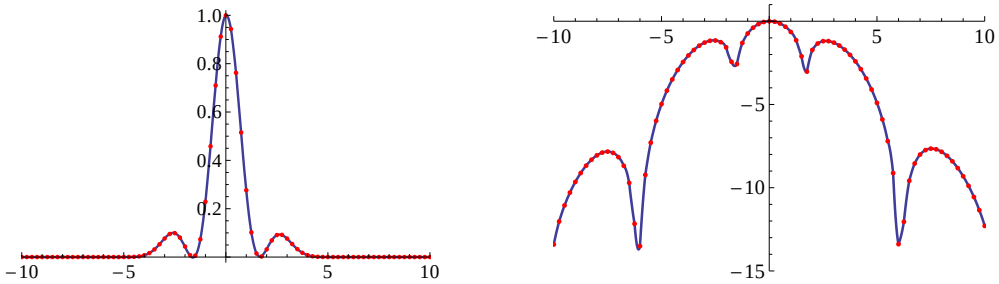


Figure : Left: Plot of $|\Psi_{HH}(A)|^2$ for the first harmonic mapped to $\mathbb{R}^3$. Right: Plot of $\log|\Psi_{HH}(A)|^2$.

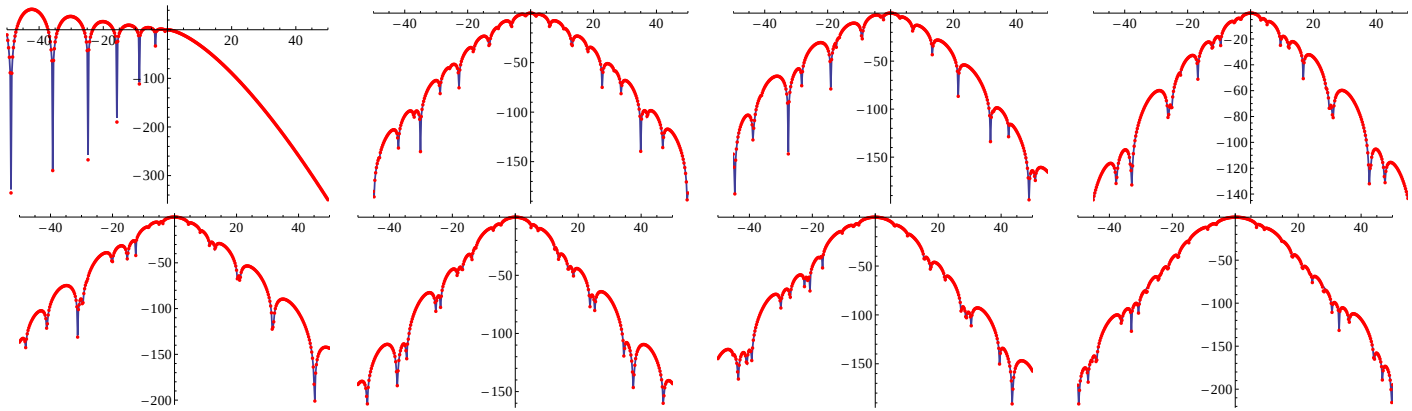


Figure : Plot of $\log|\Psi_{HH}(A)|$ for the first eight spherical harmonics mapped to $\mathbb{R}^3$.

More evidence and a conjecture

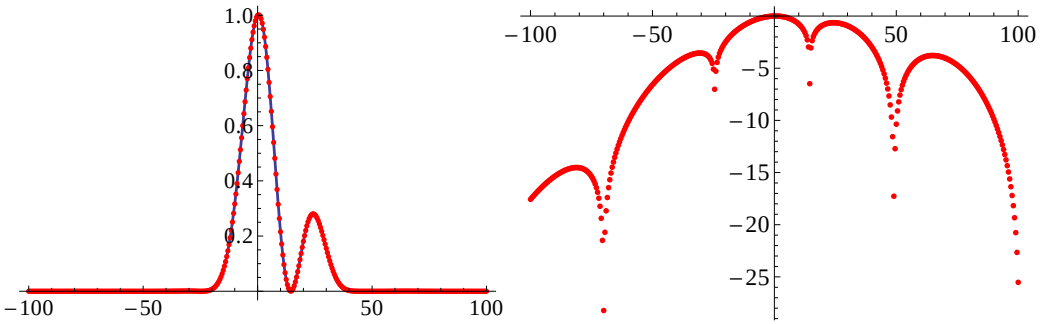


Figure : $|\Psi_{HH}(A)|^2$ (left) and $\log|\Psi_{HH}(A)|$ (right) as a function of A, the overall size of a Gaussian deformation $m(r) = A\,(e^{-r^2} - m_0(r))$ constructed to be orthogonal to the zero mode of the three-sphere.

