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Lattice QCD Input to Axion Cosmology

Evan Berkowitz

Lattice for Beyond the Standard Model Physics Workshop
Lawrence Livermore National Laboratory

Thursday, April 23rd 2015

arXiv:1504.XXXX, E. Berkowitz, M. Buchoff, E. Rinaldi

LLNL-PRES-669910
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Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344

Outline

- Introduction
- Whence axions?
- What is the over-closure bound?
- Inputs to the over-closure bound from lattice QCD
- Outlook

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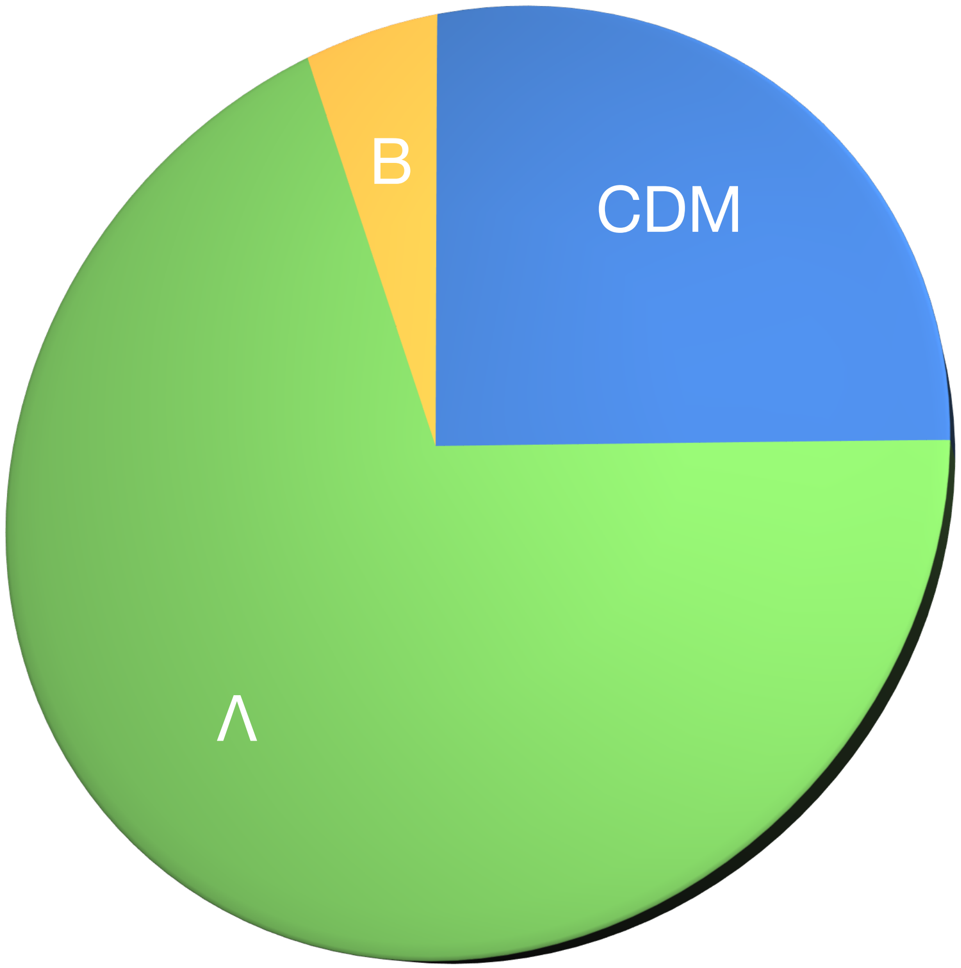
- Whence axions?
- What is the over-closure bound?
- Inputs to the over-closure bound from lattice QCD
  pure glue (for now)
- Outlook

Big Idea

- Axions were originally proposed to deal with the Strong CP Problem, also form a plausible DM candidate.
  - Calculating the axion energy density requires nonperturbative QCD input.
- Being sought in ADMX (LLNL, UW) & CAST (CERN) with large discovery potential in the next few years.
- Requiring $\Omega_a \leq \Omega_{CDM}$ yields a lower bound on the axion mass today.
  Preskill, Wise & Wilczek, Phys Lett B **120** (1983) 127-132

The Economist, 19 Dec 2006



Ω_{tot} = 1.000(7)
PDG 2014 via
P.A.R. Ade, et al., (Planck Collab. 2013 XVI), arXiv: 1303.5076v1.

QCD Theta Term

- QCD has a parameter, θ.
    - Controls QCD CP violation.

$$\mathcal{L}_{\rm QCD} \ni \theta \, \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

CP Violating

QCD Theta Term

- QCD has a parameter, θ.
  - Controls QCD CP violation.
  - Topological.
- θ can take any value in (-π,π].
- Neutron EDM ≲ 3 10^{-26}e•cm

Baker et al., PRL 97, 131801 (2006) / hep-ex/0602020

  - ⟹ |θ| ≲ 10^{-10}

$$\mathcal{L}_{\mathrm{QCD}} \ni \theta \; \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

$$Q = \frac{1}{32\pi^2} \int d^4x \; \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \in \mathbb{Z}$$

$$e^{iS} \propto e^{iQ\theta}$$

**Strong CP Problem:
Why is θ so small?**

(Some) Resolutions of the Strong CP Problem

- Just declare CP to be good in the strong sector
    - Weak sector can reintroduce the problem

- $m_u = 0$

$$\bar{q}\left(i\not{D} - m e^{i\theta'\gamma_5}\right) q$$

  'tHooft PRL **37** 8 (1976)
  Jackiw & Rebbi, PRL **37** 127 (1976)
  Callan, Dashen & Gross PLB **63** 335 (1976)
  Kaplan & Manohar PRL **56** 2004 (1986)

    - $m_u \neq 0$

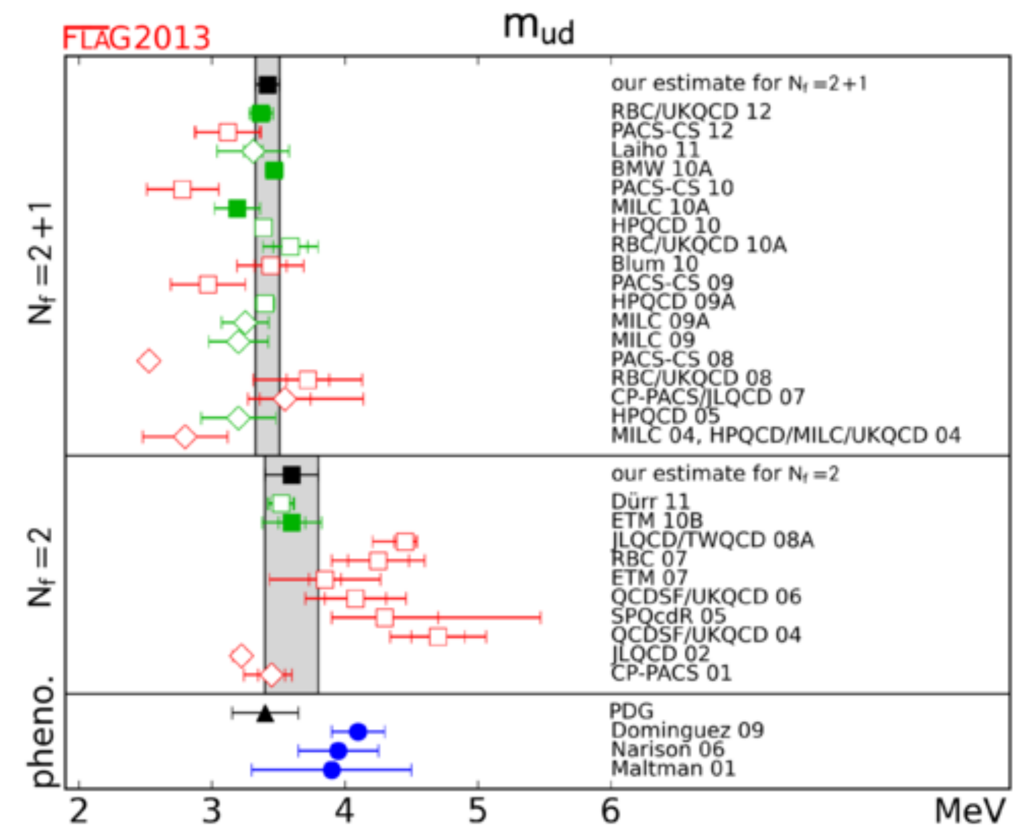
      Gasser & Leutwyler PhysRept **87** 77-169 (1982)

- Additional Peccei-Quinn symmetry & axions

  Peccei & Quinn: PRL **38** (1977) 1440, PR **D16** (1977) 1791

    - Fine tuning problem can be reintroduced via high-dimensional operators at Planck scale.

      Holman *et al.* arXiv:hep-ph/9203206
      Cheung arXiv:1003.0941



Colangelo *et al.* (FLAG) arXiv:1011.4408

Axions

Peccei & Quinn: PRL **38** (1977) 1440, PR **D16** (1977) 1791

- Couple to topological charge

$$\mathcal{L}_{\rm QCD} \ni \theta \, \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- Otherwise have shift symmetry.

- Amenable to effective theory treatment

- PQ symmetry can break before or after inflation.

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- Couple to topological charge

$$\mathcal{L}_{\text{axions}} = \frac{1}{2}\left(\partial_\mu a\right)^2 + \left(\frac{a}{f_a} + \theta\right)\ \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- Otherwise have shift symmetry.

$$a \to a + \alpha$$

- Amenable to effective theory treatment

$$V_{\text{eff}} \sim \cos\left(\theta + c\langle a\rangle\right)$$

$$m_a^2 f_a^2 = \left.\frac{\partial^2 F}{\partial\theta^2}\right|_{\theta=0}$$

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$$V_{\text{eff}} \sim \cos\left(\theta + c\langle a\rangle\right)$$

$$m_a^2 f_a^2 = \chi$$

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Axion mass

QCD Topological Susceptibility

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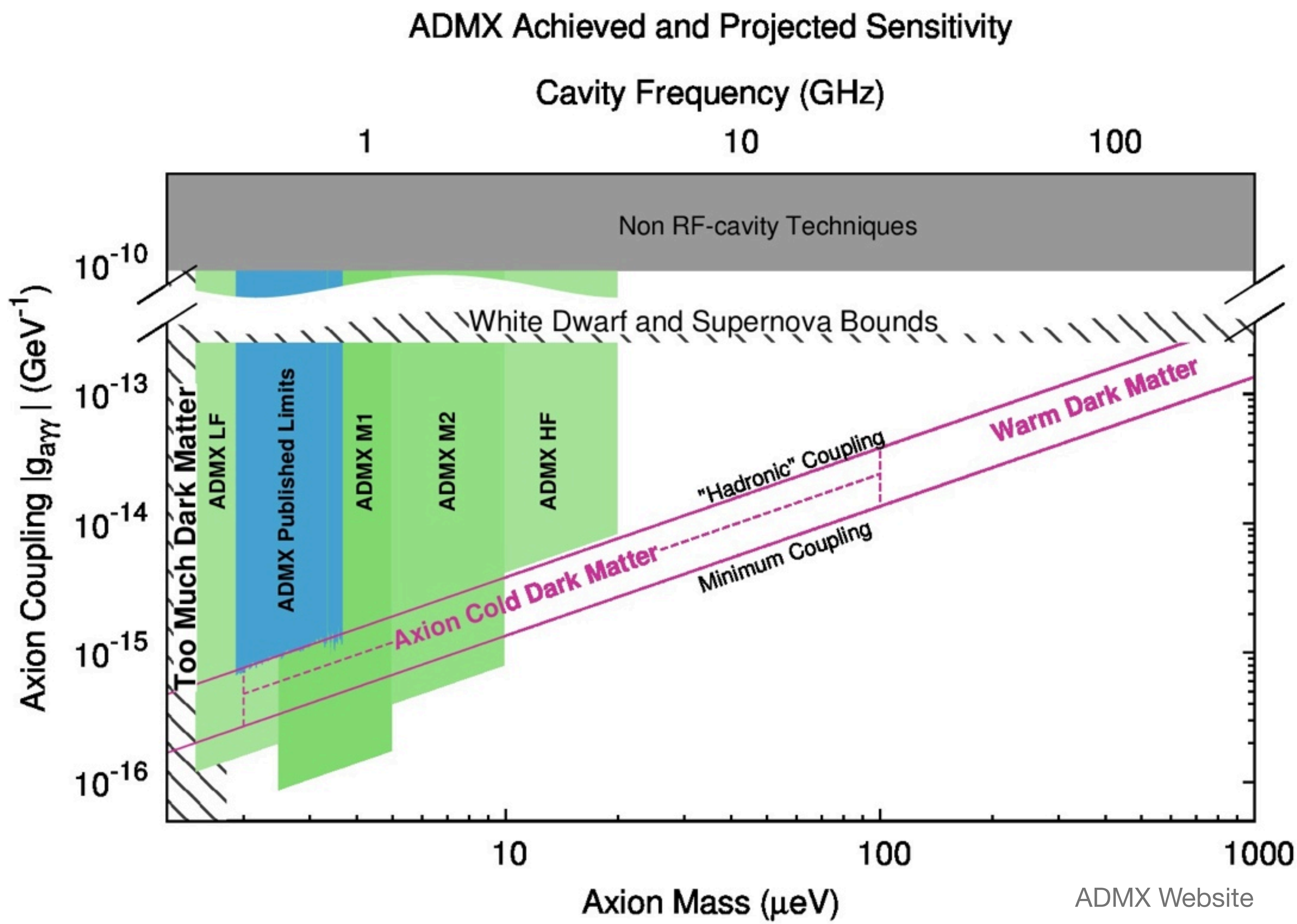
Axion mass

QCD Topological Susceptibility

- PQ symmetry can break before or after inflation.

