Synthesis and Phenomenology of large Nuclear Dark Matter (and Twin Higgs asides)

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Work with E. Hardy, J. March-Russell, S. West
arXiv 1411.3739 and arXiv 1504.05419
and J. March-Russell, I. García García (work in progress)

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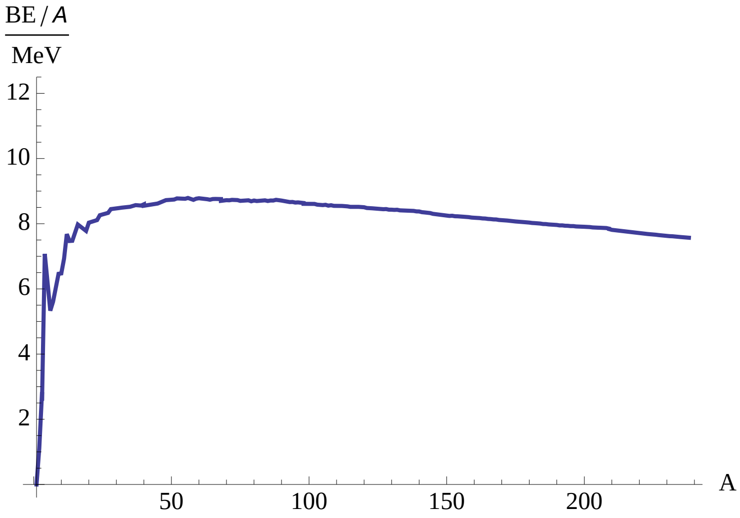
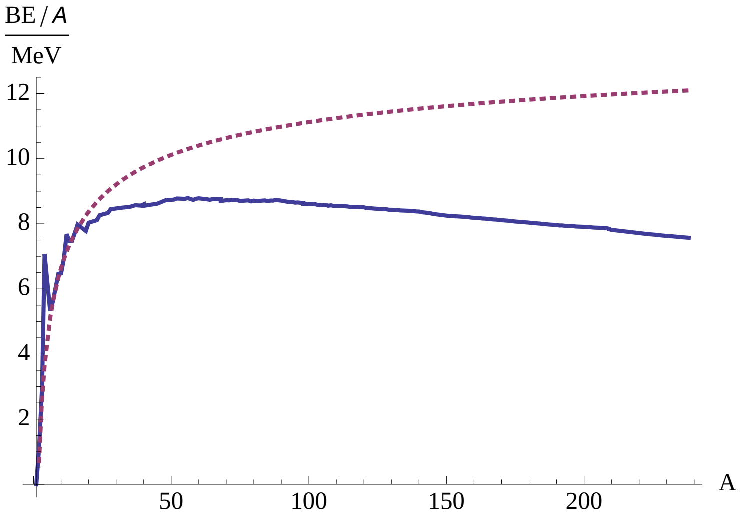
Large composite DM states

- Standard model: example of conserved baryon number, attractive interactions leading to multitude of large, stable bound states (nuclei)
- What if a similar thing happens for dark matter?
- Possibilities:
  - Number distribution over DM states
  - States with large spin
  - Structure on scales $\gg 1/m$ — form factors in scattering, possibility of larger cross sections
  - Coherent enhancement of interactions
  - Inelastic processes — fusions, fissions, excited states
  - 'Late-time' ($T \ll m$) synthesis — can achieve very heavy ($\gtrsim 100\,\mathrm{TeV}$) DM from thermal freeze-out
- Earlier example of Q-balls — non-topological solitons of scalar fields
- Related work: Krnjaic et al, Detmold et al, Wise et al

