Nucleon and Nuclear Operators in a Model-Independent Treatment of WIMP Direct Detection

- ***Galilean invariant effective theory formulation***
- ***Nuclear embedding***
- ***Implications for experiment: number and type***

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Lattice for BSM Physics

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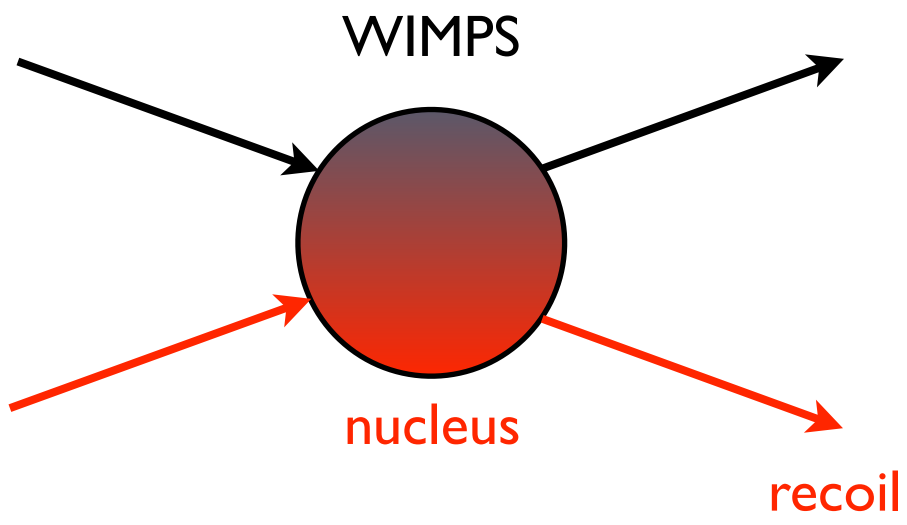
U.S. DEPARTMENT OF ENERGY

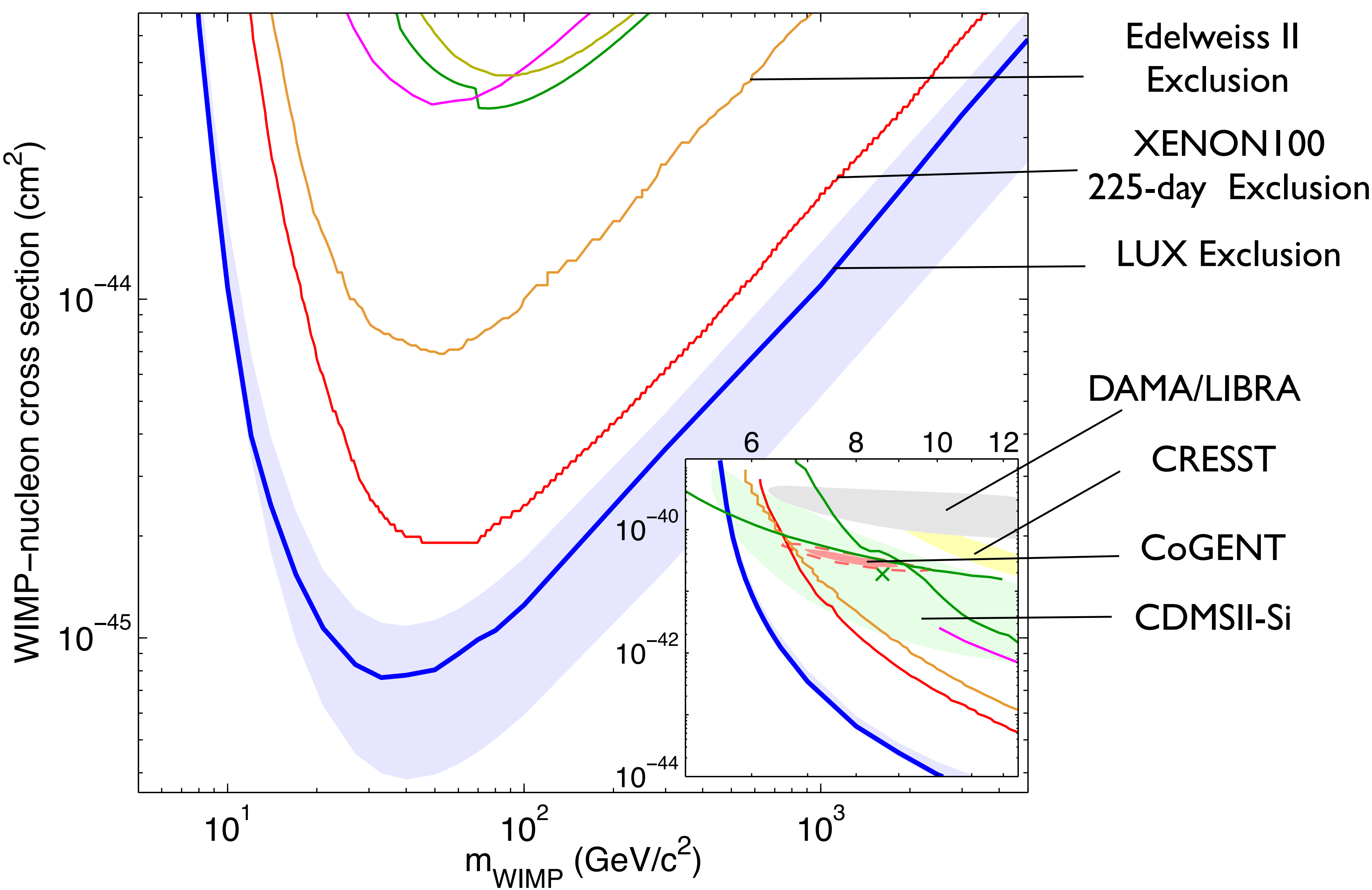
BERKELEY LAB

Direct detection: highly exclusive low-energy experiment, testing HE theory, involving a complex nuclear target

- what in principle can be measured?
- how can one formula the problem so that three communities (HE theory, HE experiment, NP) can communicate in an efficient way?
- how can one minimize uncertainties?



isoscalar charge interaction: but what if nature made other choices?



LUX (Xe): arXiv:1310.8214

How are these comparisons among experiments done?

We know some basic parameters

- WIMP velocity relative to our rest frame $\sim 10^{-3}$
- if mass is on the weak scale, WIMP momentum transfers in elastic scattering can range to $q_{max} \sim 2v_{\mathrm{WIMP}}\mu_T \sim$ 200 MeV/c
- WIMP kinetic energy ~ 30 keV: nuclear excitation (in most cases) not posible
- $R_{NUC} \sim 1.2\, A^{1/3}$ f $\Longrightarrow$ $q_{max}\, R \sim 3.2 \Leftrightarrow 6.0$ for F ⇔ Xe: the WIMP can "see" the structure of the nucleus

Astrophysics factors from particle/nuclear physics reasonably well.
Particle/nuclear factorization?