Conjecture: *The partition function of any mass deformation for which the three-sphere zero mode harmonic is fixed is bounded.*

Extensions of higher-spin dS_4/CFT_3

Can add dynamical Chern-Simons fields. Proposed to be dual to fancier higher-spin theory in de Sitter.

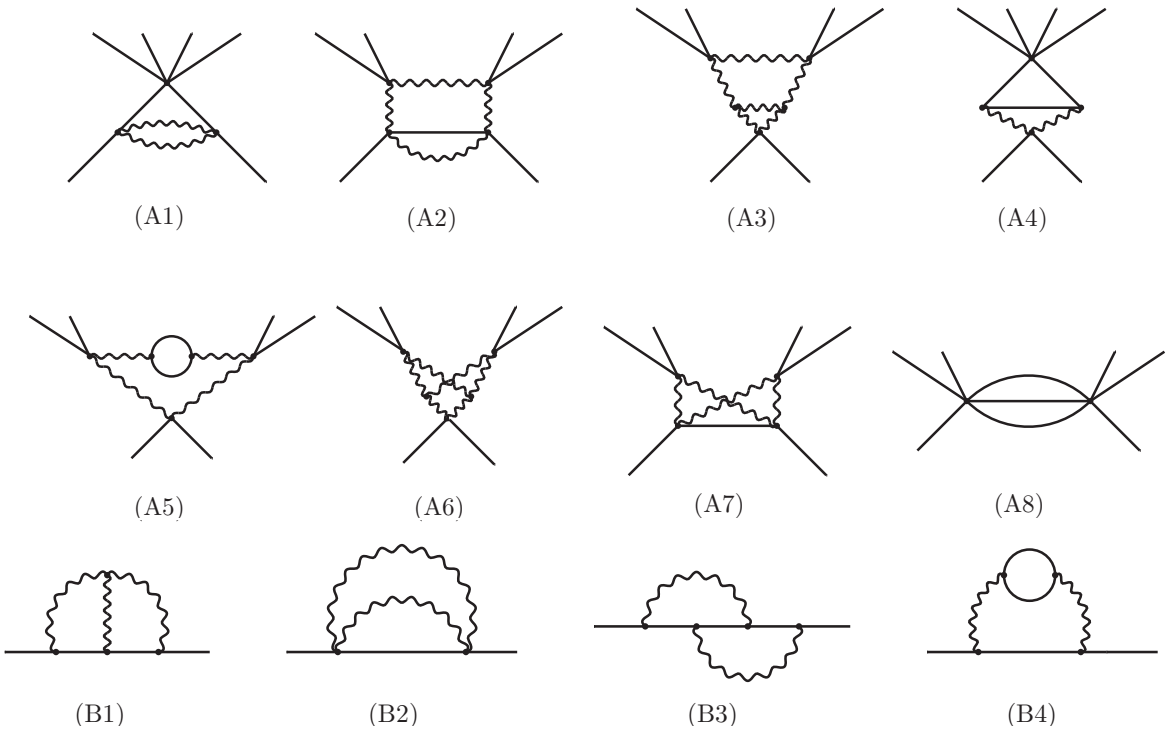
Consider the $Sp(N)$ Chern-Simons-boson theory deformed by marginal triple-trace interaction:

$$S_{CS} = -\frac{ik}{8\pi}\int d^3x\,\epsilon^{\mu\nu\rho}\left(A^a_\mu\partial_\nu A^a_\rho + \frac{1}{3}f^{abc}A^a_\mu A^b_\nu A^c_\rho\right),$$

$$S_B = \int d^3x\left(\Omega_{ij}(D_\mu\chi)_i(D^\mu\chi)_j + N\frac{\lambda_6^b}{3!}\left(\frac{\Omega_{ij}\chi_i\chi_j}{N}\right)^3\right), \quad D_\mu \equiv \partial_\mu + A_\mu.$$