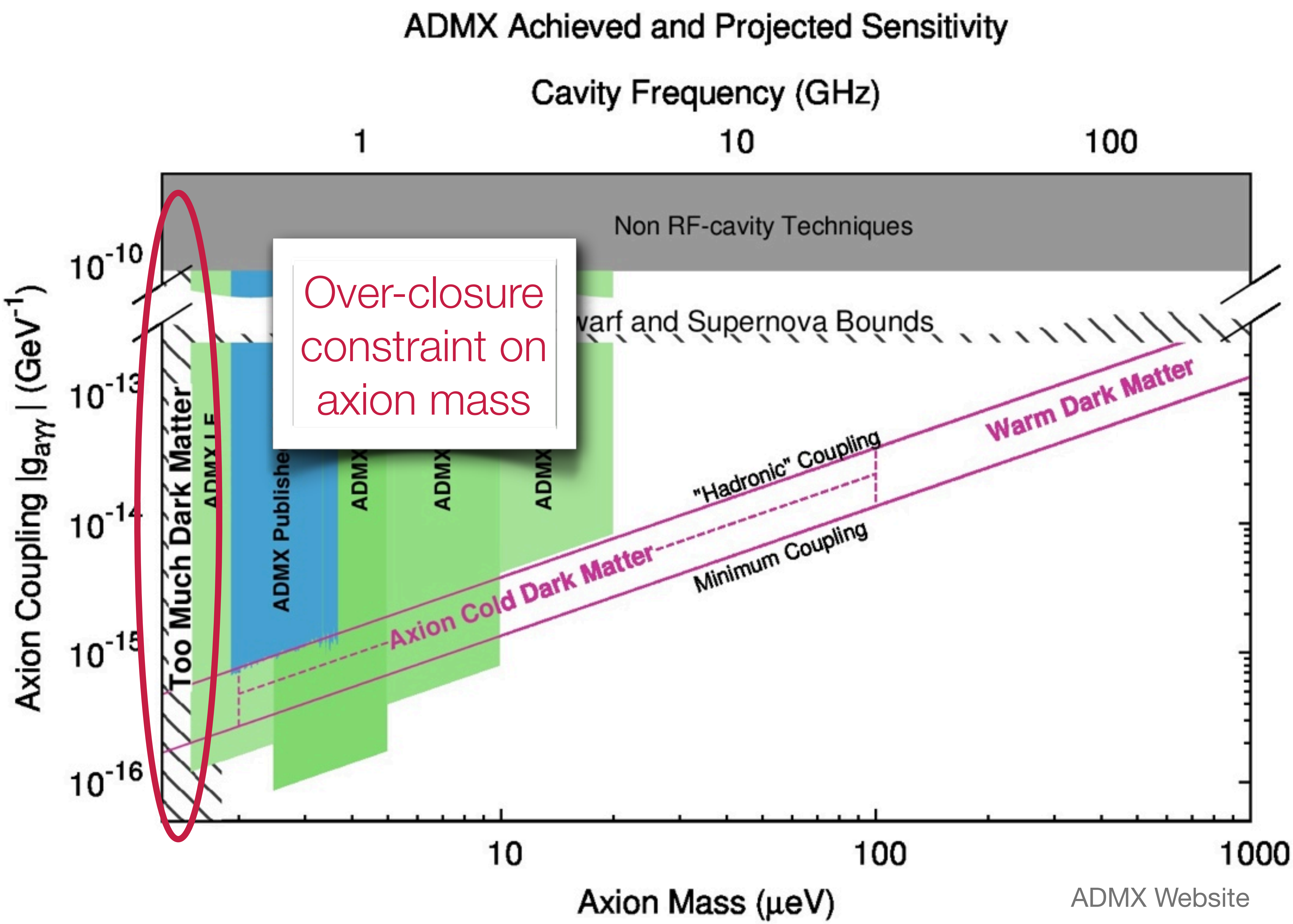
Average over initial θ

Current Axion Constraints



ADMX Website

Current Axion Constraints

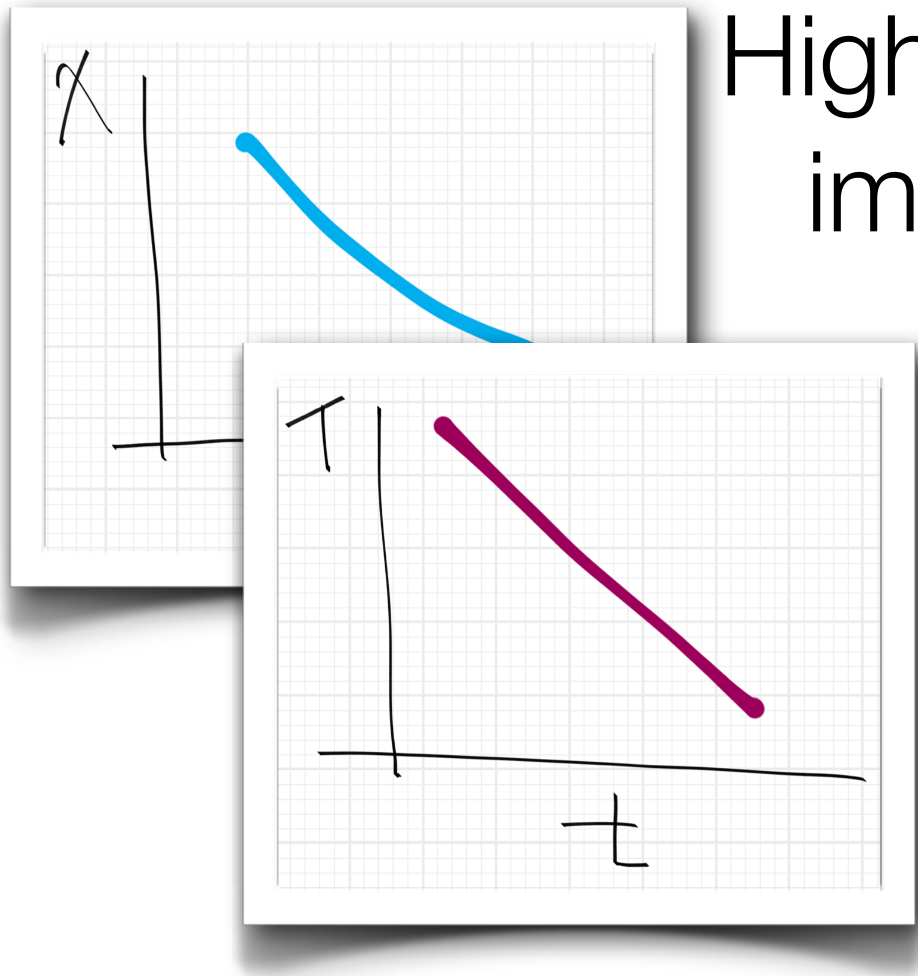


ADMX Website

The Over-Closure Bound

High temperature arguments imply χ vanishes as $T \to \infty$

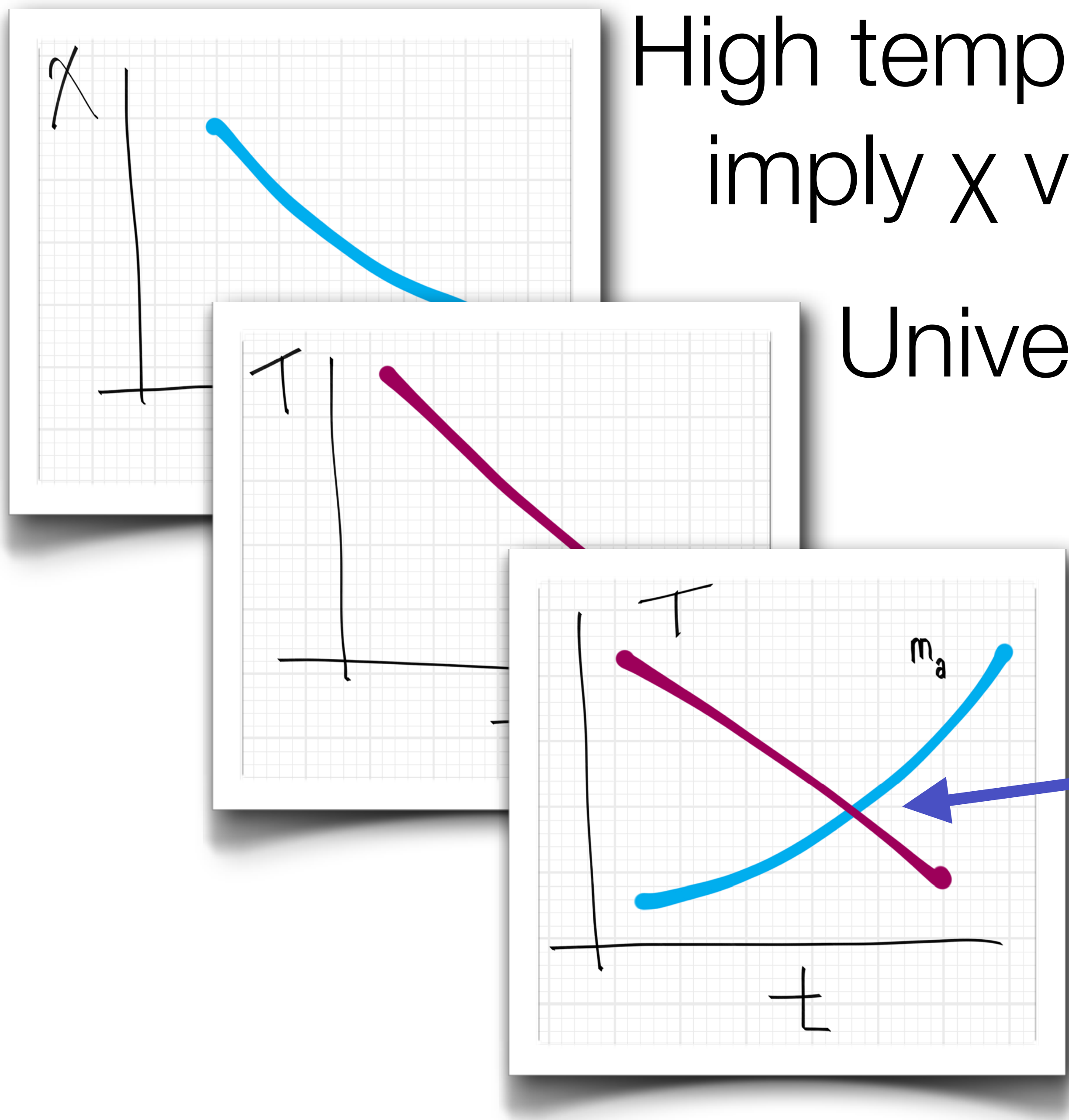
The Over-Closure Bound



High temperature arguments imply χ vanishes as T→∞

Universe cools as it expands

The Over-Closure Bound



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Universe cools as it expands

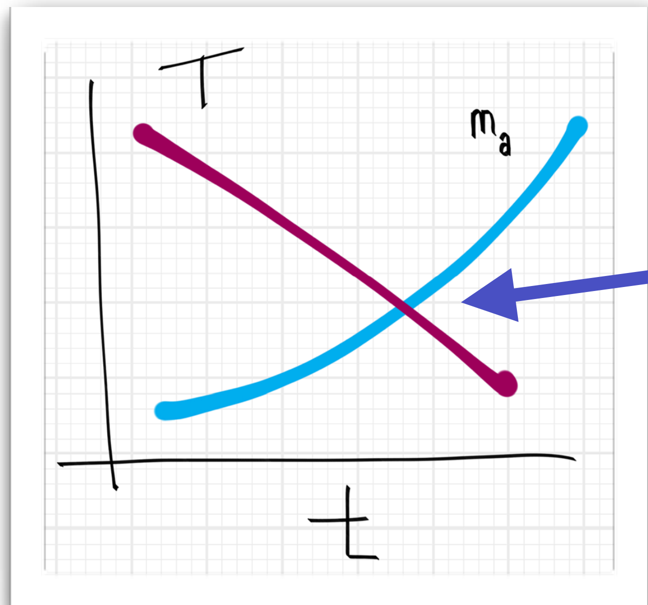
Axions freeze out when

$3H \sim m_a$

$T_1 \approx 5.5\ T_c$

H: Hubble constant

The Over-Closure Bound



Axions freeze out when

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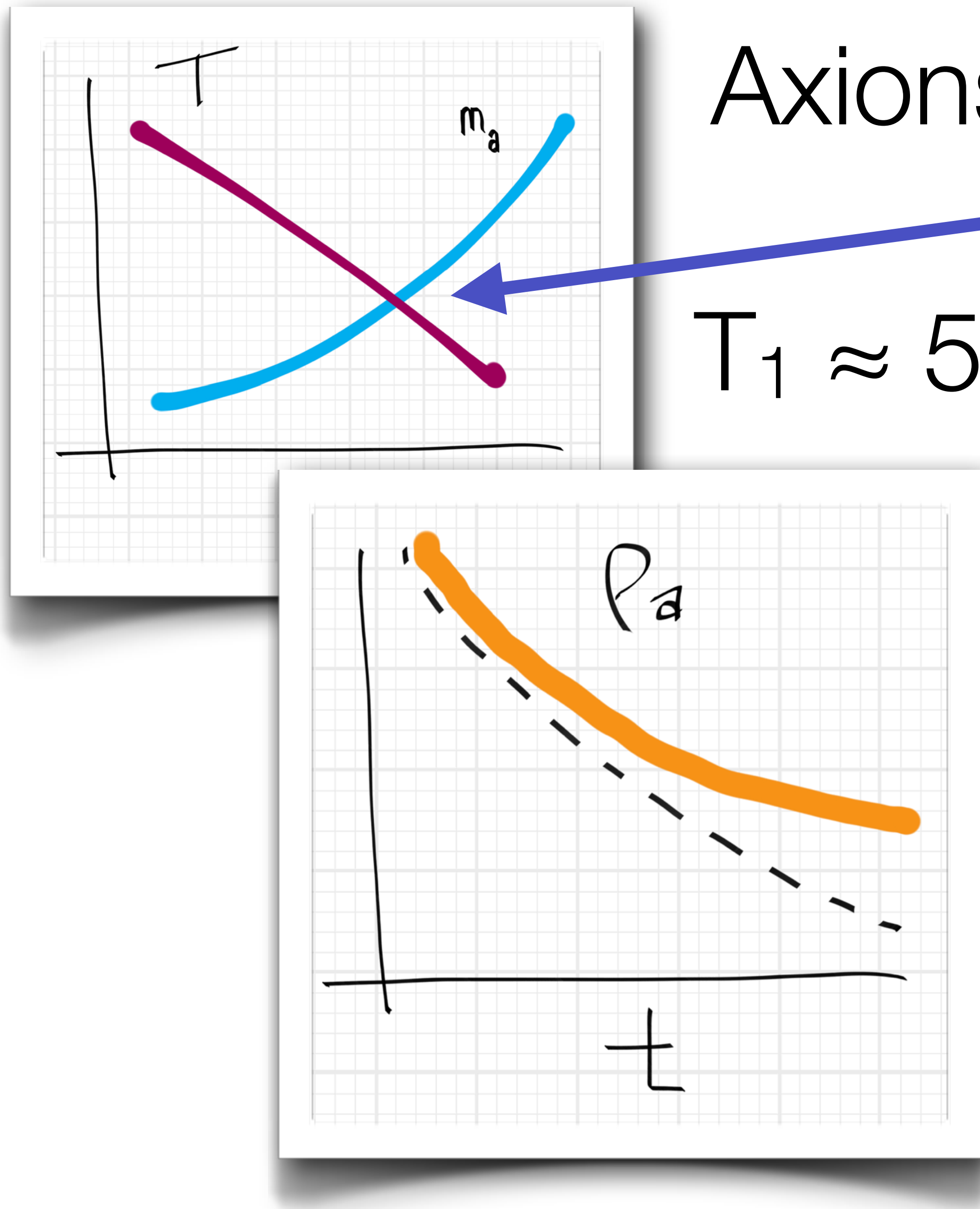
$T_1 \approx 5.5\ T_c$ from models

The Over-Closure Bound



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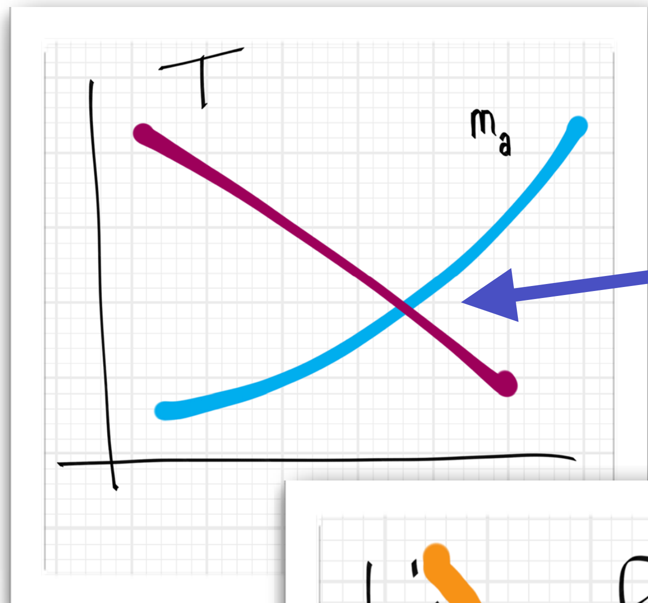
Axions continue to get heavier after they freeze out!

$$\rho(t) \neq \left(\frac{a(t_{\text{freezeout}})}{a(t)}\right)^3 \rho\left(t_{\text{freezeout}}\right)$$

The Over-Closure Bound



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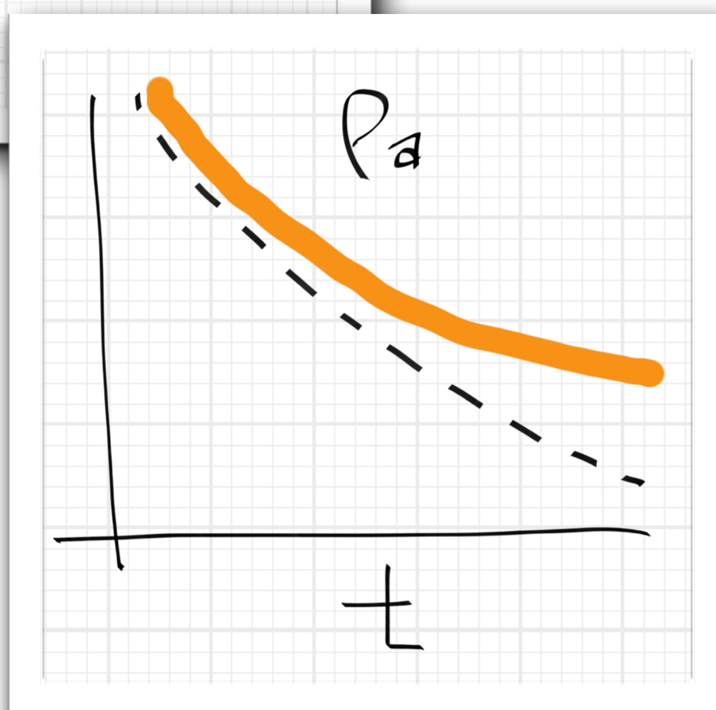