Astrophysics

$$\frac{dR}{dE_R} = N_N \frac{\rho_0}{m_W} \int_{v_{min}} d\mathbf{v} f(\mathbf{v}) v \frac{d\sigma}{dE_R}$$

Particle+nuclear physics

$$v_{min} = \sqrt{\frac{m_N E_{th}}{2\mu^2}}$$

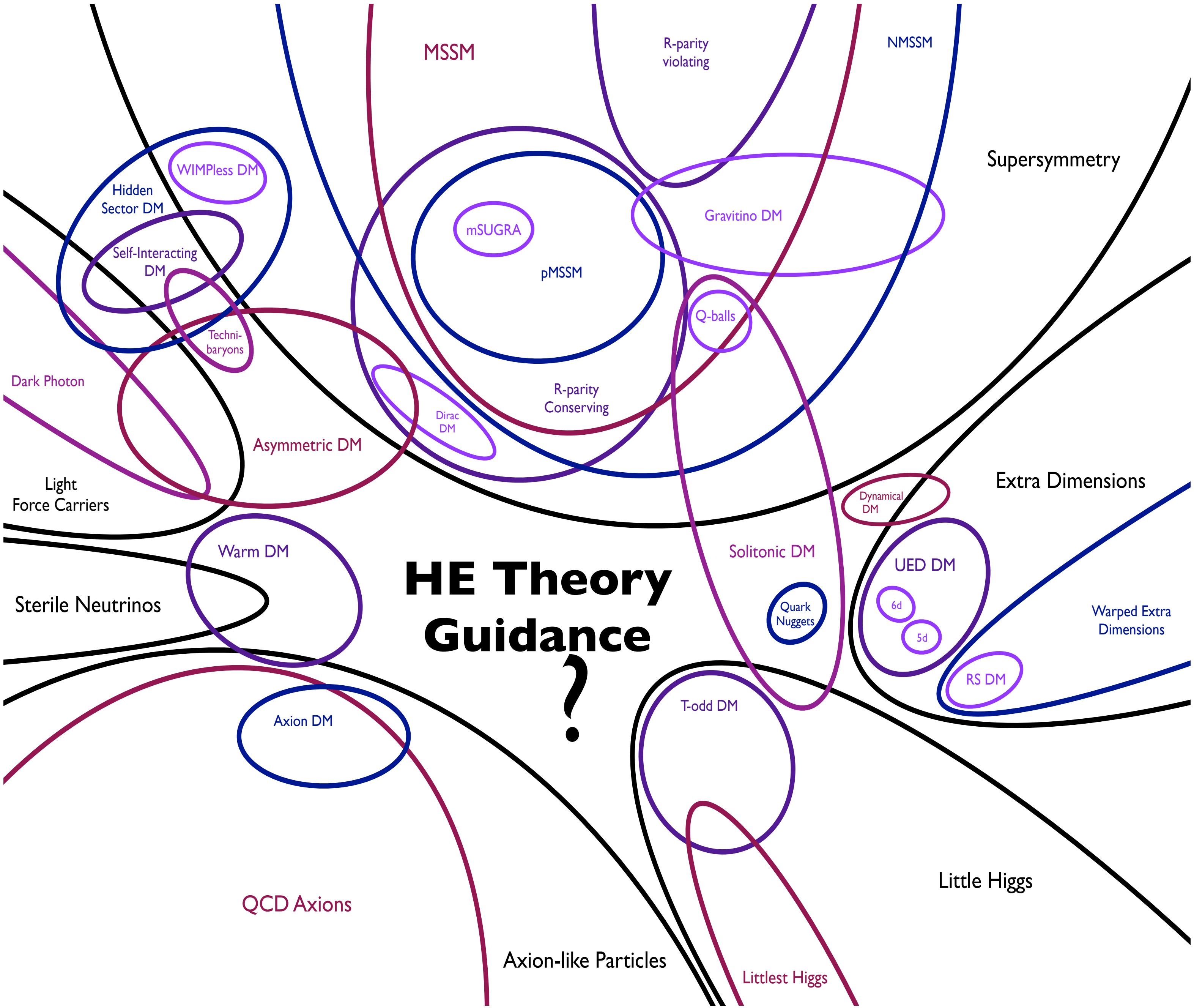
$N_N =$ number of target nuclei in detector

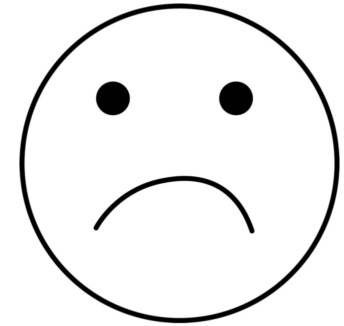
$\rho_0 =$ Milky Way dark matter density

