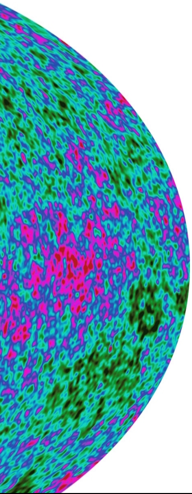
Lattice Holographic Cosmology



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Lattice for Beyond the Standard Model Physics
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Introduction

- The scale of inflation could be as high as 10^{14} GeV and as such it is highest energy scale which is directly observable.
- This is much larger than any energy scale we could achieve with accelerators.

➠ The physics of the Early Universe is a unique probe of physics **beyond the standard model**.

Introduction

- The purpose of this work is to use Lattice in order to study and further develop holographic models for the very early Universe.
- Holography maps gravitational dynamics of our four dimensional universe to observables of a three dimensional QFT with no gravity.
- These observables can then be computed putting the theory on the lattice.

Reference

This talk is based on work in progress with

- Evan Berkovitz, Philip Powel, Enrico Rinaldi, Pavlos Vranas (from LLNL)
- Masanori Hanada (from YITP Kyoto and SITP Stanford)
- Andreas Jüttner, Antonin Portelli, Francesco Sanfilippo (from SHEP, Southampton)

Outline

1. Gauge/gravity duality

2. Holographic cosmology

3. Conclusions

Outline

Gauge/gravity duality: a primer

A conjectured equivalence:

Gravity in $(d+1)$ dimensions $\Leftrightarrow$ QFT in d dimensions

- In this talk we will focus exclusive on $d = 3$:
  a 4-dimensional universe and its dual $3d$ QFT.
- The idea of "holography" has its origins in black holes physics and concrete realisations appeared in string theory.
- No familiarity with black holes and string theory is needed to understand the duality.

QFT in a nutshell

- All information about a QFT is encoded in correlators functions.
- Textbooks usually discuss correlation functions of elementary fields.
- We will instead consider correlation functions of gauge invariant composite operators.
- For example, such operators are the energy-momentum tensor $T_{\mu\nu}$, or scalar operators such as $\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}$, ($F^{\mu\nu}$ is the field strength of Yang-Mills field), $\bar{\psi}\psi$, etc.

Gauge/gravity duality in a nutshell

Gauge/gravity duality provides a way to

- obtain QFT correlation functions of gauge invariant operator by doing a gravitational computation, or conversely
- obtain the behaviour of a gravitational system using correlation functions of gauge invariant operators.

Gauge/gravity duality in a nutshell

- An important feature of the duality is that this is a weak/strong duality.

weakly coupled gravity $\Leftrightarrow$ strongly coupled QFT

- One can obtain QFT correlation functions of gauge invariant operators at strong coupling by solving Einstein equations.

Strongly coupled gravity $\Leftrightarrow$ weakly coupled QFT

- The extra dimension represents the energy scale at which we probe the theory.

Radial evolution is an RG flow.

Which theories?

- The QFT should admit a 't Hooft large N limit.
- The best understood examples involves Conformal Field Theories and Anti-de Sitter space times, hence the name AdS/CFT correspondence.
- Here we will focus on super-renormalizable theories with Lagrangian

$$S = \frac{1}{g_{YM}^2} \int d^3x \mathrm{tr} \left[\frac{1}{2} F_{ij}^I F^{Iij} + \frac{1}{2} (D\phi^J)^2 + \lambda_{J_1 J_2 J_3 J_4} \phi^{J_1} \phi^{J_2} \phi^{J_3} \phi^{J_4} \right].$$

All fields are massless and in the adjoint of $SU(N)$, $\lambda_{J_1 J_2 J_3 J_4}$, are dimensionless couplings while g_{YM}^2 has mass dimension 1.

Holography and Lattice

- The holographic dualities are still conjectural relations.
- There is however a well defined procedure, the method of holographic renormalization [de Haro, Solodukhin, KS (2000)], that one can use to compute renormalised correlation functions of gauge invariant operators doing a gravitational computation.
- These are then predictions for the correlators of the QFT at strong coupling.
- Lattice gauge theory may provide us a way to obtain quantitative tests of the duality.

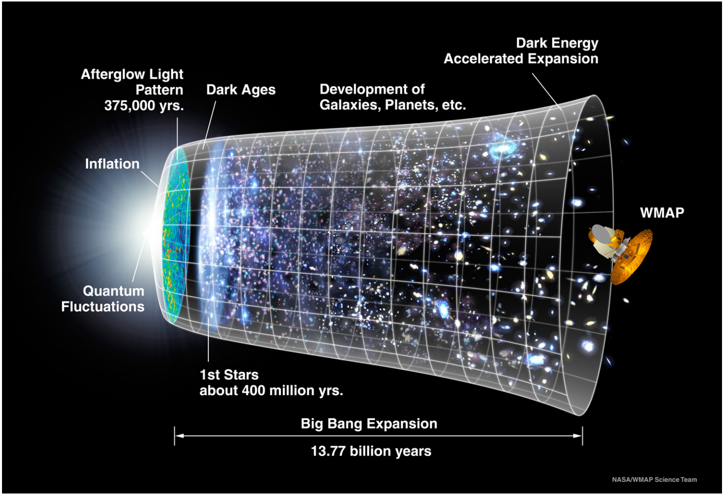
Outline

References

- This part of the talk is based on work done with Paul McFadden and Adam Bzowski (2009 - on-going).
- Related work:
  [Hull (1998)] ... (E-branes)
  [Witten (2001)] [Strominger (2001)] ... (dS/CFT correspondence)
  [Maldacena (2002)] ... (wavefunction of the universe)
  [Hartle, Hawking, Hertog (2012)] ... (quantum cosmology)