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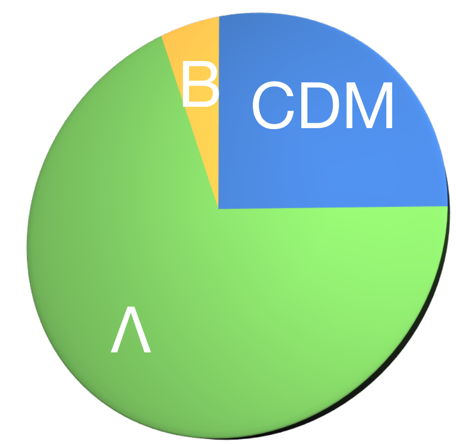
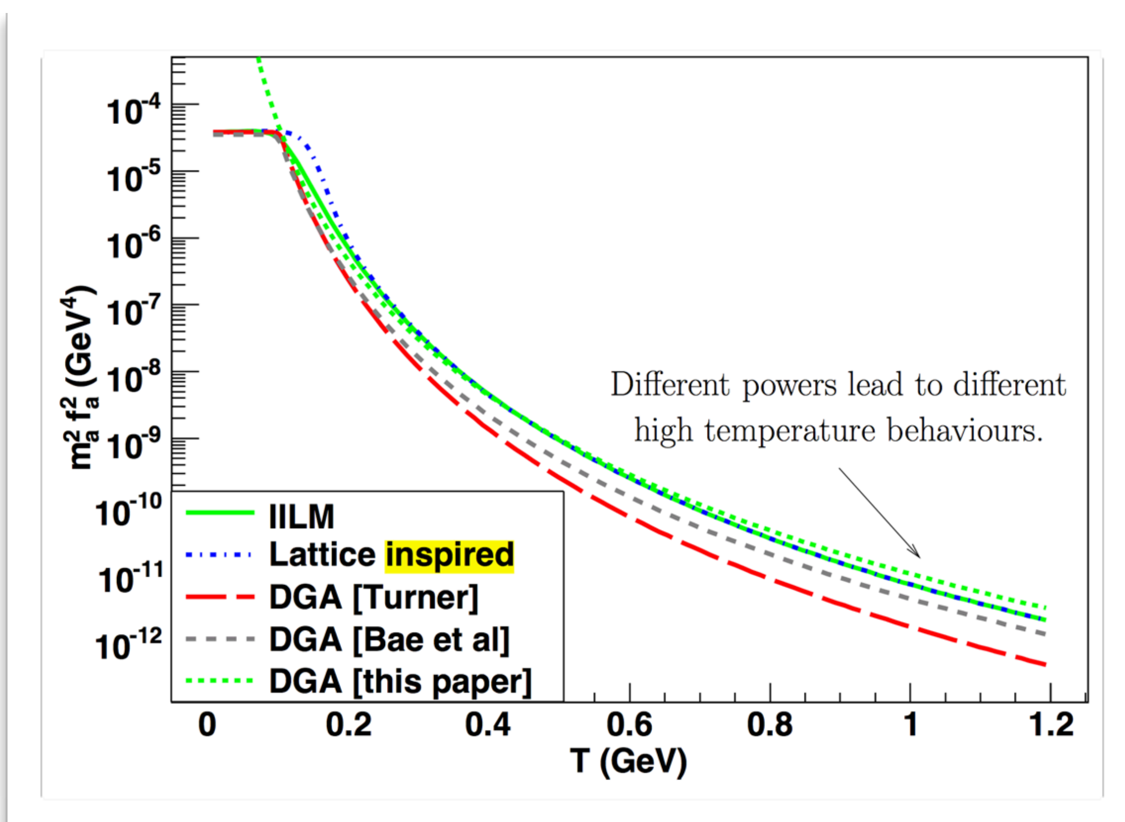
$$\frac{\rho(t)R^3}{m_a(t)} = \#\ \text{axions in a fixed comoving volume}$$

The Over-Closure Bound As It Stands Today

- Value of ρ at freeze-out given by FRW equations, EOS and $m_a(T)$
- χPT today yields $m_a = \frac{\sqrt{m_u m_d}}{m_u + m_d} \frac{f_\pi m_\pi}{f_a}$
- $m_a(T)$ is provided by models.

$$\frac{\rho}{\rho_c} < \Omega_{\mathrm{CDM}} = 0.12$$

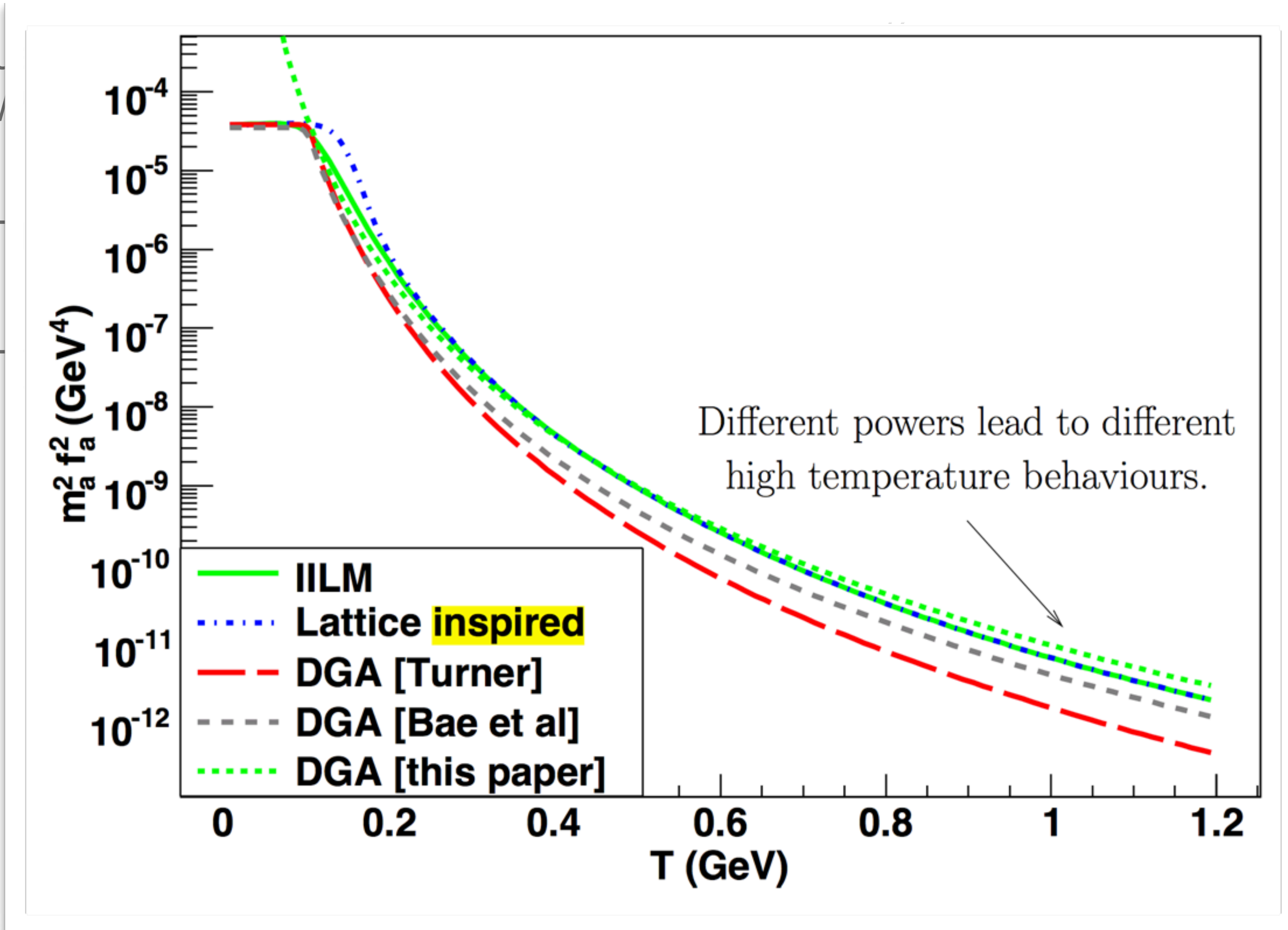




Wantz & Shellard, arXiv:0910.1066

The Over-Closure Bound As It Stands Today

- Valu
  FRV
- χPT
- m_a(



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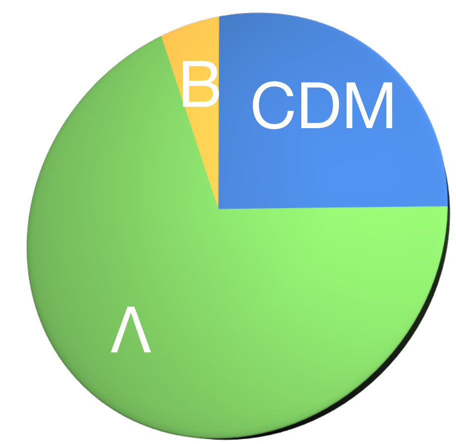

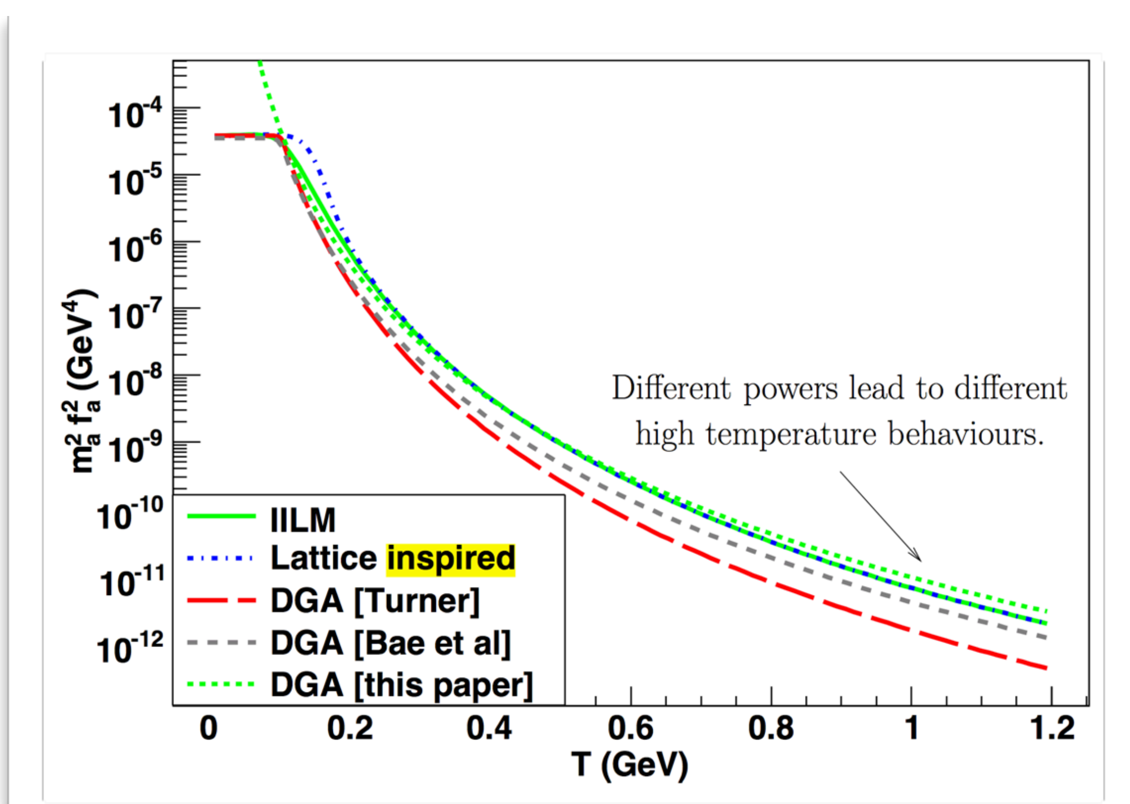

$f_a \leq (2.8 \pm 2)\ 10^{11}$ GeV

$m_a \geq 21 \pm 2$ μeV

(± uncontrolled systematic)

Wantz & Shellard, arXiv:0910.1066

The Over-Closure Bound As It Stands Today

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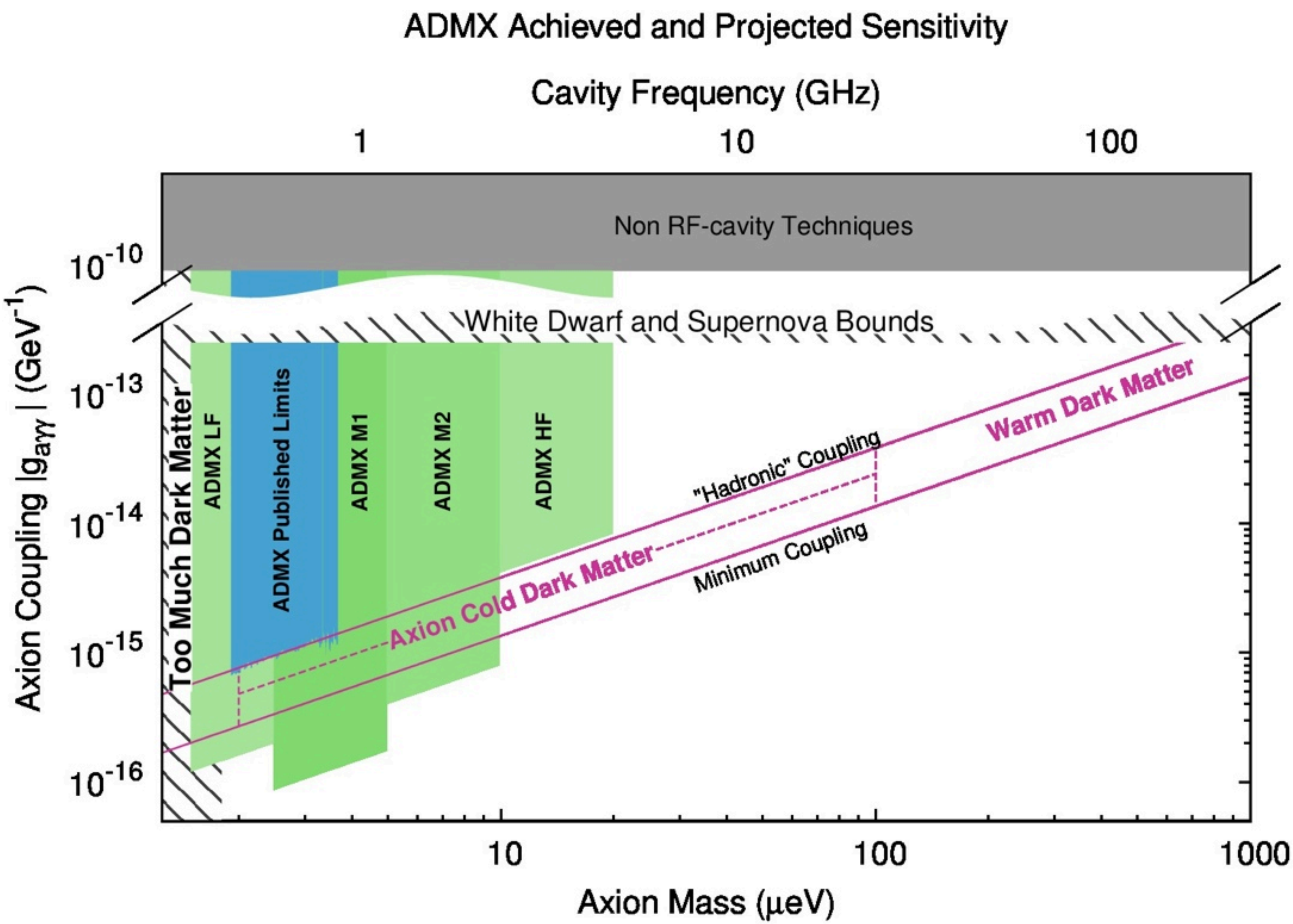