$f(\mathrm{v}) =$ WIMP velocity distribution, Earth frame

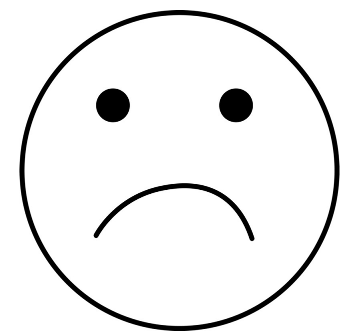
$m_W =$ WIMP mass

$\sigma =$ WIMP – nucleus elastic scattering cross section



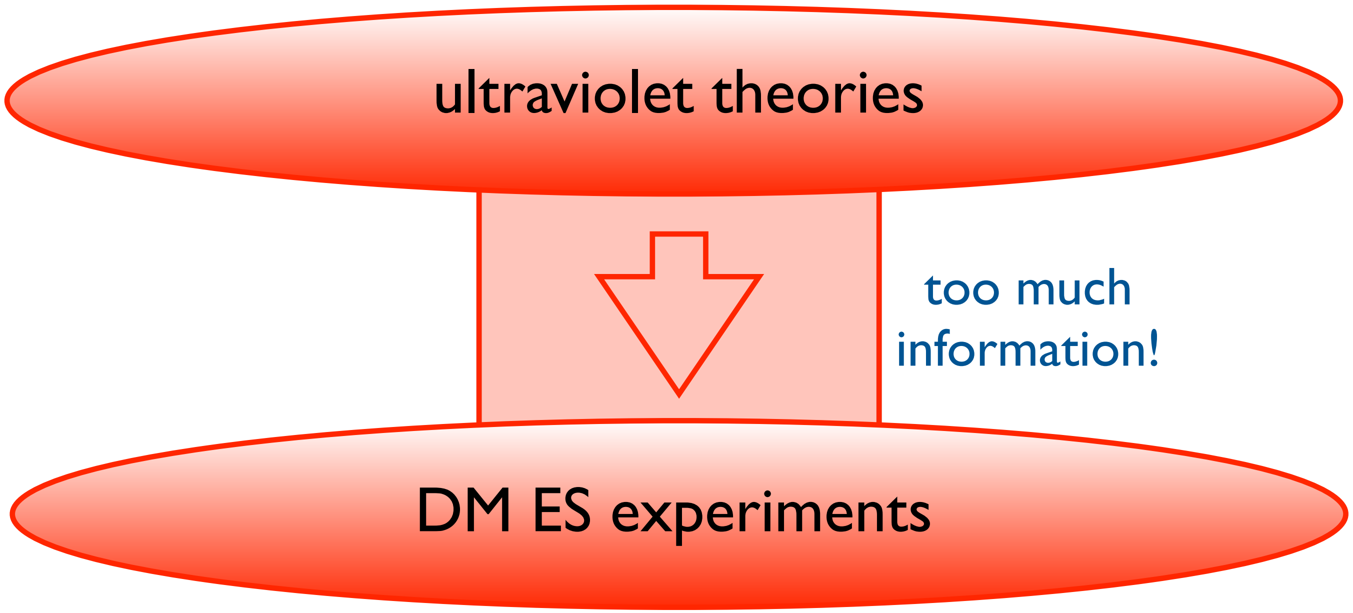


Nuclear theorist



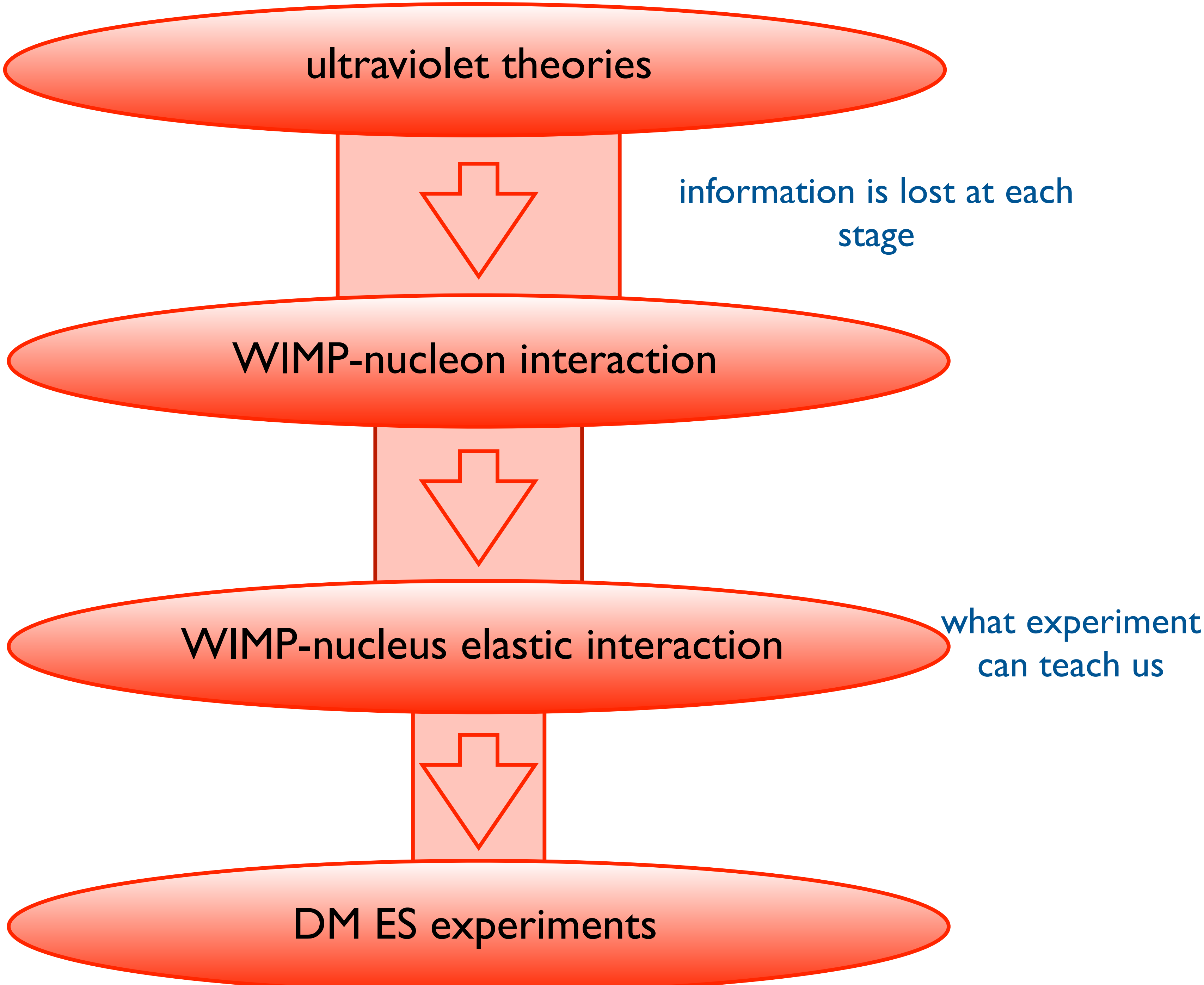
DM experimentalist

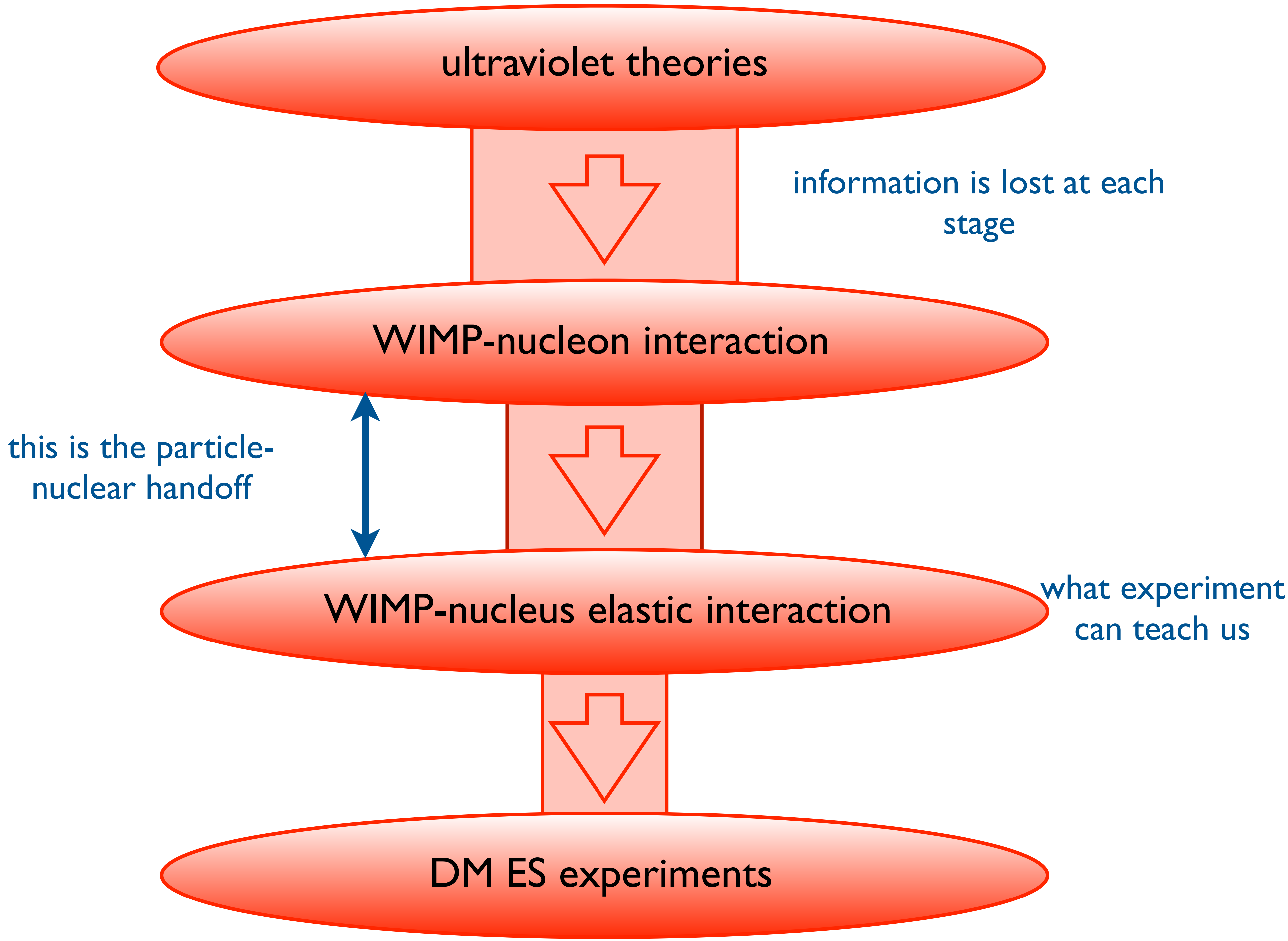
from Tim Tait

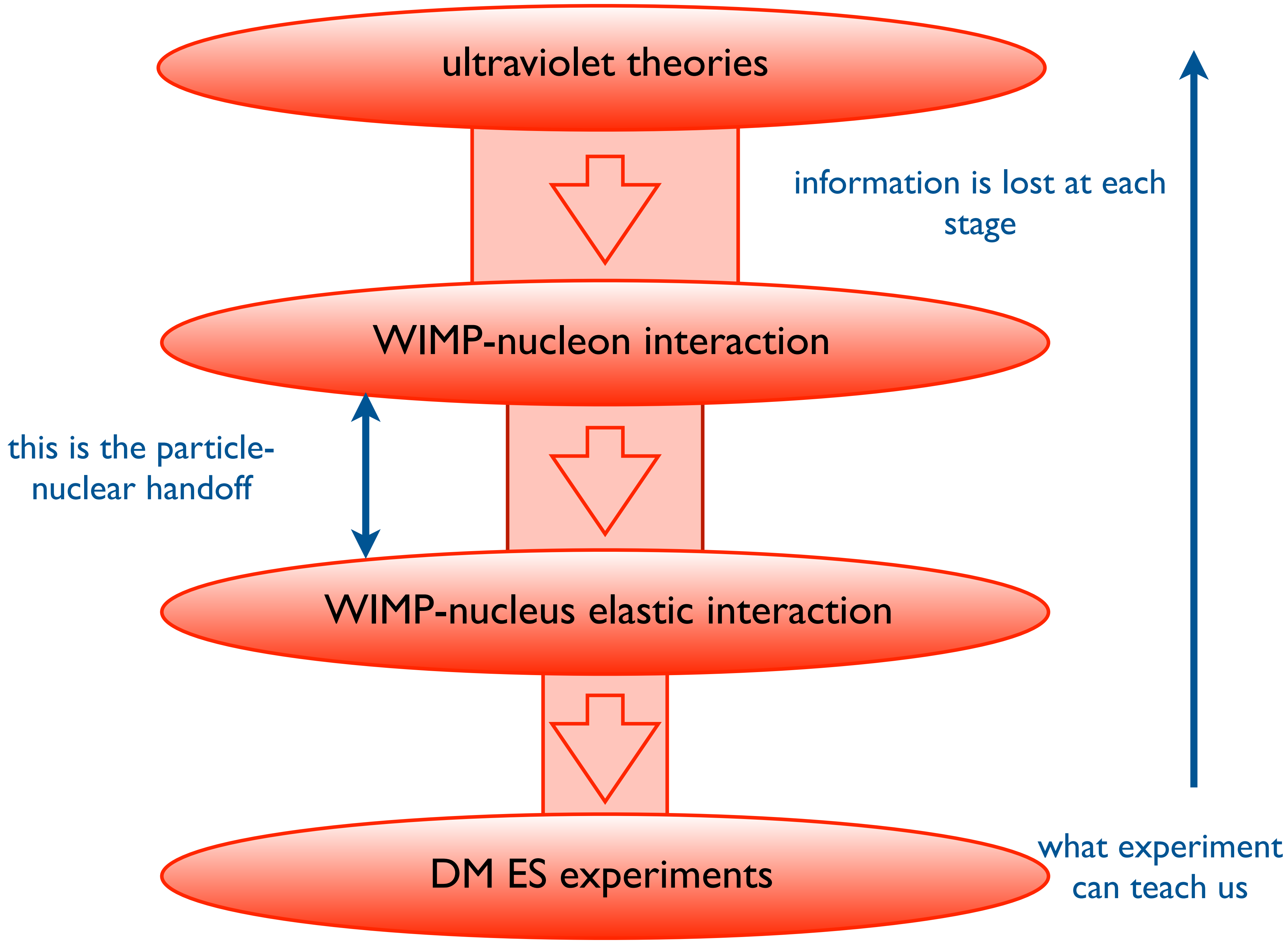


Really not practical in any comprehensive way,
tedious to repeat for multiple candidate ultraviolet theories

Better to represent the physics - the filtering process - in effective theory




ultraviolet theories
information is lost at each stage
WIMP-nucleon interaction
this is the particle-nuclear handoff
WIMP-nucleus elastic interaction
what experiment can teach us
DM ES experiments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DM ES experiments
what experiment can teach us

- Experiments are frequently analyzed and compared in a formalism in which the nucleus is treated as a point particle

$$\text{S.I.} \quad \Rightarrow \quad \langle g.s.| \sum_{i=1}^{A} (a_0^F + a_1^F \tau_3(i)) \; |g.s.\rangle$$

$$\text{S.D.} \quad \Rightarrow \quad \langle g.s.| \sum_{i=1}^{A} \vec{\sigma}(i) \; (a_0^{GT} + a_1^{GT} \tau_3(i)) \; |g.s.\rangle$$

This treatment is simpler than that we must use to describe standard-model electroweak nuclear reactions

As in standard electroweak nuclear interactions:
what responses can be generated from an linear couplings to charge, spin, velocity (covariance)?

charges:

	even	odd
vector	C_0	C_1
axial	C_0^5	C_1^5

currents:

	even	odd	even	odd	even	odd
axial spin	L_0^5	L_1^5	$T_2^{5\mathrm{el}}$	$T_1^{5\mathrm{el}}$	$T_2^{5\mathrm{mag}}$	$T_1^{5\mathrm{mag}}$