SM nuclei

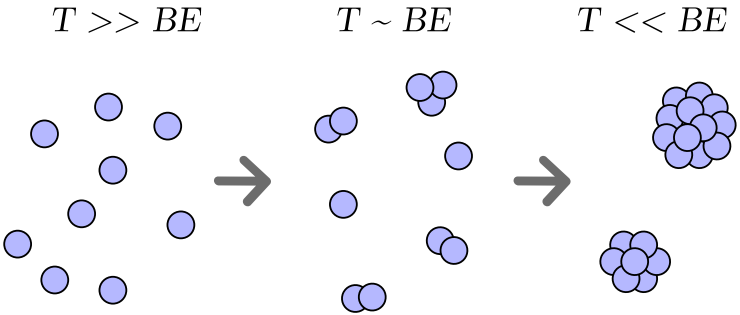


SM nuclei without Coulomb repulsion



Dark nucleosynthesis

Free energy $F = E - T\,S$:
large $T \Rightarrow$ everything dissociated
small $T \Rightarrow$ large states favoured
Assume asymmetric



Freeze-out of fusions

- Equal sizes: $A + A \to 2A$

$$\frac{\Gamma}{H} \sim \frac{\langle \sigma v \rangle n_A}{H} \sim \frac{\sigma_1 v_1 n_0}{H} A^{2/3} A^{-1/2} A^{-1} = \frac{\sigma_1 v_1 n_0}{H} A^{-5/6}$$

  With $M_A = AM_1$,

$$\frac{\sigma_1 v_1 n_0}{H} \sim 2\times 10^7 \left(\frac{1\,\mathrm{GeV\,fm^{-3}}}{\rho_b}\right)^{2/3} \left(\frac{T}{1\,\mathrm{MeV}}\right)^{3/2} \left(\frac{M_1}{1\,\mathrm{GeV}}\right)^{-5/6}$$

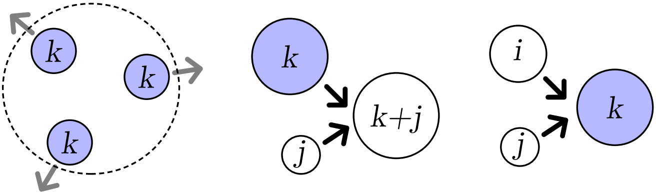
  so build-up to $A \sim 5 \times 10^8$ may be possible

- Small + large: $1 + A \to (1 + A)$. Rate for A of these is

$$\Gamma \sim \langle \sigma v \rangle n_k \frac{k}{A} \sim \sigma_1 v_1 n_0 \frac{1}{k^{1/2}} A^{2/3} A^{-1} = \frac{\sigma_1 v_1 n_0}{k^{1/2}} A^{-1/3}$$

Aggregation process

$$\frac{dn_k}{dt} + 3Hn_k = -\sum_{j\geq 1} \langle \sigma v \rangle_{j,k} n_j n_k + \frac{1}{2} \sum_{i+j=k} \langle \sigma v \rangle_{i,j} n_i n_j$$



In terms of yields, $n_k(t)/s(t) \equiv Y_k(t) \equiv Y_0 y_k(w(t)), \quad (\sum y_k = 1)$

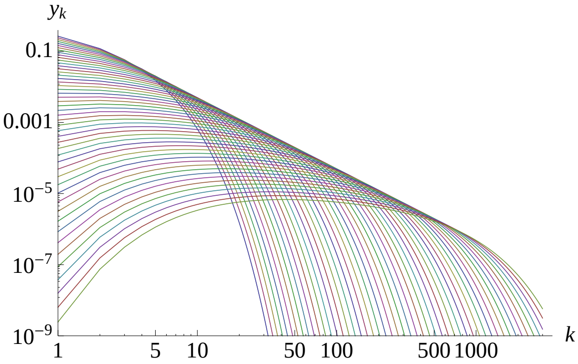
where new 'time' w is $$\frac{dw}{dt} = Y_0 \sigma_1 s(t) f(T_d(t)) = n_0 \sigma_1 v_1$$

Then, $$\frac{dy_k}{dw} = -y_k \sum_j R_{k,j} y_j + \frac{1}{2} \sum_{i+j=k} R_{i,j} y_i y_j$$

Scaling solution

$$\langle \sigma v \rangle_{i,j} \sim (\text{radius}_i + \text{radius}_j)^2 v_{\text{rel}}$$

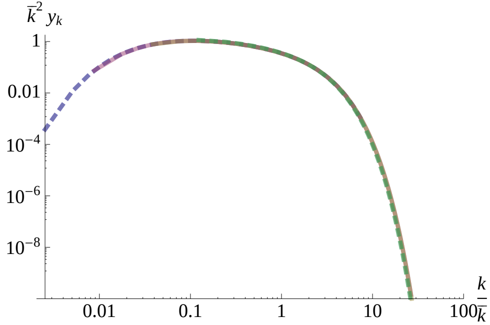
$$R_{i,j} = (i^{2/3} + j^{2/3})(i^{-1/2} + j^{-1/2}) \quad , \quad R_{\lambda i, \lambda j} = \lambda^{1/6} R_{i,j}$$



Number distributions at equally-spaced $\log w$ values, up to $w = 75$.