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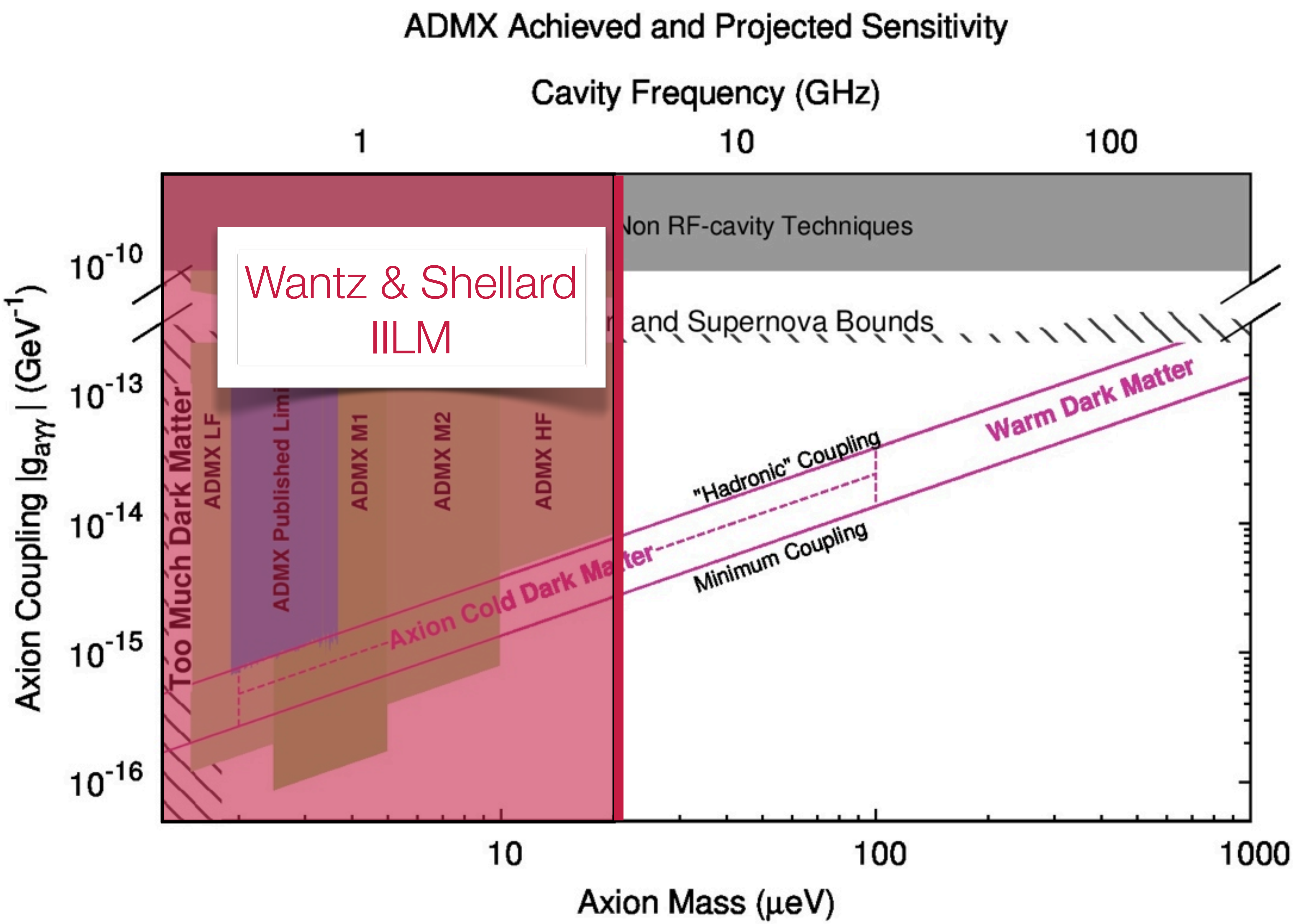

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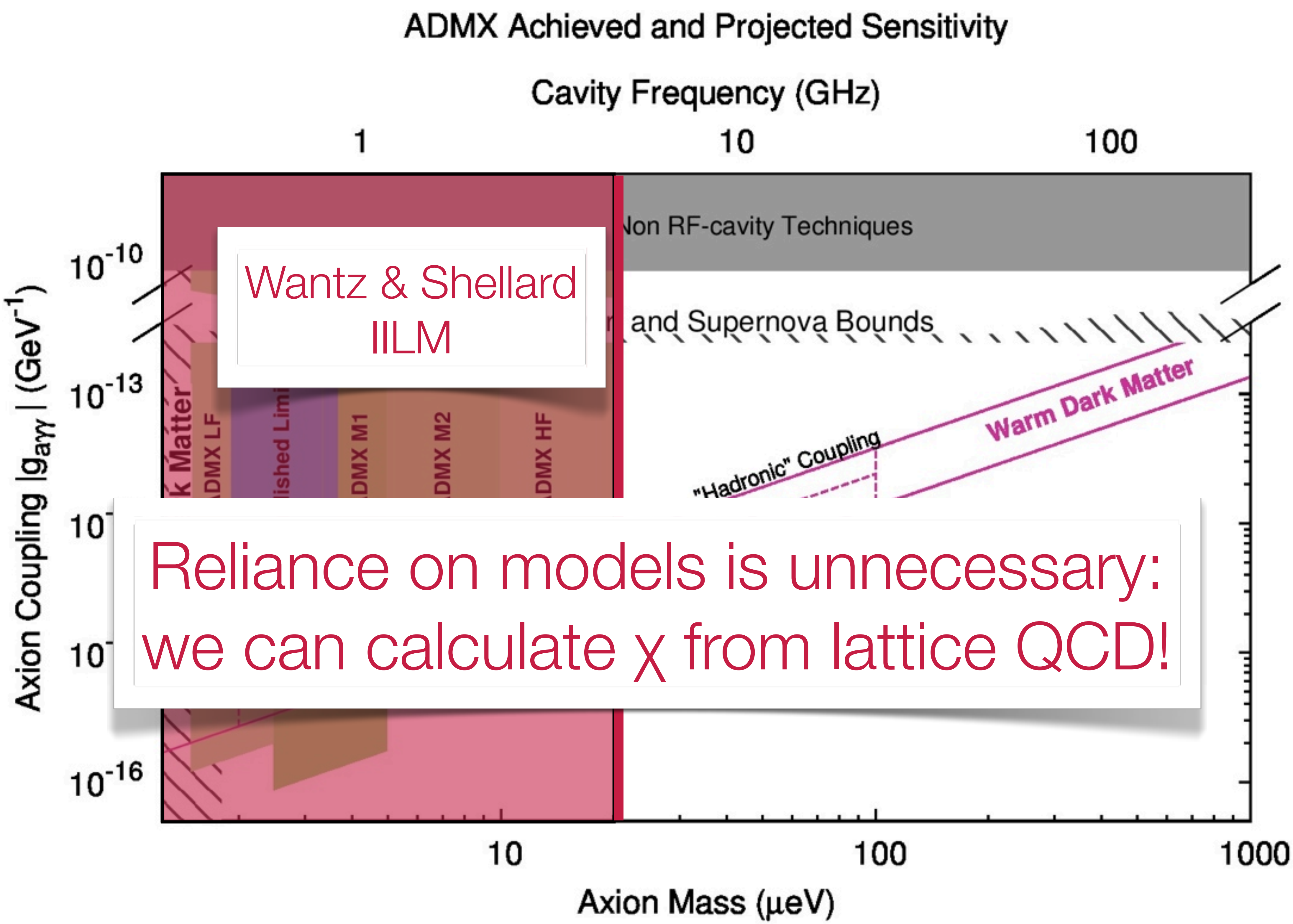

Wantz & Shellard
IILM

Reliance on models is unnecessary:
we can calculate χ from lattice QCD!

Overview of Lattice Ensembles & Measurements

- SU(3) YM with Wilson plaquette action
- T between 1.2 and 2.5
- N_σ between 48 and 144 (larger at higher T)
- N_τ either 6 or 8
- Between 14000 and 52000 measurements
  - Combined hot & cold starts
  - Cut of 3000 cfg.s for thermalization
  - 10 compound sweeps of 1 heatbath step and 8 over-relaxation steps

$$\frac{1}{32\pi^2}\sum_x \epsilon^{\mu\nu\rho\sigma}\Box_{\mu\nu}\Box_{\rho\sigma}$$

$Q_{\mathbb{R}}$ raw measurement

$Q_{\mathbb{Z}}$ naïve rounding

Q_a artifact corrected
Lucini & Teper, hep-lat/0103027

Q_f globally fit
del Debbio *et al.*, hep-th/0204125

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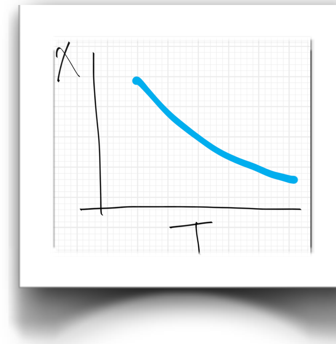
del Debbio *et al.*, hep-th/0204125

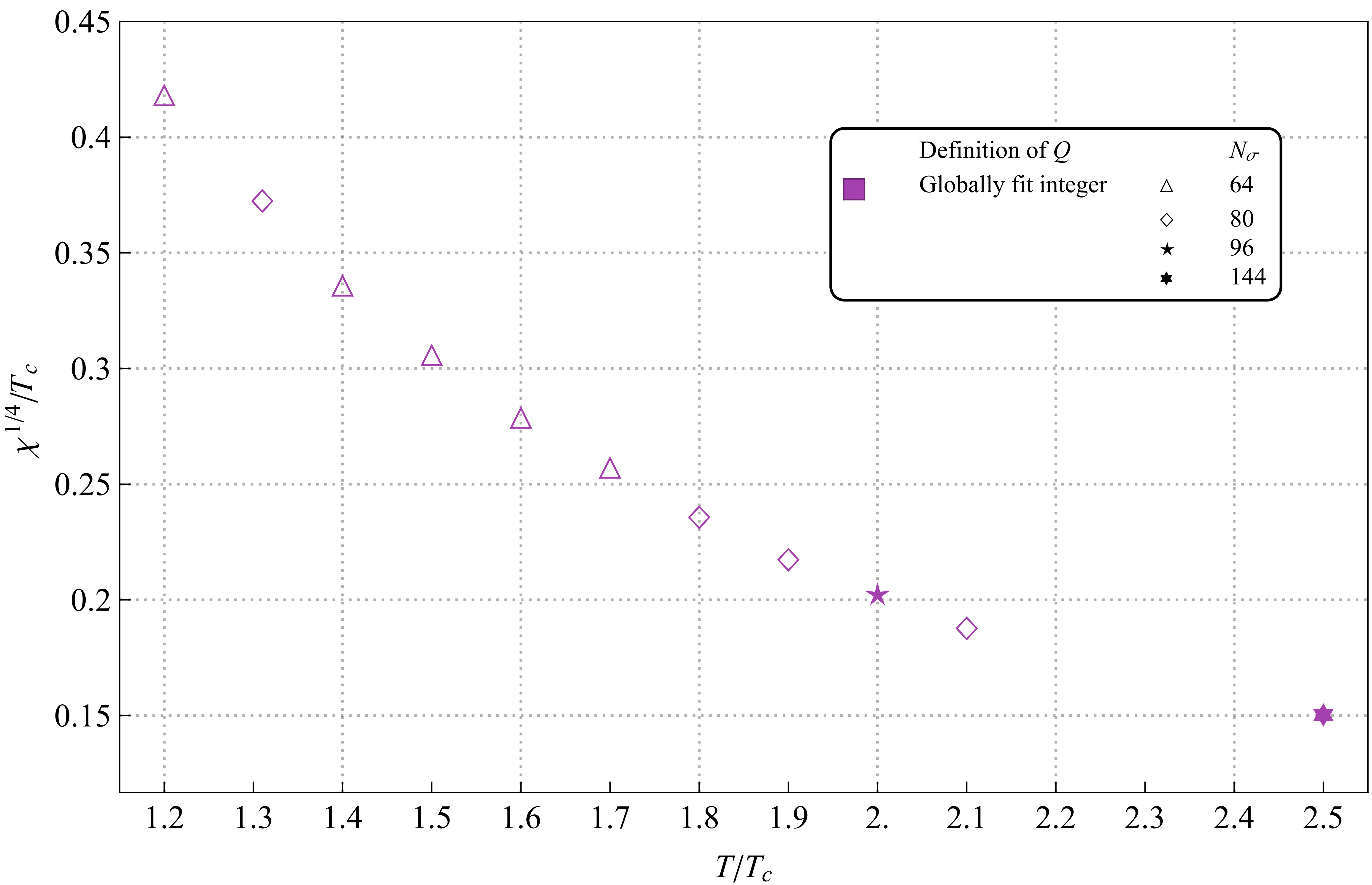
Essentially no discretization or finite volume corrections



Pure glue measurements show χ vanishes as T→∞

Best Lattice Results

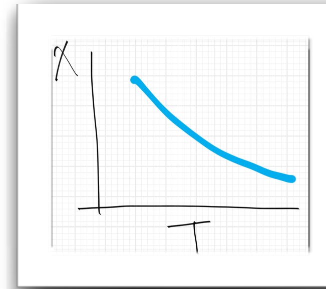
Berkowitz, Buchoff & Rinaldi, arXiv:1505.XXXX



Compare with Gattringer et al., arXiv:hep-lat/0203013 up to T/T_c = 1.31

DIGM Best Fit & Extrapolation

Berkowitz, Buchoff & Rinaldi, arXiv:1505.XXXX



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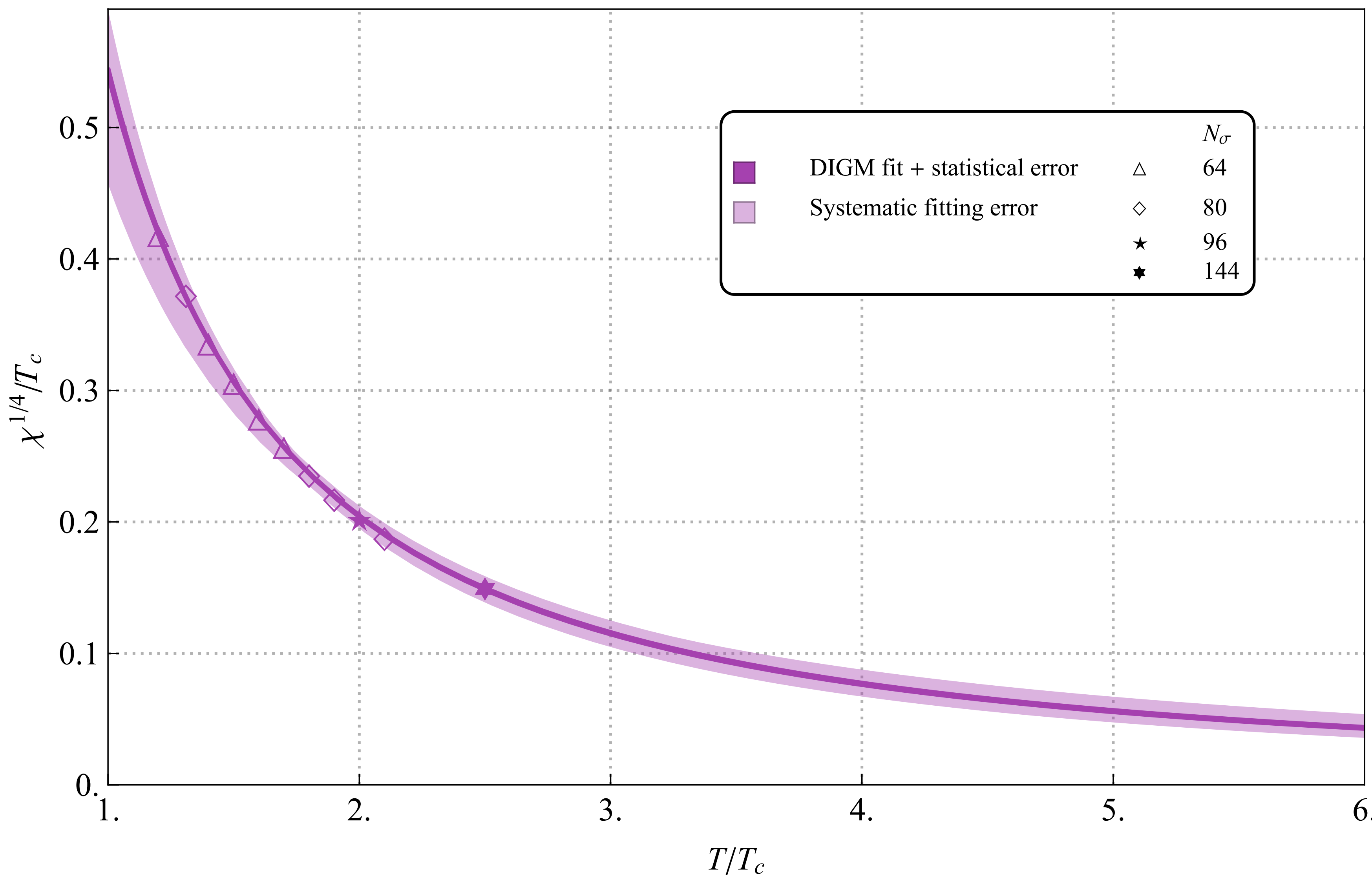
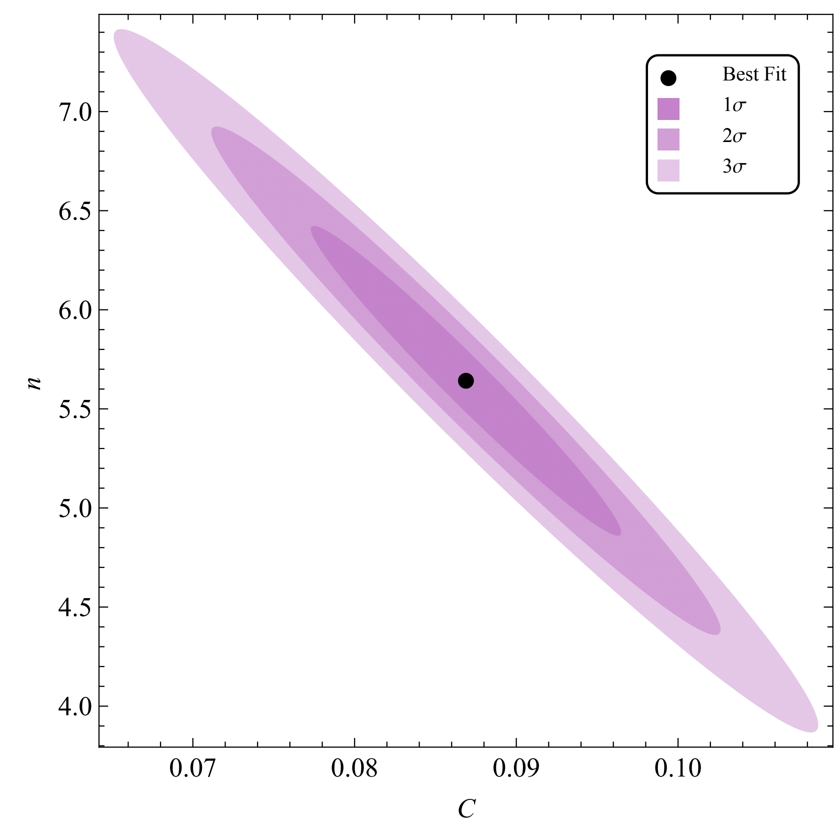