vector velocity	L_0	L_1	T_2^{el}	T_1^{el}	T_2^{mag}	T_1^{mag}
vector spin – velocity	L_0	L_1	T_2^{el}	T_1^{el}	T_2^{mag}	T_1^{mag}

(where we list only the leading multipoles in J above)

Answer: those allowed by symmetry

Response constrained by good parity of nuclear g.s.

	even	odd
vector	C_0	
axial		C_1^5

	even	odd	even	odd	even	odd
axial spin		L_1^5		$T_1^{5\mathrm{el}}$	$T_2^{5\mathrm{mag}}$	
vector velocity	L_0		T_2^{el}			T_1^{mag}
vector spin − velocity	L_0		T_2^{el}			T_1^{mag}

Response constrained by good parity and time reversal of nuclear g.s.

	even	odd
vector	C_0	
axial		

	even	odd	even	odd	even	odd
axial spin		L_1^5		$T_1^{5\mathrm{el}}$		
vector velocity						T_1^{mag}
vector spin $-$ velocity	L_0		T_2^{el}			

The resulting table of allowed responses has six entries (not two)

The union rules state:
Interactions allow by symmetries must be included in a proper effective theory

- This suggests more can be learned about ultraviolet theories from ES than is generally assumed - that's good
- But what quantum mechanics are we missing? What are these additional responses?

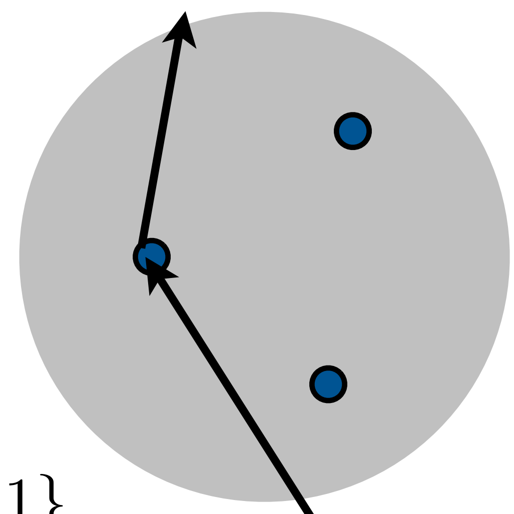
They are the responses connected with velocity-dependent interactions - theories that have derivative couplings - previously assumed to be small

Let's take an example: consider $\sum_{i=1}^{A} \vec{S}_\chi \cdot \vec{v}^{\perp}(i)$

the velocity is defined by Galilean invariance $\vec{v}^{\perp}(i) = \vec{v}_\chi - \vec{v}_N(i)$

- In the point-nucleus limit $\vec{S}_\chi \cdot \vec{v}_{\mathrm{WIMP}} \sum_{i=1}^{A} 1(i)$

where $\vec{v}_{\mathrm{WIMP}} \sim 10^{-3}$.