  [Trivedi et al][Garriga et al] [Coriano et al] .... [Arkani-Hamed, Maldacena]

Timeline of the Universe



Credit: NASA / WMAP Science Team

Holographic cosmology

- In holographic cosmology the four dimensional Universe is described via the dual QFT.
- This framework accommodates both conventional inflation (strong coupled dual QFT) and qualitatively new models for the very early Universe (weakly coupled QFT).

- While holography for cosmology is less understood theoretically (there is no string theory construction), this is a case where one can make a direct contact with observations.
- Comparing predictions with current data one may either provide evidence or falsify these theories.

Holographic Universe

- In holographic cosmology:

  Cosmological evolution = inverse RG flow

- The dual QFT should have a strongly coupled UV fixed point corresponding to the current dark energy era.

- In the IR the theory should either flow to:
- an IR fixed point (corresponding to de Sitter inflation), or
- a phase governed by a super-renormalizable theory of the type we discussed earlier (corresponding to power-law inflation).

Holographic inflation

We focus from now on the IR of the theory (inflationary era).

- If the QFT is strongly coupled in the IR then this would correspond to perturbative gravity in the early Universe.
  - Such theories correspond to conventional inflationary models.
  - Checking the QFT predictions (to be discussed shortly) against standard cosmological perturbation theory would provide a test of holography.
- If the QFT is weakly coupled in the IR then this would correspond to a non-geometric phase in the early Universe.
  - These are qualitative new modes for the very early Universe.
  - Checking the QFT predictions against observations one can either obtain support or falsify these models.

Take-home message

- If one can simulate 3d QFTs which in IR have either a fixed point or become super-renormalizable then one has interesting holographic models for the very early universe.
- Depending on the nature of the IR theory (strongly or weakly coupled) one can either test holography or obtain predictions that can be checked against observations.

How do we make predictions?

- We need to provide formulae that relate cosmological observables with QFT correlation functions.
- The Cosmic Microwave Background (CMB) carries information about the very early Universe.
- Two of the main observables, currently measured by satellites (such as Planck) and other missions, are the power spectra and non-Gaussianities and there are explicit holographic formulae for them.
- Here I will focus on the scalar power spectrum.

Holographic formula for the scalar power spectrum

➢ The scalar power spectrum is given by

$$\Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{4\pi^2} \frac{1}{\text{Im}\,\langle T(q)T(-q)\rangle},$$

where $T = T^i_i$ is the trace of the energy momentum tensor T_{ij} and we Fourier transformed to momentum space. The imaginary part is taken after the analytic continuation,

$$q \to -iq, \qquad N \to -iN$$

➠ It suffices to compute the 2-point function of the energy-momentum tensor in the IR in order to compare with observations.

Confronting with data

- In cosmology there are very few observables so the way we check the theory against data is different than in high energy physics.
- The main question one addresses is:

> Given a set of models, which one is preferred by the data?

- One way to answer this is to check how well the model fits the data: what is the probability for obtaining the data given the model.
- A better way is to compute the so-called Bayesian Evidence: what is the probability for the model given the data.

Protocol

1. Choose a model with desired IR behaviour.
2. Compute 2-point function of the energy momentum tensor.
3. Insert in holographic formula to obtain the holographic prediction.
4. Compute Bayesian Evidence to check whether the model is ruled in or out.

A simple model

We are currently putting on lattice the following model:

- A non-minimally coupled massless scalar field in the adjoint of $SU(N)$ with ϕ^4 self-interaction

$$S = \int d^3x \mathrm{Tr} \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{2}\phi^4 \right),$$

and energy momentum tensor

$$T_{ij} = \mathrm{Tr} \left(\partial_i \phi \partial_j \phi - \frac{1}{2}\delta_{ij}((\partial\phi)^2 + \lambda\phi^4) + \xi(\delta_{ij}\Box - \partial_i\partial_j)\phi^2 \right)$$

- The perturbative answer to 2-loops is known [Coriano, Delle Rose, KS (to appear)]
- The fit of the perturbative model to data is in progress [Flauger, KS (to appear)]

Another interesting model

- ➢ $SU(N)$ gauge theory coupled to N_ϕ conformally coupled massless scalars (without self-interaction).

- Perturbative answer for pure glue to 2-loops is known [Coriano, Delle Rose, KS (to appear)].
- We would need about 300 scalars in order to satisfy the constrain that gravitational waves have not been observed so far.
- In this model confinement of YM is linked with the resolution of the Big Bang singularity.

Outline

Conclusions

- If one can simulate 3d QFTs which in IR have either a fixed point or become super-renormalizable then one has interesting holographic models for the very early universe.
- Depending on the nature of the IR theory (strong or weakly coupled) one can either test holography or obtain predictions that can be checked against observations.
- It suffices to compute the 2-point function of the energy-momentum tensor in the IR in order to compare with observations.
- There are interesting new models of the early universe where input from lattice gauge theory would make a difference.