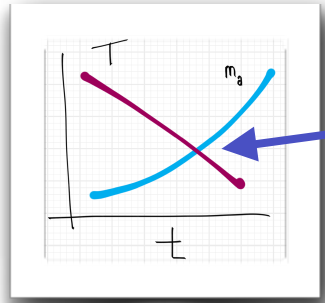
$$\frac{\chi}{T_c^4} = \frac{C}{(T/T_c)^n}$$



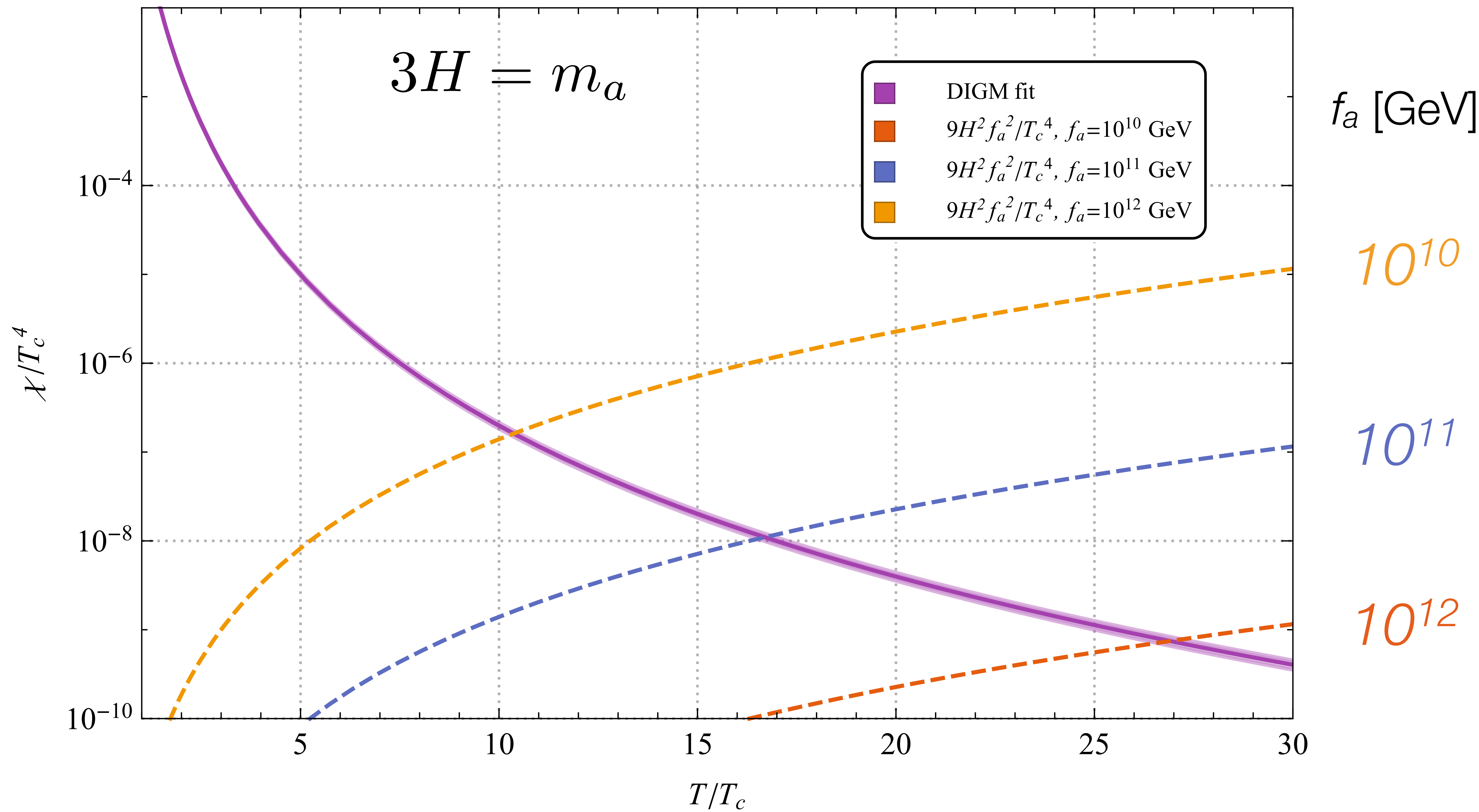
	C	n
Best Fit	0.0869	5.64
Covariance Matrix		
C	$2.\times10^{-6}$	$5.\times10^{-5}$
n	$5.\times10^{-5}$	0.001

Axion Freezeout

Berkowitz, Buchoff & Rinaldi, arXiv:1505.XXXX

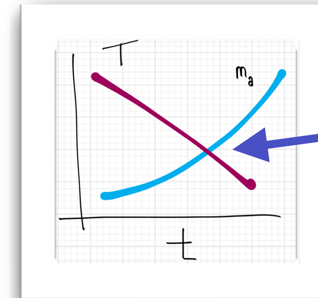


Axions freeze out when
3H ~ m_a
$T_1 \approx 5.5\ T_c$ from models



Axion Freezeout

Berkowitz, Buchoff & Rinaldi, arXiv:1505.XXXX



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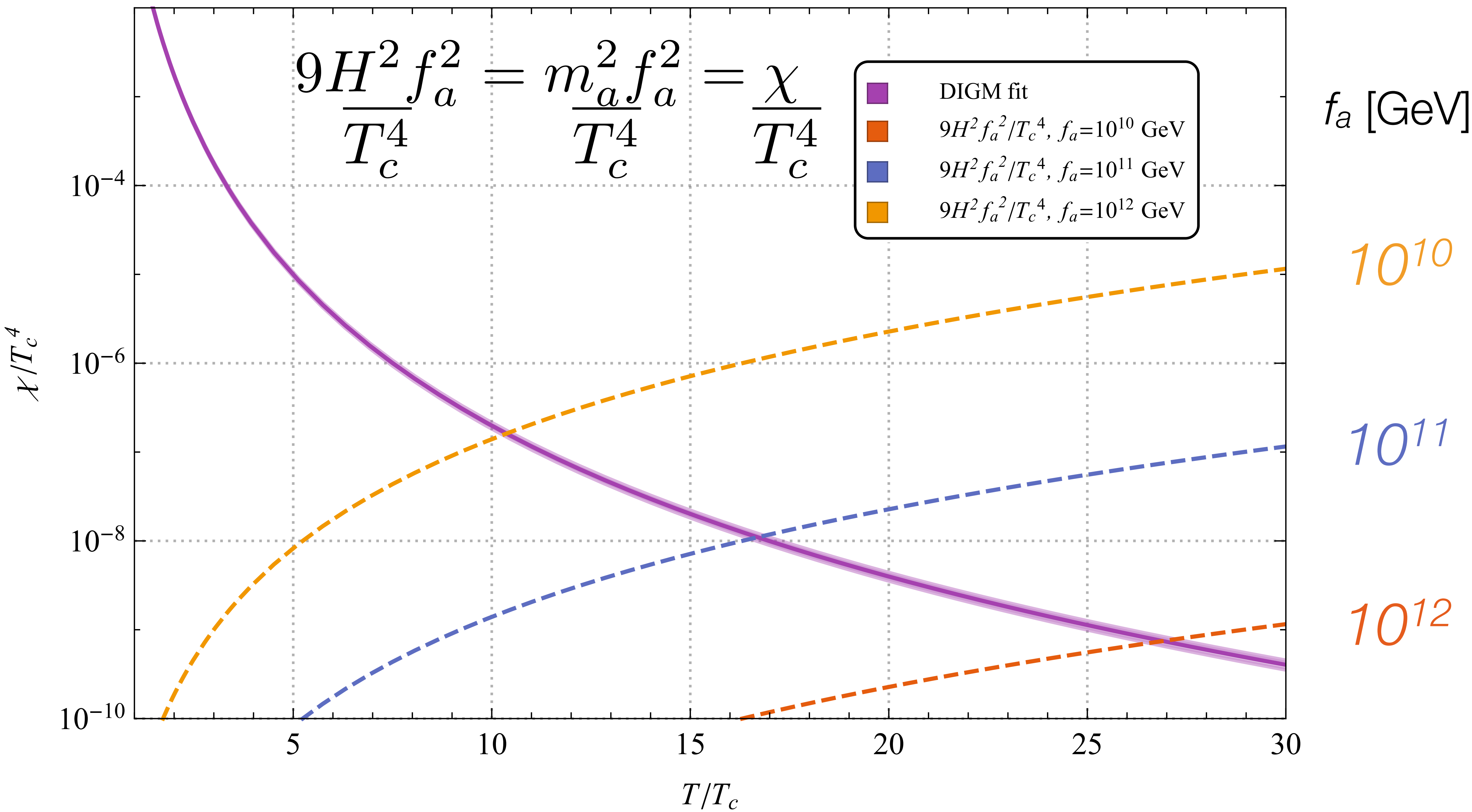
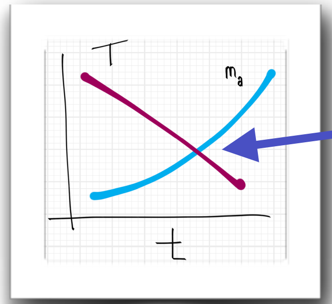


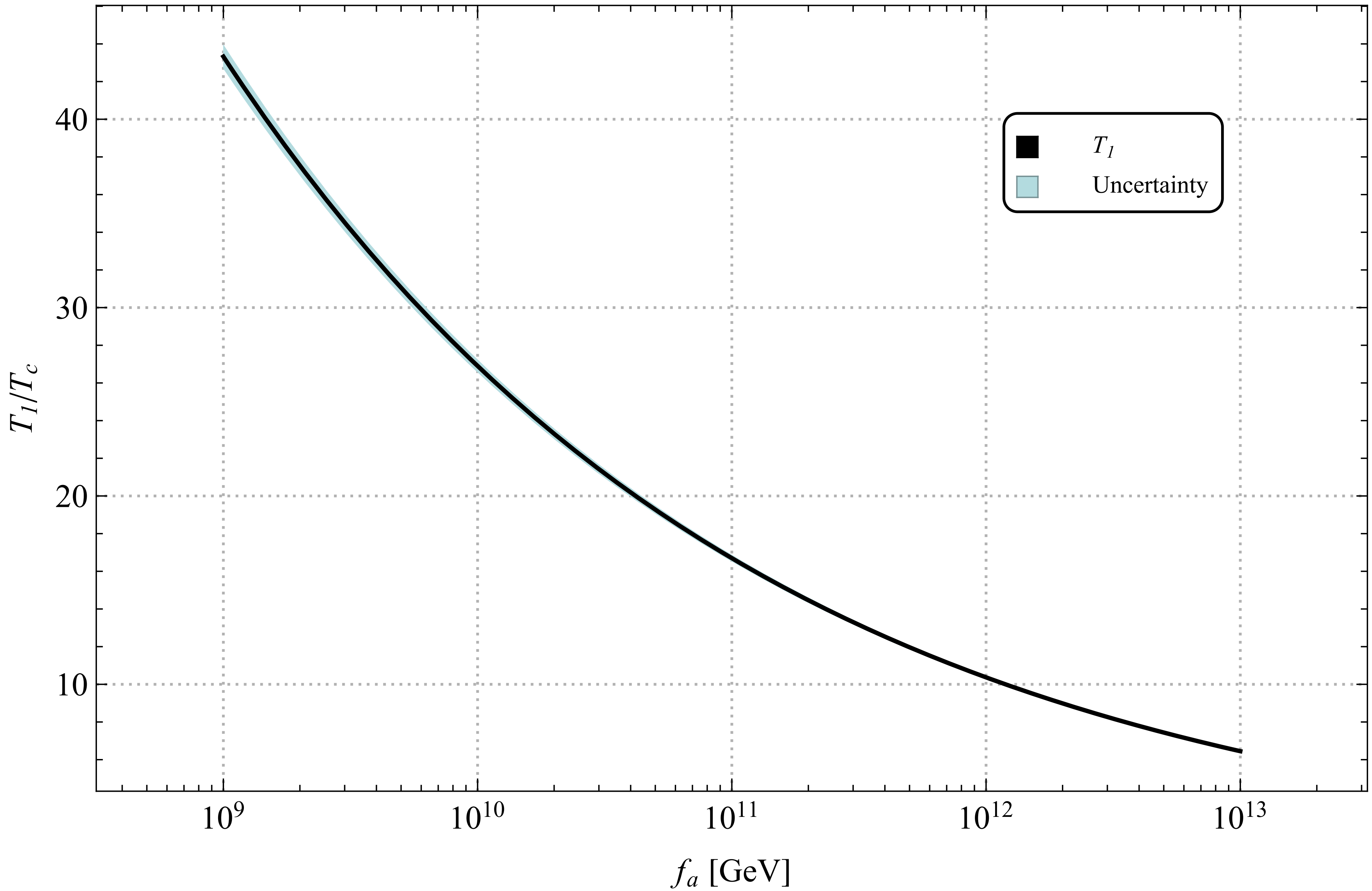
$$H^2 = \frac{\pi^2}{90}\frac{1}{m_P^2}\, g_{*R}(T)\, T^4$$

Axion Freezeout

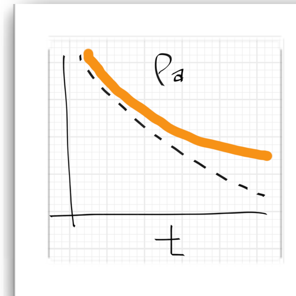
Berkowitz, Buchoff & Rinaldi, arXiv:1505.XXXX



Freezeout temperature T_1 for
$3H = m_a$
from pure glue



Pure Glue Axion Density



Axion density at freezeout controls axion density today

$$\frac{\rho(t)R^3}{m_a(t)} = \#\text{ axions in a fixed comoving volume}$$

$$\rho(T_\gamma) = \rho(T_1)\frac{m_a(T_\gamma)}{m_a(T_1)}\left(\frac{R(T_1)}{R(T_\gamma)}\right)^3$$

$$T_\gamma = 2.73\mathrm{K}$$

$T_1 = T_1(f_a)$ as just shown

$$m_a(T_1) = \frac{\sqrt{\chi}}{f_a}$$

$$m_a(T_\gamma) = \frac{1}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi m_\pi$$ χPT

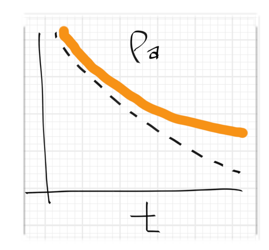
$R(T)$ from cosmology

$$\rho(T_1) = \frac{1}{2}m_a^2 f_a^2 \theta_1^2$$

θ_1 random if PQ-breaking is after inflation.

$$\langle\theta_1^2\rangle = \frac{\pi^2}{3}$$

Pure Glue Axion Density



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$$T_\gamma = 2.73\text{K}$$

$T_1 = T_1(f_a)$ as just shown $\qquad m_a(T_1) = \frac{\sqrt{\chi}}{f_a}$

Rely on our lattice calculation

$$m_a(T_\gamma) = \frac{1}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi m_\pi$$ χPT

$R(T)$ from cosmology

$$\rho(T_1) = \frac{1}{2} m_a^2 f_a^2 \theta_1^2$$ θ_1 random if PQ-breaking is after inflation.