- But in reality

$\{\vec{v}^{\perp}(i), i = 1, \ldots A\} \rightarrow \{\vec{v}_{\mathrm{WIMP}}; \quad \vec{\dot{v}}(i), i = 1, \ldots, A-1\}$

and $\vec{\dot{v}}(i) \sim 10^{-1}$: SI/SD retains the least important term

Parameter counting in the effective theory

- These velocities hide: the $\vec{\dot{v}}(i)$ carry odd parity and cannot contribute by themselves to elastic nuclear matrix elements.

- But in elastic scattering, momentum transfers are significant. The full velocity operator is

$$e^{i\vec{q}\cdot\vec{r}(i)}\vec{\dot{v}}(i) \quad \text{where} \quad \vec{q}\cdot\vec{r}(i) \sim 1$$

- We can combine the two vector nuclear operators $\vec{r}(i)$, $\vec{\dot{v}}$ to form a scalar, vector, and tensor. To first order in $\vec{q}$ for the new "SD" case

$$-\frac{1}{i}q\vec{r}\times\vec{\dot{v}} = -\frac{1}{i}\frac{q}{m_N}\vec{r}\times\vec{\dot{p}} = -\frac{q}{m_N}\vec{\ell}(i)$$

$\vec{\ell}(i)$ is a new dimensionless operator. And we deduce an instruction for the ET that is not obvious. Internal nucleon velocities are encoded

$$\dot{v} \sim 10^{-1} \sim \frac{q}{m_N}$$

Galilean invariant effective theory

- The most general Hermitian WIMP-nucleon interaction can be constructed from the four variables

$$\vec{S}_\chi \quad \vec{S}_N \quad \vec{v}^\perp \quad \frac{q}{m_N}$$

- This interaction (filter #1) constructed to 2nd order in velocities

$$\begin{aligned}
H_{ET} = &\left[a_1 + a_2\ \vec{v}^\perp \cdot \vec{v}^\perp + a_5\ i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right)\right] + \vec{S}_N \cdot \left[a_3\ i\frac{\vec{q}}{m_N} \times \vec{v}^\perp + a_4\ \vec{S}_\chi + a_6\ \frac{\vec{q}}{m_N}\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right] \\
+ &\left[a_8\ \vec{S}_\chi \cdot \vec{v}^\perp\right] + \vec{S}_N \cdot \left[a_7\ \vec{v}^\perp + a_9\ i\frac{\vec{q}}{m_N} \times \vec{S}_\chi\right] \qquad \text{(parity odd)} \\
+ &\left[a_{11}\ i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right] + \vec{S}_N \cdot \left[a_{10}\ i\frac{\vec{q}}{m_N} + a_{12}\ \vec{v}^\perp \times \vec{S}_\chi\right] \qquad \text{(time and parity odd)} \\
+ &\ \vec{S}_N \cdot \left[a_{13}\ i\frac{\vec{q}}{m_N}\vec{S}_\chi \cdot \vec{v}^\perp + a_{14}\ i\vec{v}^\perp\ \vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right] \qquad \text{(time odd)}
\end{aligned}$$

The coefficients represent the information that survives at low energy from a semi-infinite set of high-energy theories

- We can then embed this in the nucleus (filter #2) to find what information survives, accessible to elastic experiments.

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2})\; W_i(q^2 b^2)$$

hard-working experimentalists try to measure

- We can then embed this in the nucleus (filter #2) to find what information survives, accessible to experiment.

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2})\, W_i(q^2 b^2)$$

WIMP tensor:
contains all of the DM particle physics

depends on two "velocities"

$$\vec{v}^{\perp\,2} \sim 10^{-6} \qquad \frac{\vec{q}}{m_N}^{2} \sim \langle v_{\rm internucleon}\rangle^2 \sim 10^{-2}$$

- We can then embed this in the nucleus (filter #2) to find what information survives, accessible to experiment.

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2})\, W_i(q^2 b^2)$$

Nuclear tensor:
"nuclear knob" that can be turned by the experimentalists to deconstruct dark matter

Game - vary the W_i to determine the R_i:
change the nuclear charge, spin, isospin, and any other relevant nuclear properties that can help

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2})\, W_i(q^2 b^2)$$

$$W_1 \sim \langle J| \sum_{i=1}^{A} 1(i)\, |J\rangle^2$$

take q → 0
suppress isospin

the S.I. response

contributes for J=0 nuclear targets

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2})\, W_i(q^2 b^2)$$

$$W_2 \sim \langle J| \sum_{i=1}^{A} \hat{q}\cdot\vec{\sigma}(i)\ |J\rangle^2$$

take q → 0
suppress isospin

$$W_3 \sim \langle J| \sum_{i=1}^{A} \hat{q}\times\vec{\sigma}(i)\ |J\rangle^2$$

the S.D. response (J>0) ....

but split into two components, as the longitudinal and transverse responses are independent, coupled to different particle physics

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2}) \, W_i(q^2 b^2)$$

$$W_4 \sim \langle J| \sum_{i=1}^{A} \vec{\ell}(i) \, |J\rangle^2$$

take q → 0
suppress isospin

A second type of vector (requires J>0) response, with selection rules very different from the spin response

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2})\, W_i(q^2 b^2)$$

$$W_5 \sim \langle J|\ \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \vec{\ell}(i)\ |J\rangle^2$$

take q → 0
suppress isospin

A second type of scalar response, with coherence properties very different from the simple charge operator

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2})\; W_i(q^2 b^2)$$

take q → 0
suppress isospin

$$W_6 \sim \langle J|\ \sum_{i=1}^{A} \left[\vec{r}(i) \otimes \left(\vec{\sigma}(i) \times \frac{1}{i} \vec{\nabla}(i) \right)_1 \right]_2\ |J\rangle^2$$

A exotic tensor response: in principle interactions can be constructed where no elastic scattering occurs unless J is at least 1

The point-nucleus world is a very simple one

Generally any derivative coupling is seen most easily in the new responses

$$R_M^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) = c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{\vec{q}^{\,2}}{m_N^2}\vec{v}_T^{\perp 2} c_5^\tau c_5^{\tau'} + \vec{v}_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right]$$

$$R_{\Phi''}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) = \frac{\vec{q}^{\,2}}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^{\tau'}\right)$$

$$R_{\Phi'' M}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) = c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}$$

$$R_{\tilde{\Phi}'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) = \frac{j_\chi(j_\chi+1)}{12}\left[c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right]$$

$$R_{\Sigma''}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) = \frac{\vec{q}^{\,2}}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{\vec{q}^{\,4}}{m_N^4} c_6^\tau c_6^{\tau'} + \vec{v}_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2}\vec{v}_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right]$$

$$R_{\Sigma'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) = \frac{1}{8}\left[\frac{\vec{q}^{\,2}}{m_N^2}\vec{v}_T^{\perp 2} c_3^\tau c_3^{\tau'} + \vec{v}_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{\vec{v}_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^{\tau'}\right) + \frac{\vec{q}^{\,2}}{2m_N^2}\vec{v}_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right]$$

$$R_{\Delta}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) = \frac{j_\chi(j_\chi+1)}{3}\left[\frac{\vec{q}^{\,2}}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right]$$

$$R_{\Delta\Sigma'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) = \frac{j_\chi(j_\chi+1)}{3}\left[c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right].$$

Observations:

- Six of the possible operators generate SI or SD interactions
  Six of the possible operates are seen only through new responses
  Two of the operators cannot be seen in elastic scattering

- ES can in principle give us 8 constraints on DM interactions

- This argues for a variety of detectors - or at least, continued development of a variety of detector technologies

Interactions that (effectively) cannot be seen in elastic scattering

$$\frac{P^\mu}{m_\mathrm{M}}\bar\chi\chi\bar N\gamma_\mu\gamma^5 N \quad \Big| \quad -4\frac{m_\chi}{m_\mathrm{M}}\vec v^\perp\cdot\vec S_N \quad \text{axial charge} \quad \Big| \quad -4\frac{m_\chi}{m_\mathrm{M}}\mathcal{O}_7$$