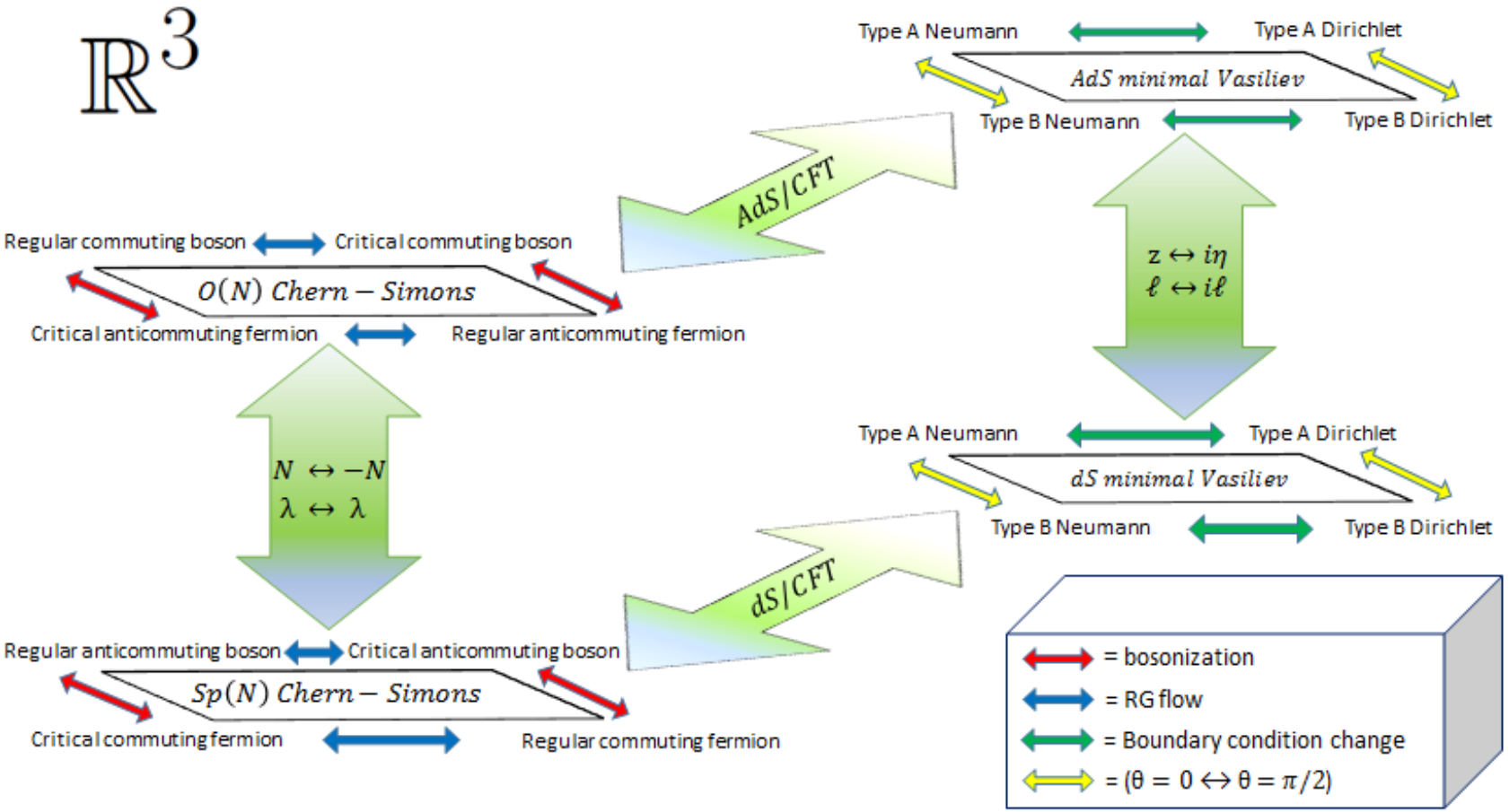
We can calculate the beta functions of this theory as a function of $\lambda = N/k$, $\lambda_6 = g_6N^2$. CS level k quantized and does not run.

Evidence for extensions on $\mathbb{R}^3$



$$\beta_{\lambda_6} = \frac{1}{16\pi^2 N^2}\left(12\lambda^4(\pm N - 1) - 20\lambda^2\lambda_6(\pm N - 1) + \lambda_6^2(\pm 3N + 22)\right)$$

Results on $\mathbb{R}^3$



An analogous set of dualities exists for the $U(N)$ gauge theories!

Calculations

- Partition function calculates bulk wavefunctionals. Can we calculate the partition function for more general deformations? Is the wavefunctional still normalizable?
- Can we calculate the partition function in the presence of a dynamical Chern-Simons term?
- Does the critical $Sp(N)$ theory exist at finite N?
- Can we calculate the partition function in the critical theory?
- Don't forget $U(N)$ theories!
- Motivations: (a) de Sitter relevant to origin and fate of our universe. We do *not* understand it. (b) Higher-spin theory is intermediate between quantum field theory and string theory.
- References: $Sp(N)$ proposal 1108.5735 , CS-$Sp(N)$ 1405.1424 , 1309.7413 , Wavefunctions and critical $Sp(N)$ discussion 1207.5517 , 1305.6321