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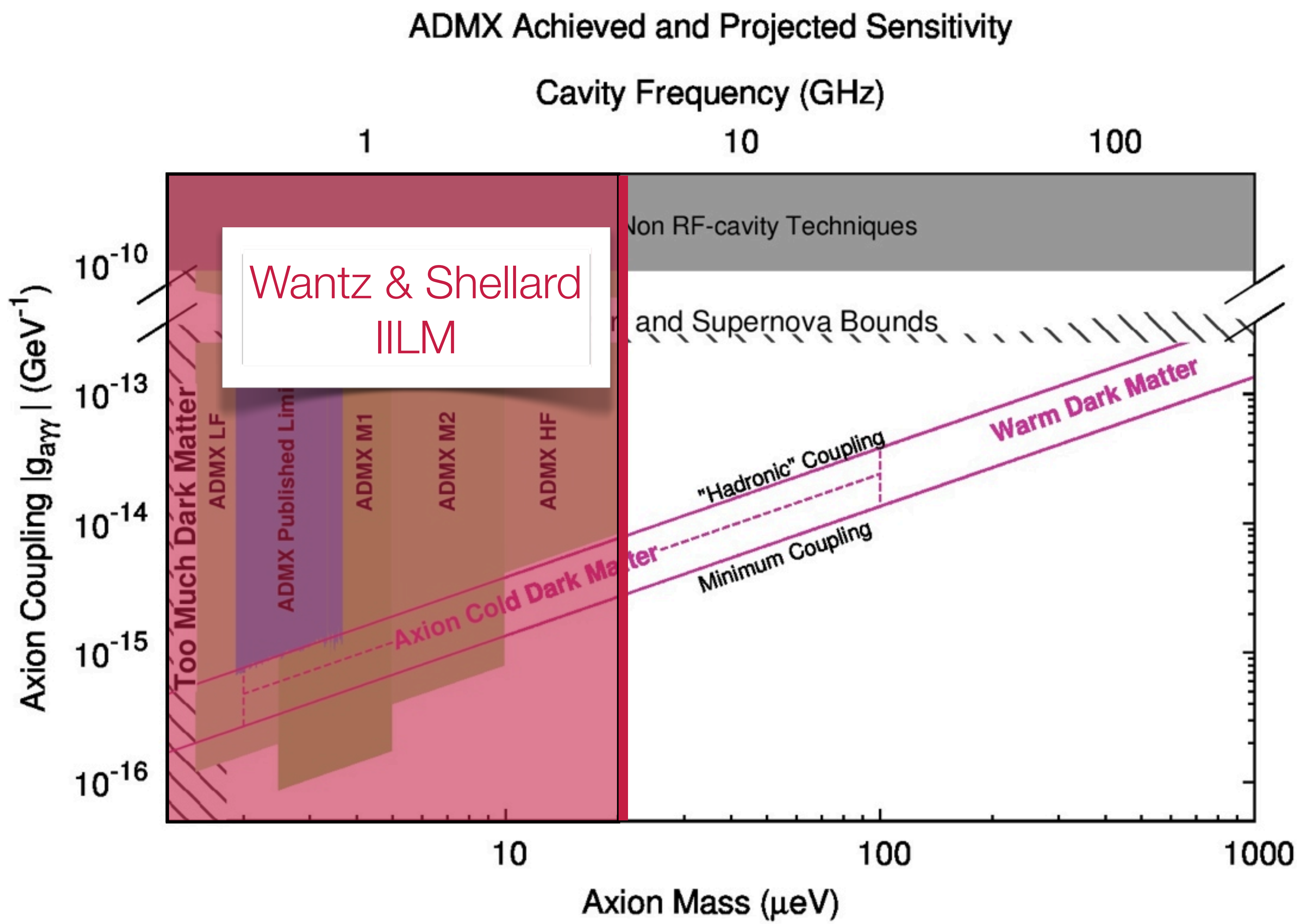
The Over-Closure Bound As It Stands Today

Wantz & Shellard, arXiv:0910.1066



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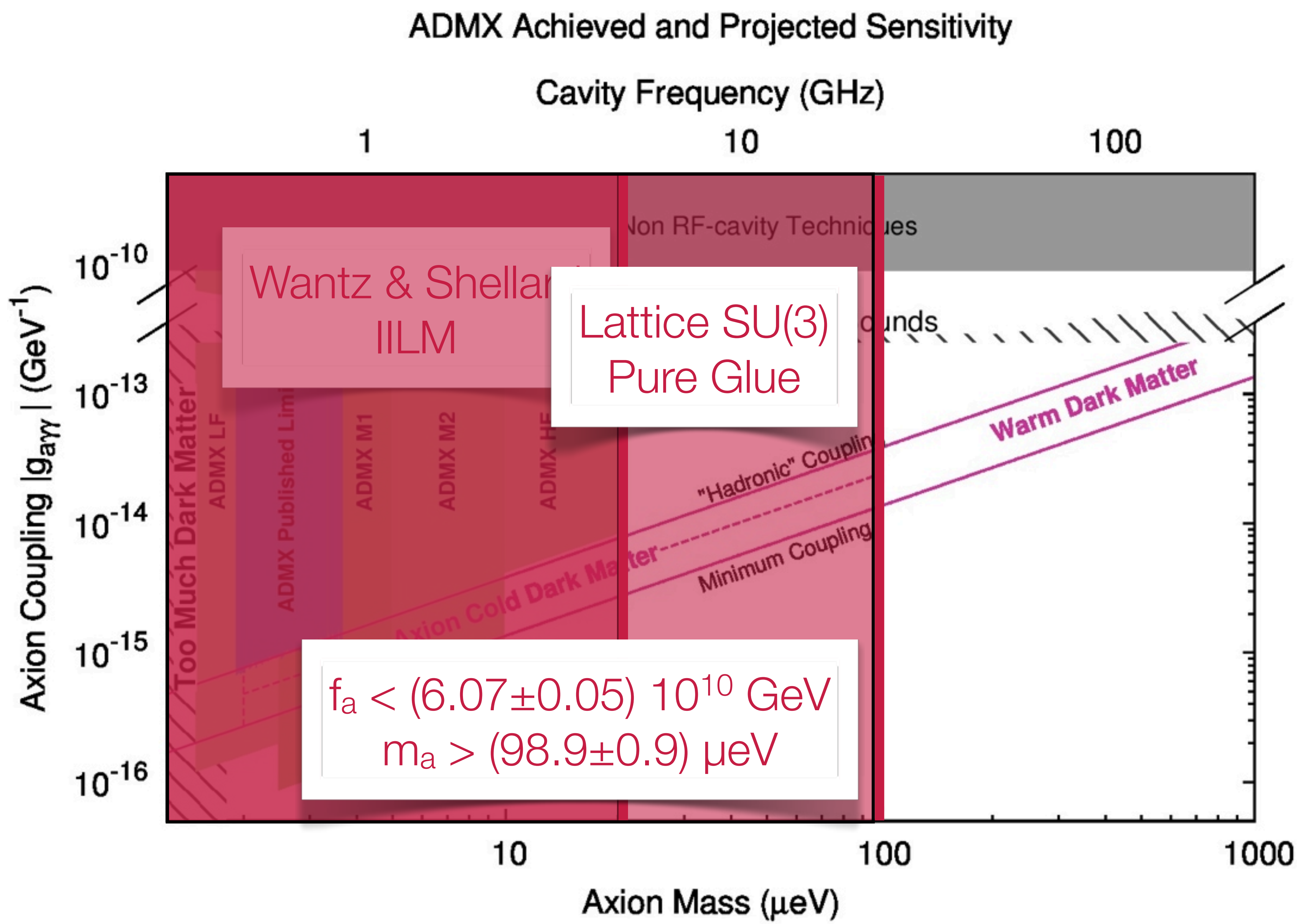


Conclusions & Outlook

The Economist, 19 Dec 2006

- PQ symmetry:
  - cleans up the Strong CP problem
  - provides a plausible, largely unconstrained DM candidate: the axion.
- Axion searches will search large swaths of interesting parameter space soon.
  - Lattice QCD can provide important nonperturbative input for calculating Ω_a
- DIGM fits outstandingly to pure glue at high temperature.
- Pure glue predicts a high over closure bound!

Future Steps

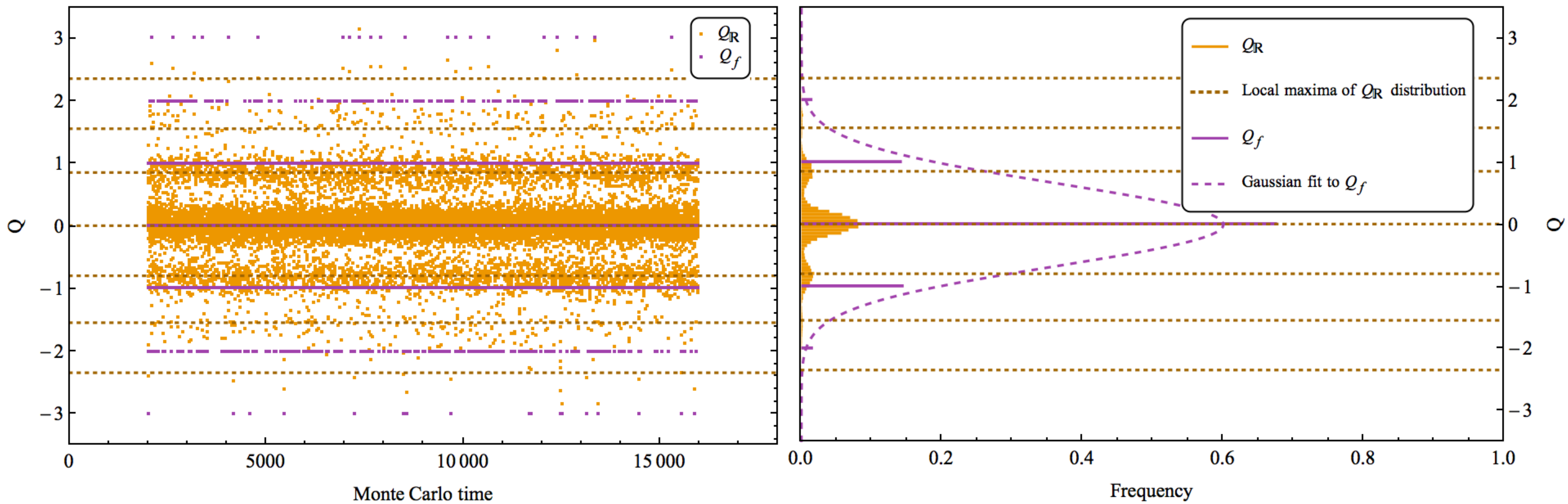
- Measure higher moments? May be able to get χ_4, χ_6.
- Incorporate quarks
  - It may be sufficient to use heavy quarks.
- Fermionic definition of Q
- Move to anisotropic lattices to alleviate finite-volume effects at high T.
- Fixed topology methods / open boundary conditions at high T.

  Aoki *et al.*, arXiv:0707.0396v2 Lüscher & Schæfer, arXiv:1105.4749
- Finite θ:
  - Imaginary-θ has no sign problem
  - Real, finite θ may be amenable to Langevin methods

Backup Slides

Example MC History

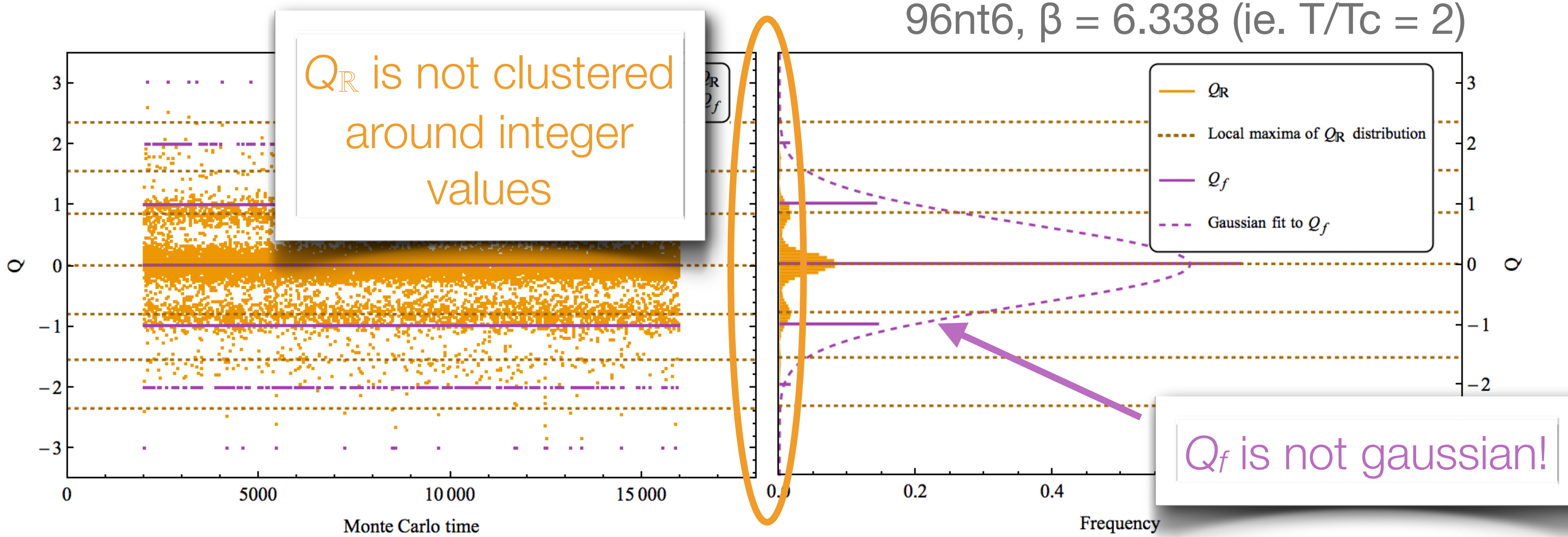
96nt6, β = 6.338 (ie. T/Tc = 2)