Interactions that effectively only contribute to the new responses

$$\bar\chi\gamma^\mu\gamma^5\chi\frac{K_\mu}{m_\mathrm{M}}\bar N N \quad \Big| \quad 4\frac{m_N}{m_\mathrm{M}}\vec v^\perp\cdot\vec S_\chi \quad \text{orbital angular momentum} \quad \Big| \quad 4\frac{m_N}{m_\mathrm{M}}\mathcal{O}_8$$

New and standard interactions coming together, but with the latter suppressed (the reverse also happens)

vector charge

$$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_\mathrm{M}}\chi\frac{K_\mu}{m_\mathrm{M}}\bar N N \quad \Big| \quad \frac{m_N}{m_\chi}\frac{\vec q^{\,2}}{m_\mathrm{M}^2}1_\chi 1_N + 4i\frac{m_N}{m_\mathrm{M}}\vec v^\perp\cdot\left(\frac{\vec q}{m_\mathrm{M}}\times\vec S_\chi\right) \quad \Big| \quad \frac{m_N}{m_\chi}\frac{\vec q^{\,2}}{m_\mathrm{M}^2}\mathcal{O}_1 - 4\frac{m_N^2}{m_\mathrm{M}^2}\mathcal{O}_5$$

orbital angular momentum

$$\text{but } \frac{\text{vector charge}}{\text{orbital angular momentum}} \sim \frac{q^2}{m_N^2}\frac{m_N}{m_\chi}Z^2 \sim 10^{-4}Z^2$$

The expanded set of responses means that comparisons between experiments in a simplified analysis may be misleading

For illustration purposes only!

DAMA/LIBRA: NaI

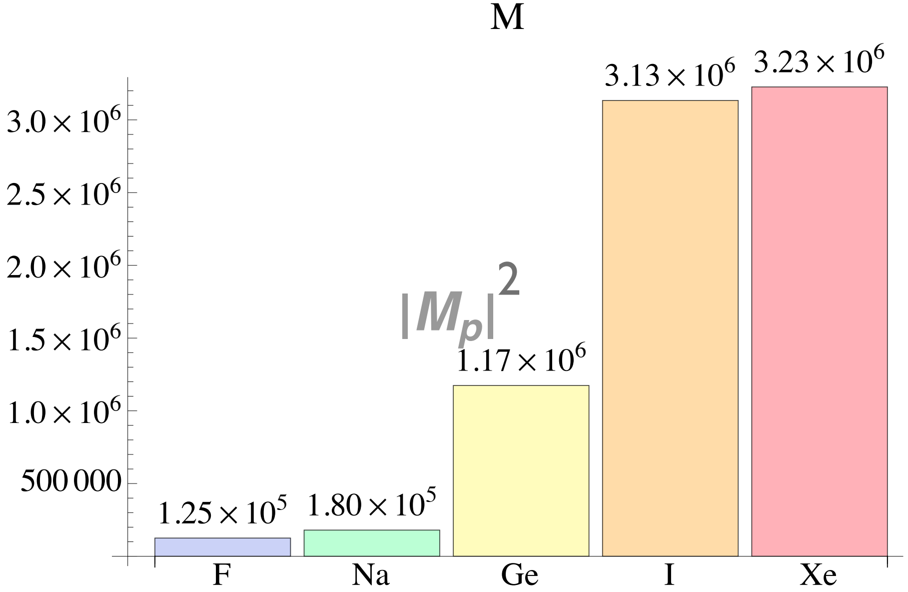
CoGENT: Ge

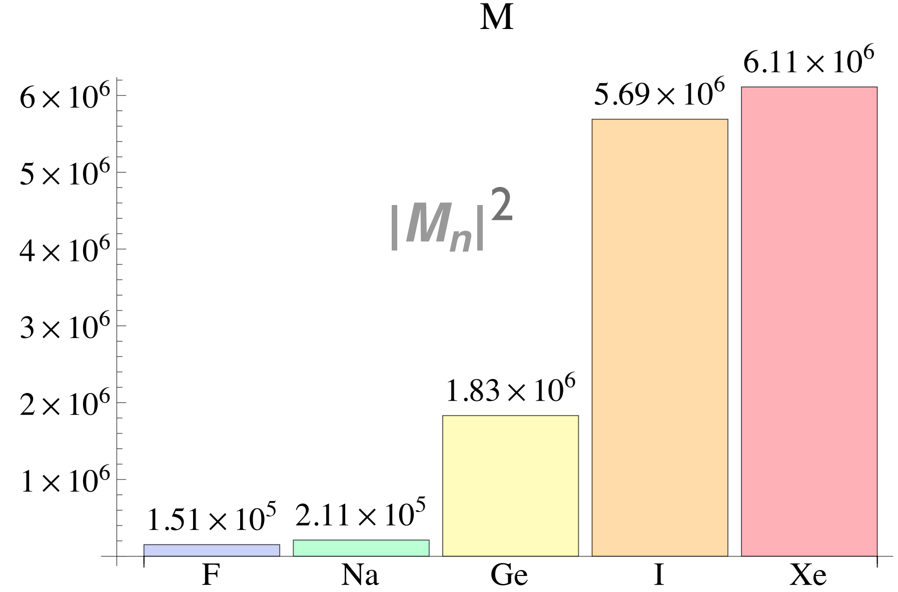
LUX: Xe

systematic?

scalar charge responses: p vs. n S.I.

(normalized to natural abundance)

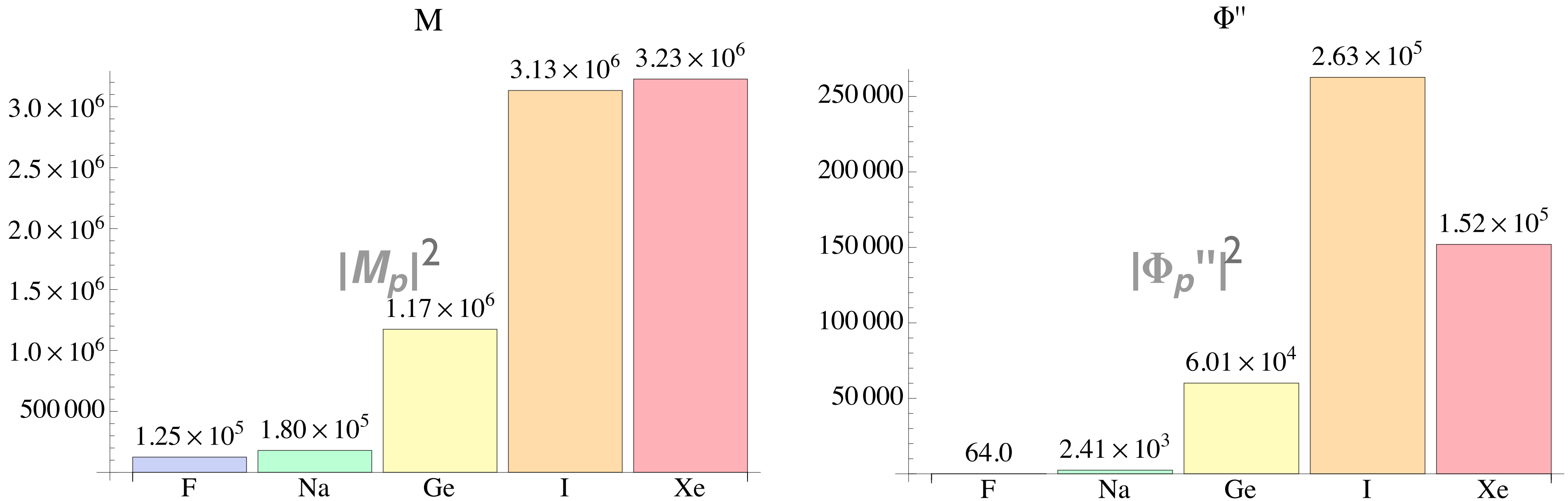




Standard SI sensitivities: LUX (Xe ~ DAMA (NaI) > CDMS-Ge

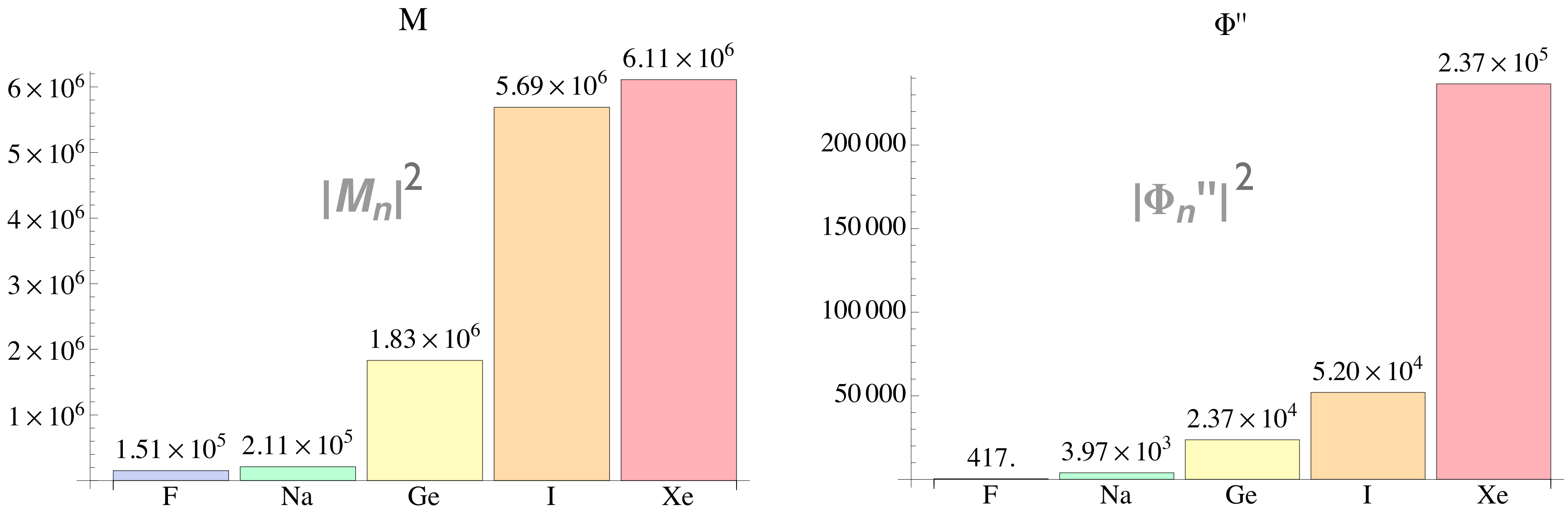
Little sensitivity to isospin (unless tuned)

Scalar operators, p: $1(i)$ vs $\vec{\sigma}(i) \cdot \vec{\ell}(i)$



LUX (Xe) ~ DAMA (NaI) ⇒ DAMA > LUX

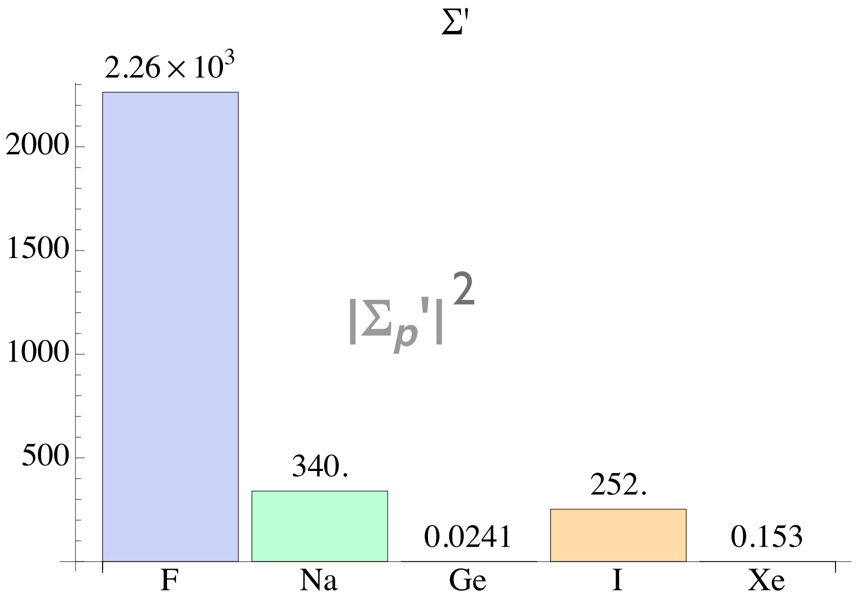
Scalar operators, n: $1(i)$ vs $\vec{\sigma}(i) \cdot \vec{\ell}(i)$

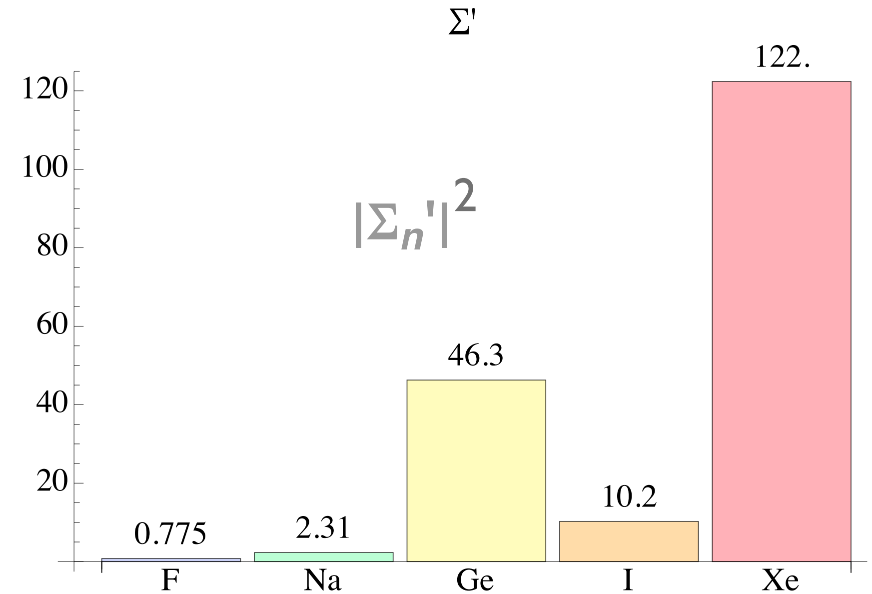


LUX (Xe) ~ DAMA (NaI) ⇒ DAMA < LUX

vector (transverse) spin response

(normalized to natural abundance)