Topology fluctuates sufficiently for measuring χ

cf. Durr *et al.*, hep-lat/0612021

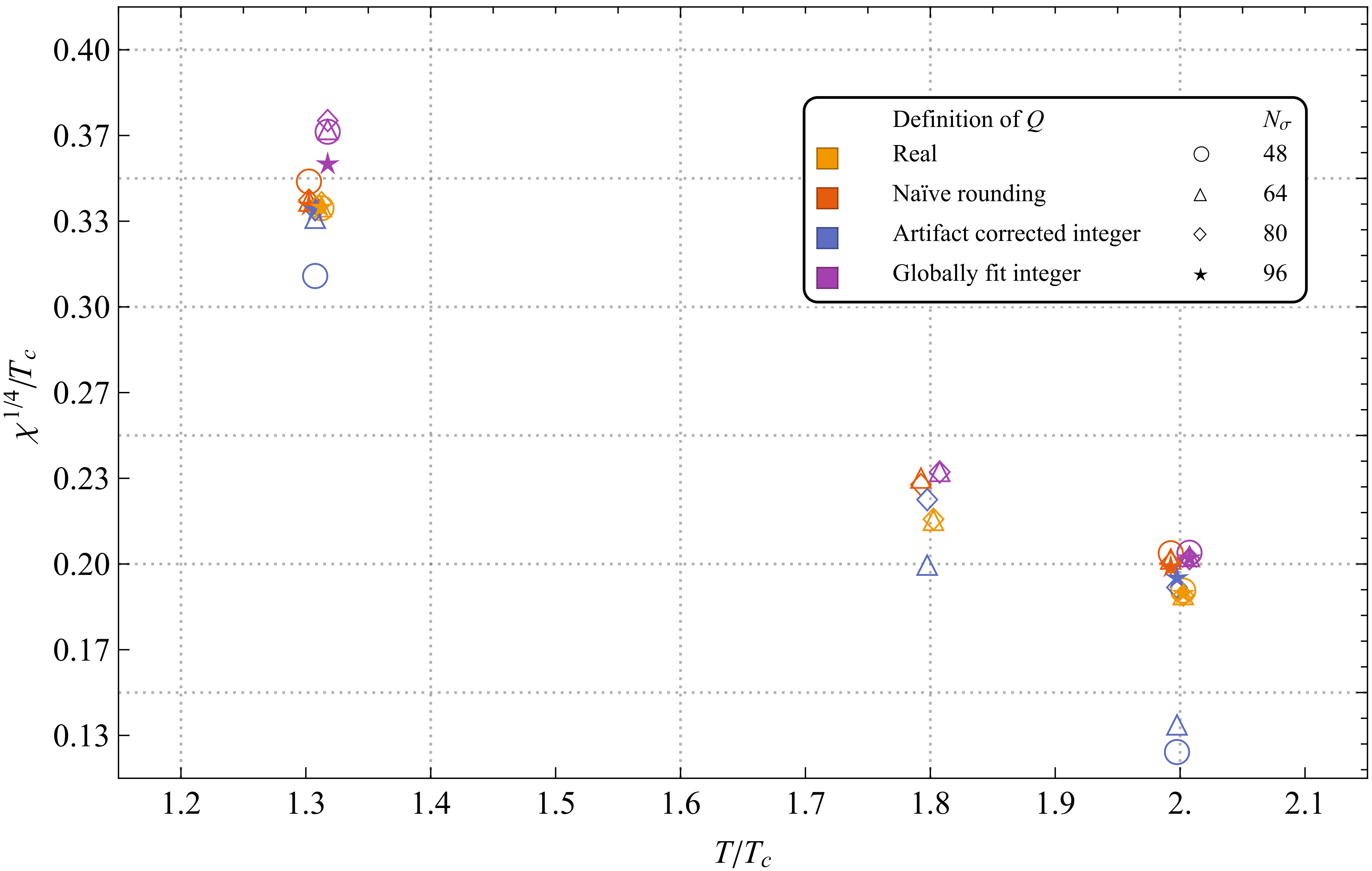
Example MC History



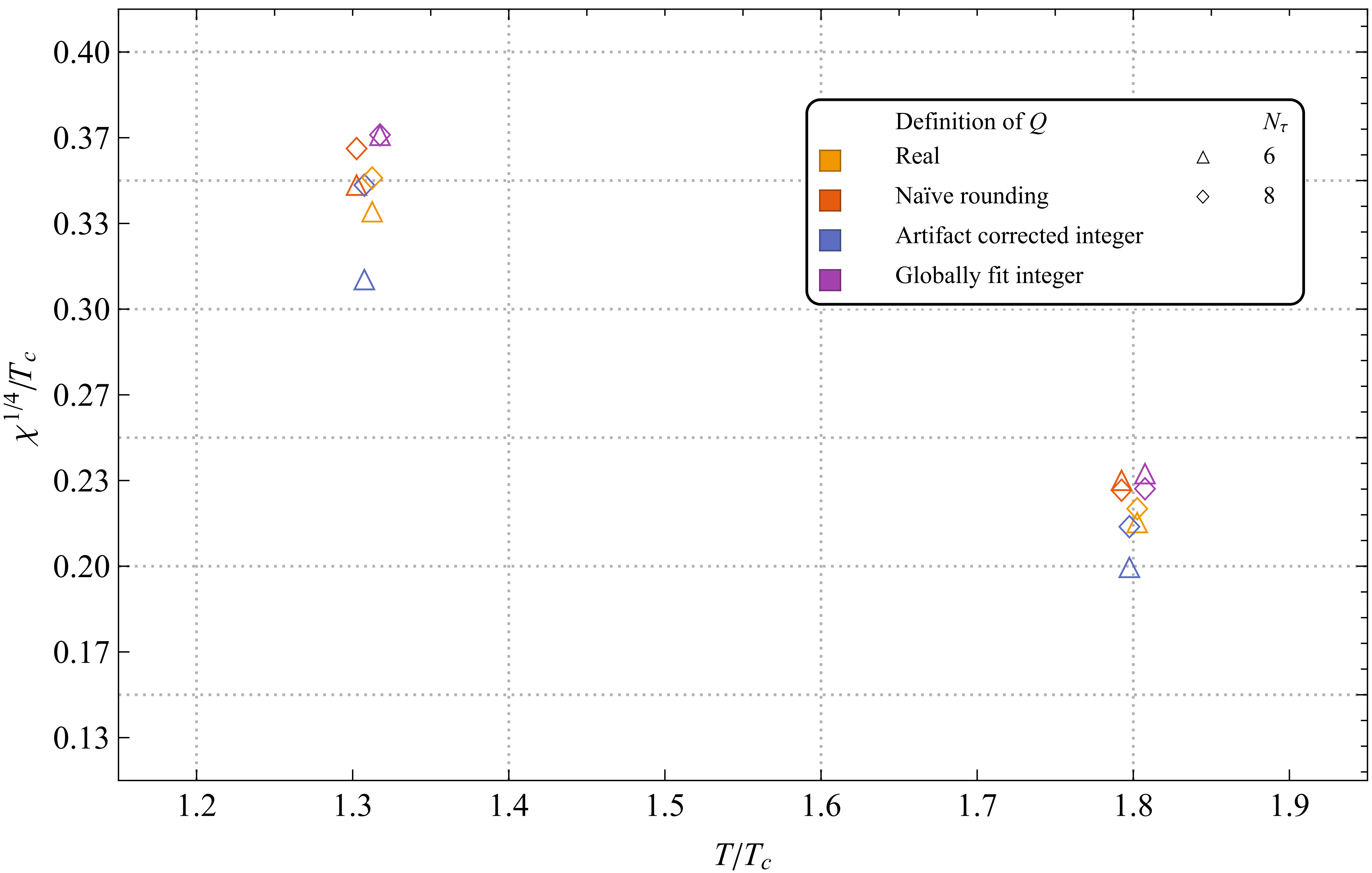
Topology fluctuates sufficiently for measuring χ

cf. Durr *et al.*, hep-lat/0612021

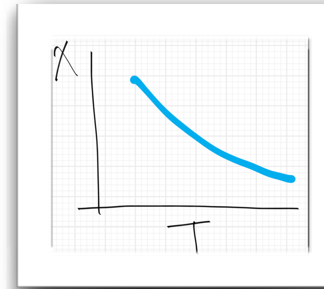
Finite Volume Effects



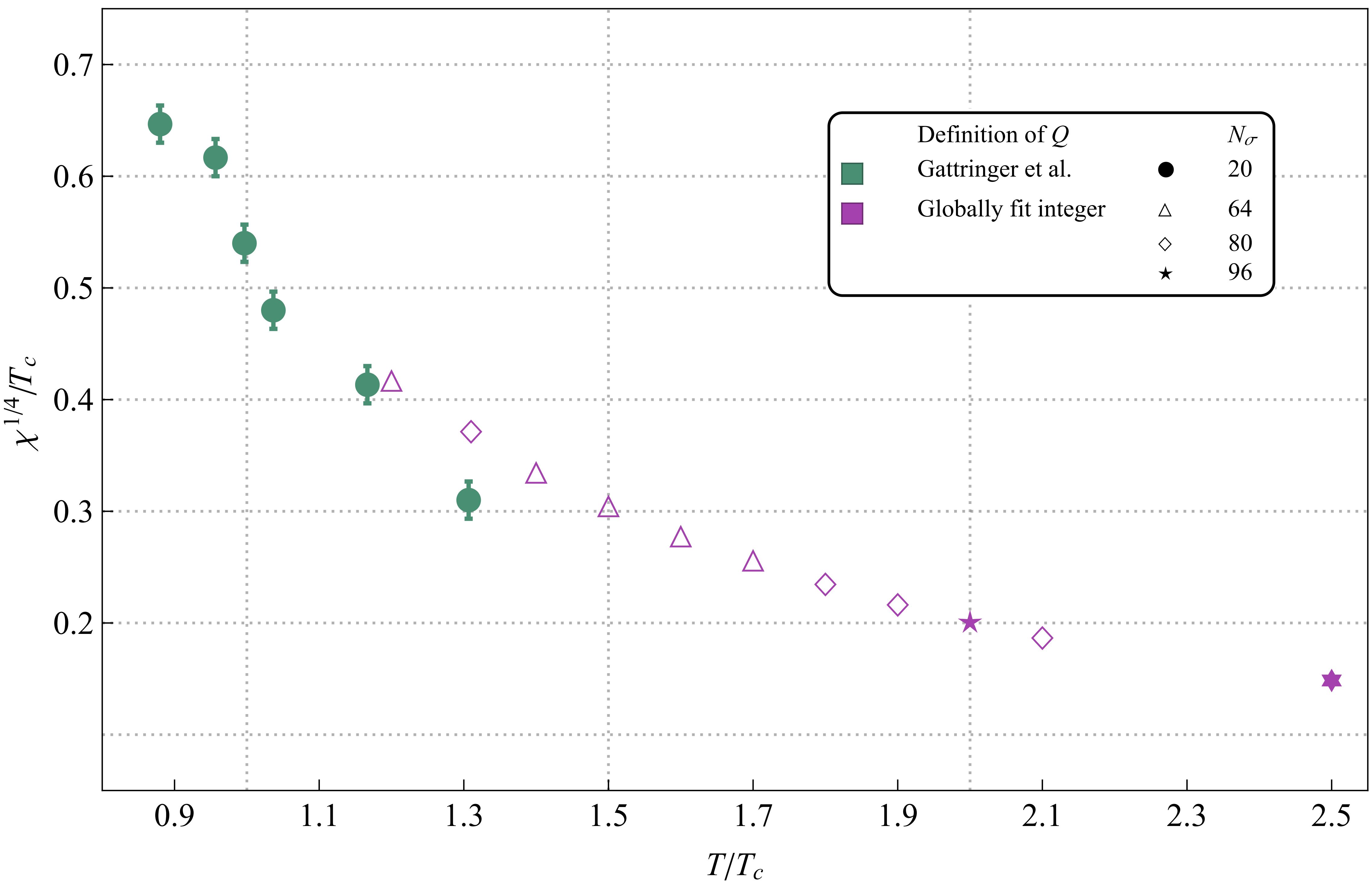
Discretization Effects



Comparison with Gattringer



Pure glue measurements show χ vanishes as T→∞



Gattringer et al., arXiv:hep-lat/0203013

Ensemble & Measurement Information

Berkowitz, Buchoff & Rinaldi, arXiv:1505.XXXX - possibly slightly outdated

T/T_c	β	$a\sqrt{\sigma}$	N_τ	N_σ	N_{meas}	central value $\chi^{1/4}/T_c \pm \delta\chi^{1/4}/T_c$ statistical measured from							
						$Q_\mathbb{R}$		$Q_\mathbb{Z}$		Q_a		Q_f	
1.2	6.00106	0.2161	6	64	12000	0.3871	0.0013	0.3881	0.0013	0.3815	0.0013	**0.4193**	**0.0014**
1.31	6.05386	0.1979	6	48	13600	0.3390	0.0011	0.3494	0.0010	0.3126	0.0010	0.3688	0.0012
				64	34000	0.3401	0.0007	0.3423	0.0007	0.3356	0.0007	0.3701	0.0007
				80	12000	0.3415	0.0011	0.3425	0.0011	0.3387	0.0011	**0.3734**	**0.0012**
				96	12000	0.3401	0.0011	0.3409	0.0011	0.3392	0.0011	0.3570	0.0011
	6.2423	0.1484	8	64	31998	0.3520	0.0010	0.3634	0.0010	0.3492	0.0010	0.3687	0.0011
				96	12000	0.3542	0.0016	0.3561	0.0016	0.3537	0.0015	0.3707	0.0017
1.4	6.0953	0.1852	6	64	52000	0.3095	0.0005	0.3154	0.0005	0.3077	0.0005	**0.3370**	**0.0006**
1.5	6.1397	0.1729	6	64	52000	0.2816	0.0005	0.2930	0.0005	0.2835	0.0005	**0.3070**	**0.0005**
1.6	6.1825	0.1621	6	64	51998	0.2568	0.0005	0.2721	0.0005	0.2586	0.0005	**0.2799**	**0.0005**
1.7	6.22365	0.1525	6	64	22000	0.2367	0.0008	0.2535	0.0008	0.2329	0.0008	**0.2583**	**0.0009**
1.8	6.26323	0.1441	6	64	22000	0.2181	0.0008	0.2346	0.0008	0.2007	0.0009	0.2372	0.0009
				80	30000	0.2184	0.0006	0.2318	0.0006	0.2260	0.0006	**0.2367**	**0.0006**
	6.47163	0.1080	8	96	12000	0.2233	0.0017	0.2305	0.0017	0.2163	0.0018	0.2311	0.0018
1.9	6.30132	0.1365	6	64	22000	0.2021	0.0008	0.2178	0.0009	0.1677	0.0011	0.2193	0.0009
				80	32000	0.2021	0.0006	0.2159	0.0006	0.2090	0.0006	**0.2184**	**0.0006**
1.99	6.55	0.0973	8	64	12795	0.2012	0.0030	0.2041	0.0036	0.1836	0.0037	0.2041	0.0036
2.0	6.338	0.1297	6	48	13600	0.1903	0.0017	0.2048	0.0020	0.1275	0.0030	0.2050	0.0020
				64	23598	0.1892	0.0010	0.2031	0.0011	0.1386	0.0014	0.2040	0.0011
				80	24000	0.1888	0.0007	0.2014	0.0008	0.1919	0.0008	0.2030	0.0008
				96	12000	0.1898	0.0009	0.2002	0.0009	0.1959	0.0009	**0.2037**	**0.0010**
2.1	6.37331	0.1235	6	80	22000	0.1773	0.0008	0.1879	0.0009	0.1746	0.0009	**0.1887**	**0.0010**
2.5	6.50204	0.1037	6	128	12000	0.1493	0.0008	0.1495	0.0011	0.1477	0.0010	0.1490	0.0011
				144	13797	0.1496	0.0007	0.1525	0.0008	0.1513	0.0008	**0.1518**	**0.0008**

$$\frac{1}{32\pi^2}\sum_x \epsilon^{\mu\nu\rho\sigma}\Box_{\mu\nu}\Box_{\rho\sigma}$$

$Q_\mathbb{R}$ raw measurement

$Q_\mathbb{Z}$ simple rounding

Q_a artifact corrected

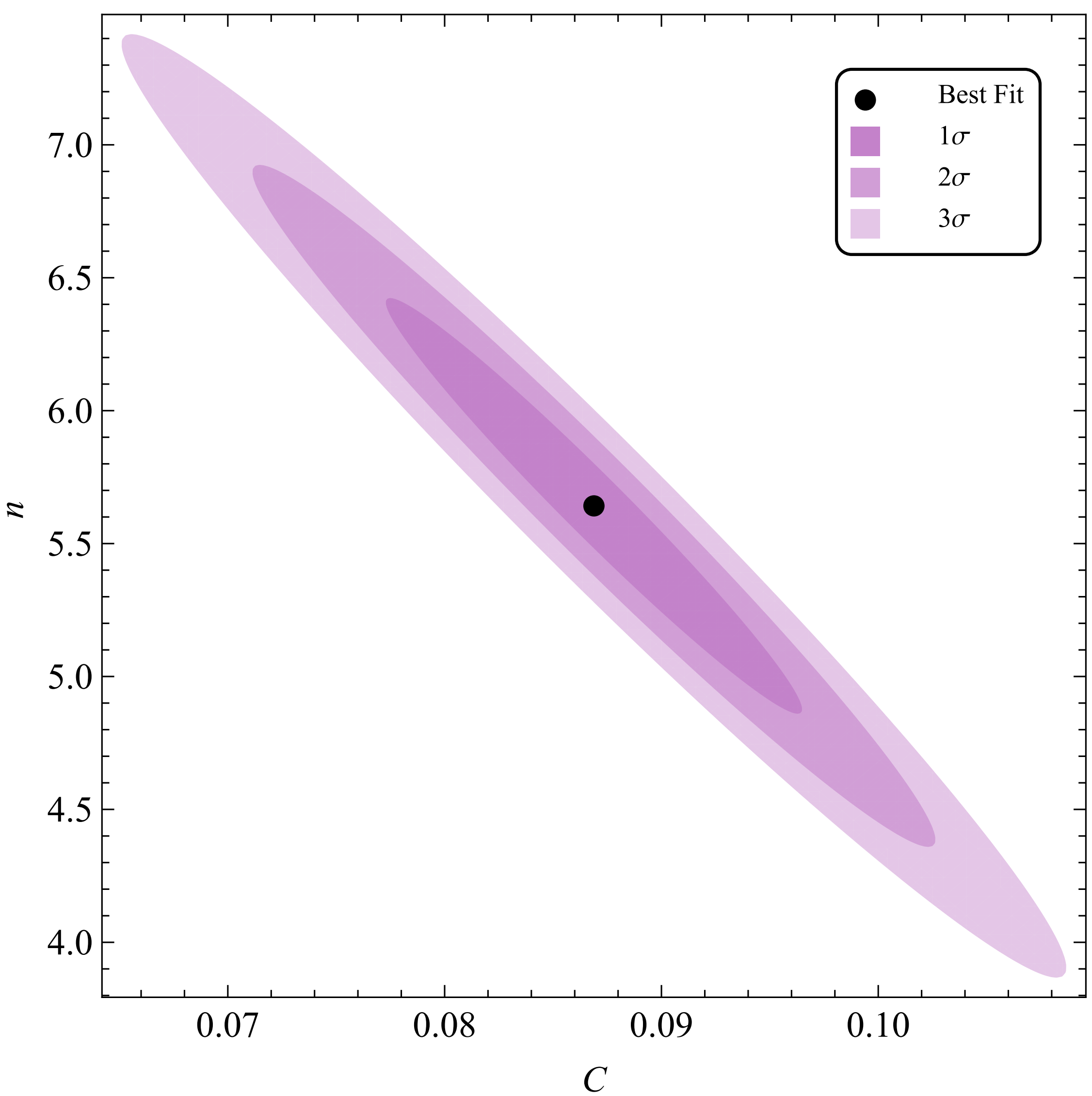
Lucini & Teper, hep-lat/0103027

Q_f globally fit

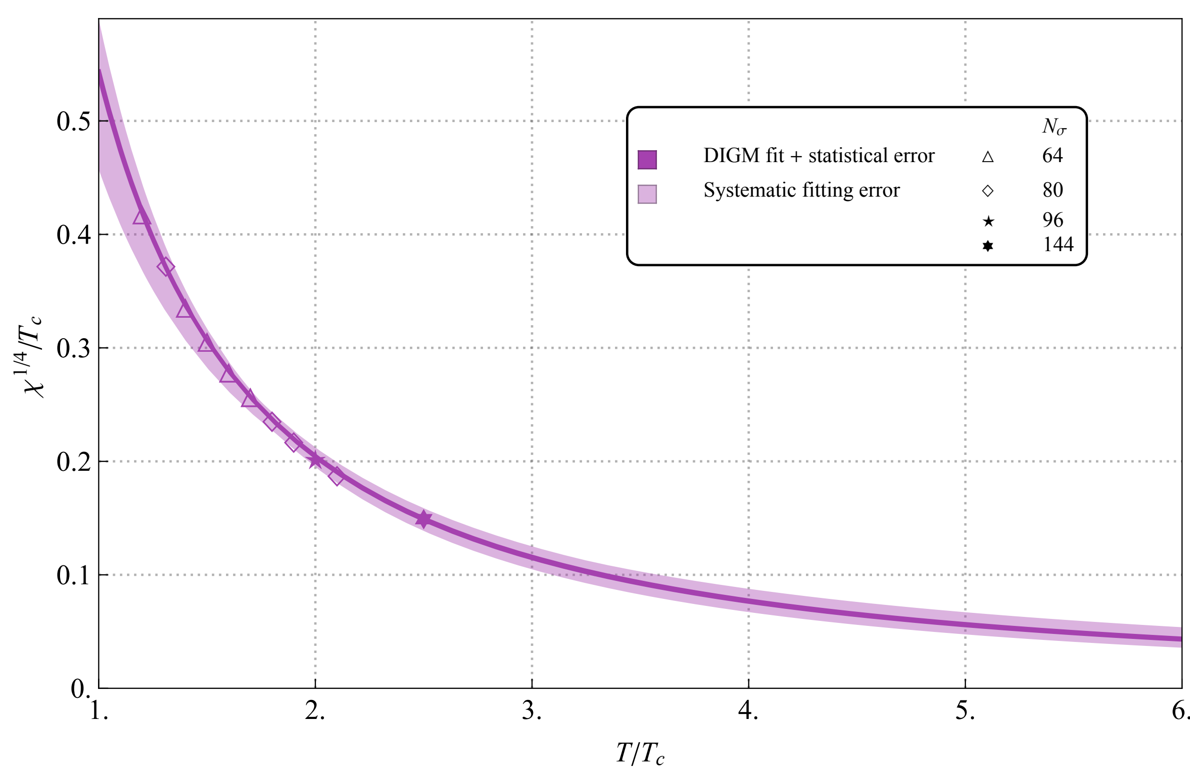
del Debbio *et al.*, hep-th/0204125

Backup Slide: DIGM Best Fit

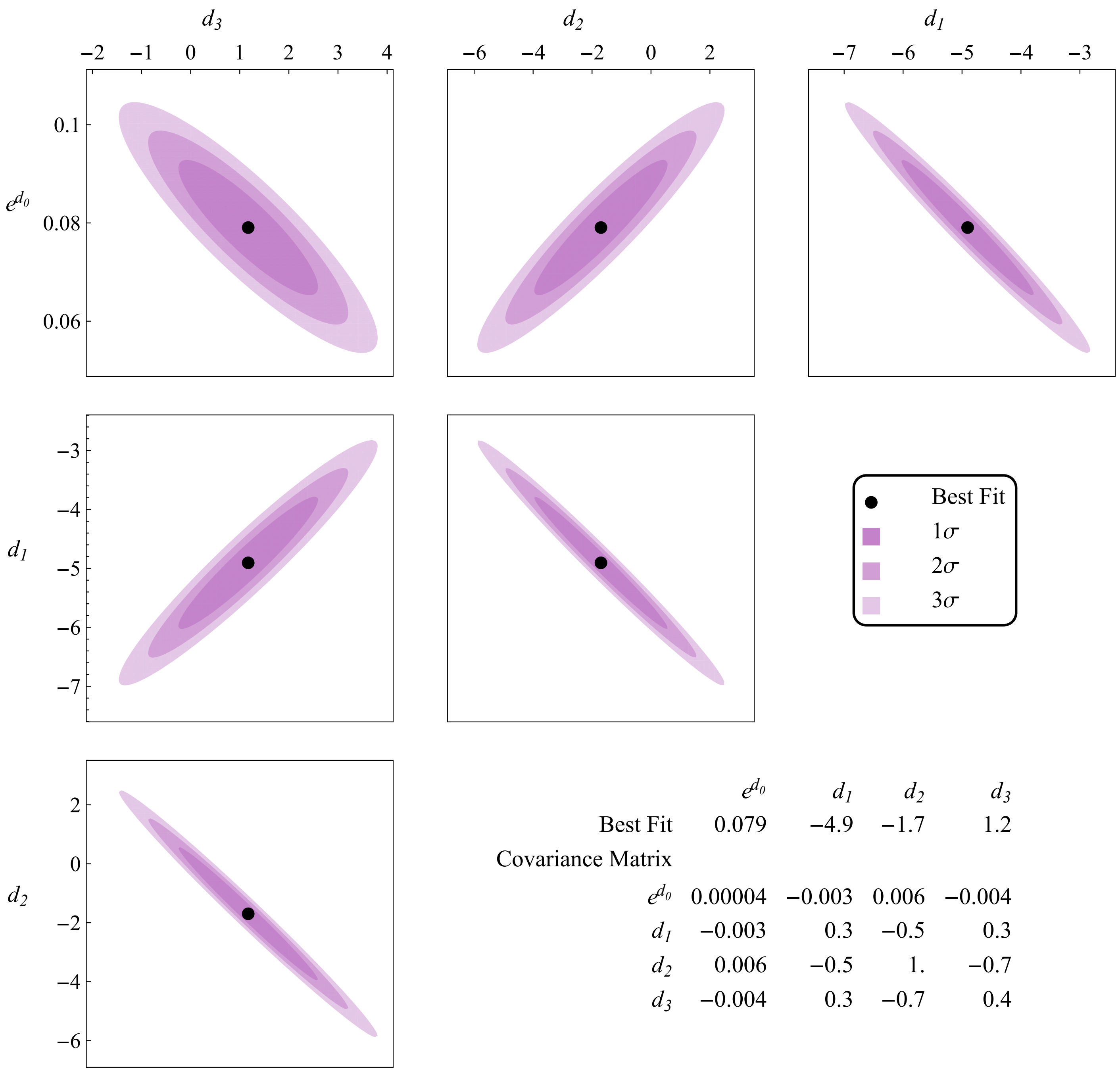
$$\frac{\chi}{T_c^4} = \frac{C}{(T/T_c)^n}$$



	C	n
Best Fit	0.0869	5.64
Covariance Matrix		
C	$2.\times10^{-6}$	$5.\times10^{-5}$
n	$5.\times10^{-5}$	0.001

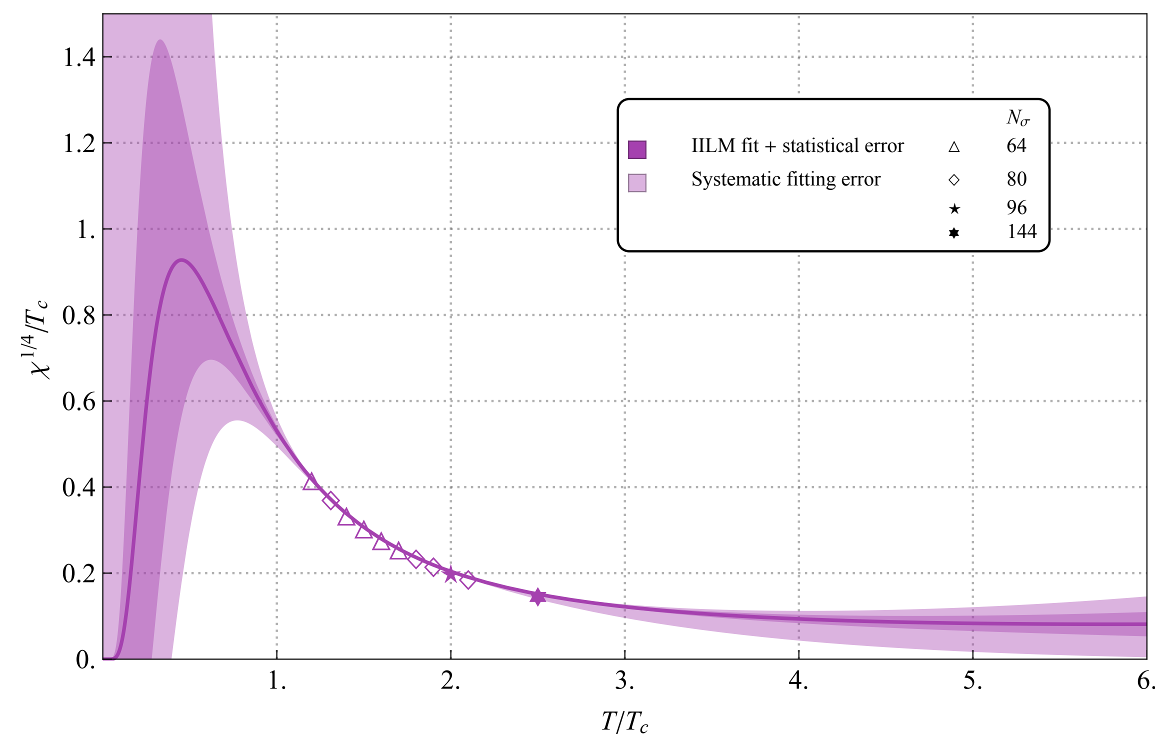


Backup Slide: IILM Best Fit



	e^{d_0}	d_1	d_2	d_3
Best Fit	0.079	−4.9	−1.7	1.2
Covariance Matrix				
e^{d_0}	0.00004	−0.003	0.006	−0.004
d_1	−0.003	0.3	−0.5	0.3
d_2	0.006	−0.5	1.	−0.7
d_3	−0.004	0.3	−0.7	0.4

$$\frac{\chi}{T_c^4} = \frac{e^{d_0}}{(T/T_c)^{-d_1}} \exp\left[d_2\left(\ln\frac{T}{T_c}\right)^2 + d_3\left(\ln\frac{T}{T_c}\right)^3\right]$$



Backup Slide: Scale Setting Systematic

- Temperature is set through $a\sigma^{1/2}(\beta_c)$ so that everything is naturally in units of T_c

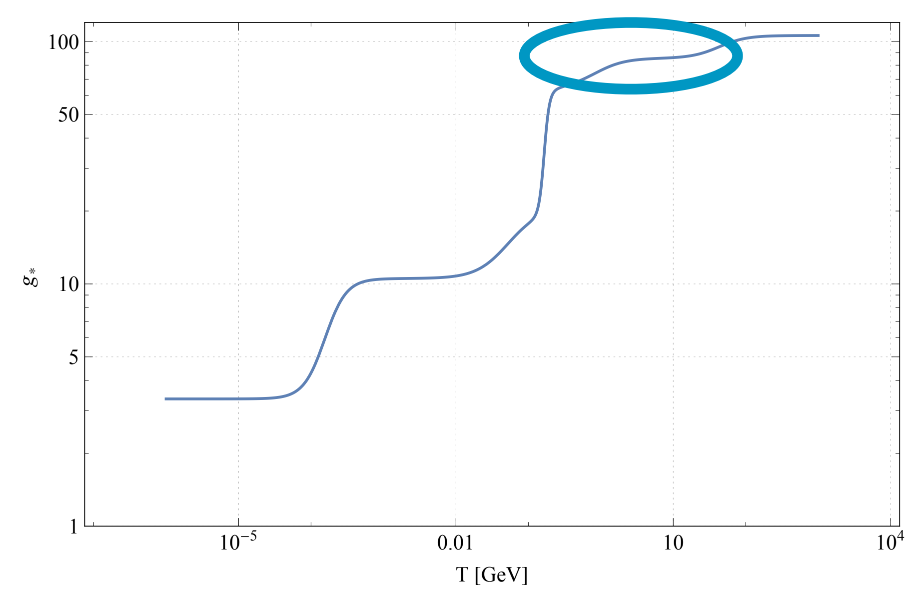
$$\frac{T}{T_c}(\beta, N_\tau) = \frac{a\sqrt{\sigma}(\beta_c)N_{\tau_c}}{a\sqrt{\sigma}(\beta)N_\tau}$$

  - $T_c^{glue} \approx 2T_c^{QCD}$

- $3H \sim m_a$ is a particularly fortuitous form

$$H^2 = \frac{\pi^2}{90}\frac{1}{m_P^2}\, g_{*R}(T)\, T^4$$

$$\frac{\chi}{T_c^4} = \frac{9H^2 f_a^2}{T_c^4} \propto g_{*R}\left(\frac{T}{T_c} T_c\right)\left(\frac{T}{T_c}\right)^4$$



Scale comes in extremely mildly

Backup Slide: Cosmological R

FRW

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)R$$

EOS

$$\rho = \rho(T)$$

$$p = p(T)$$

$$T = T(t)$$

$$R = R(T)$$

State of Universe