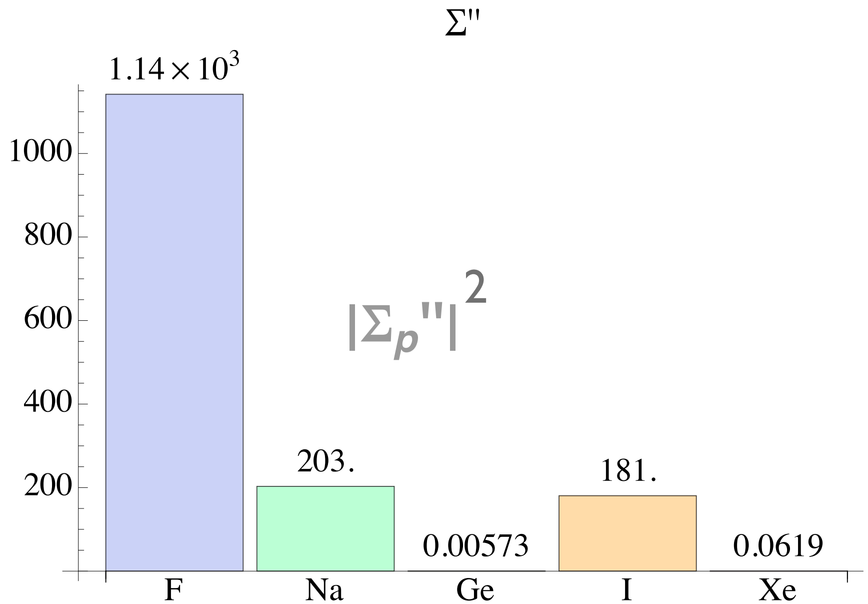


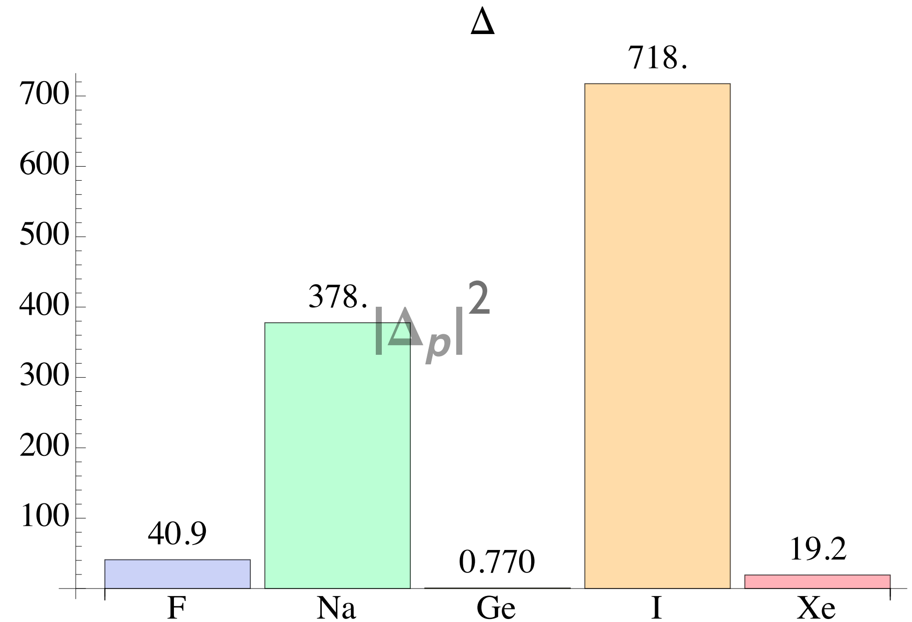
proton coupled: Picasso (F) > DAMA (NaI) ≫ CDMS-Ge & LUX

neutron coupled: LUX & CDMS-Ge ≫ DAMA ≫ Picasso

isospin

Vector, proton coupled: $\vec{\sigma}(i)$ vs. $\vec{\ell}(i)$



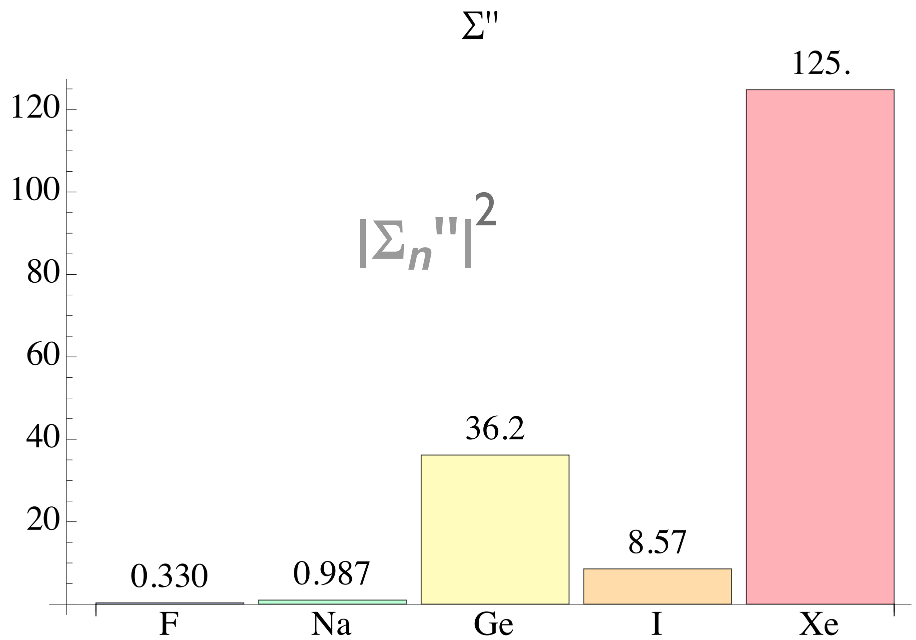


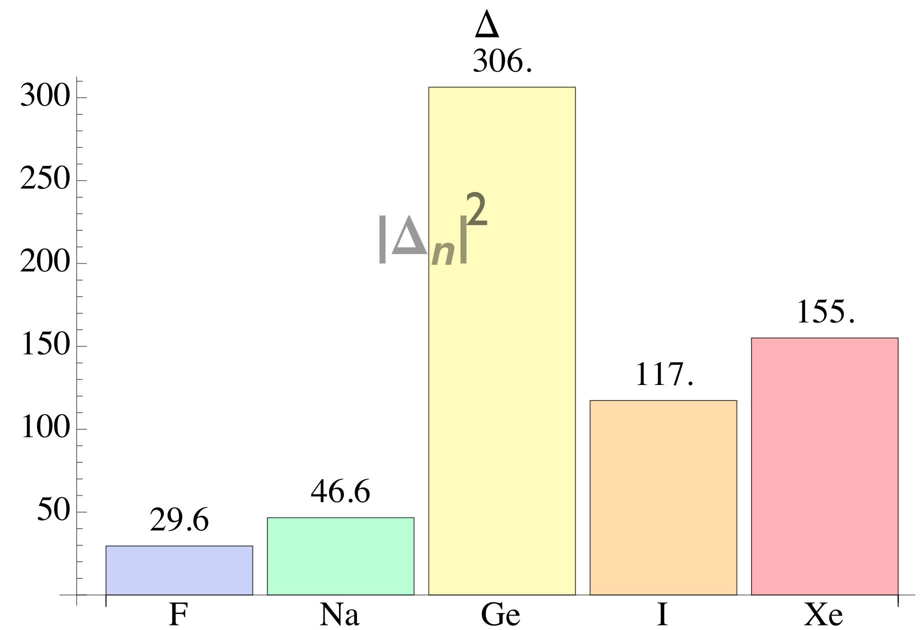
spin coupled: Picasso (F) > DAMA (NaI)

l-coupled coupled: DAMA (NaI) ≫ Picasso (F) F: $2s_{1/2}$

orbital vs. spin ambiguity

Vector, neutron coupled: $\vec{\sigma}(i)$ vs. $\vec{\ell}(i)$





spin coupled: LUX > CDMS-Ge ≫ DAMA

l-coupled coupled: CDMS-Ge > LUX ~ DAMA

orbital vs. spin ambiguity

Summary

- There is a lot of variability that can be introduced between detector responses by altering operators (and their isospins)

- Pairwise exclusion of experiments in general difficult

- But the bottom line is a favorable one: there is a lot more that can be learned from elastic scattering experiments than is apparent in conventional analysis

- This suggests we should do more experiments, not fewer

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