Scaling solution

- Shape stays the same, average size increases, $\bar{k}(w) \sim w^{6/5}$.



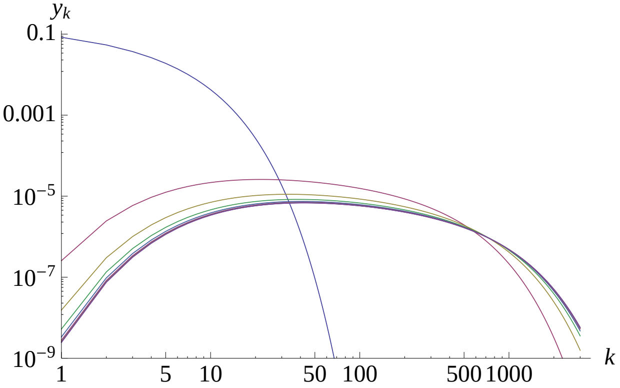
$\bar{k}^2 y_k$ versus $k/\bar{k}$, where $\bar{k} = 1/\sum_k y_k$.

- Attractor solution, depending only on large-k behaviour of kernel — reach this form (eventually) independent of initial conditions, small-k kernel.

Real-time behaviour

$$T_d \propto 1/a \quad \Rightarrow \quad w(T) \simeq \frac{2}{3}\frac{n_0\sigma_1 v_1}{H_0}\left(1 - \left(\frac{T}{T_0}\right)^{3/2}\right)$$

Most of build-up completes within one Hubble time.

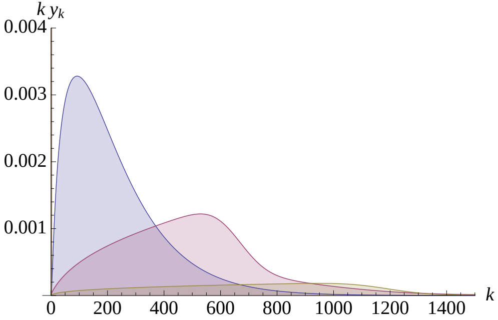


Number distributions at half *e*-folding time intervals

What if there's a bottleneck at small numbers? (cf. SM)

- If $R_{i,j}$ for small i, j is low enough, and $w_{\max}$ is small enough, never reach scaling regime
- Counter-intuitively, this can result in building up *larger* nuclei, since small + large fusions are less velocity-suppressed
- For $1 + k \to (k+1)$ fusions,

$$\frac{dk}{dw} \simeq R_{1,k} y_1 \propto k^{2/3} \Rightarrow k \sim \left(\int dw \, y_1 \right)^3$$



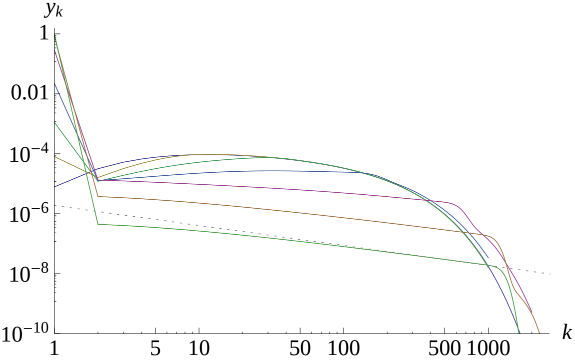
Mass distribution at $w = 25$ for $R_{1,1} = 4, 10^{-4}, 10^{-5}$

Power-law distribution

If we have a bath of small-number states throughout,

$$k \sim (w_{\max} - w_{\rm inj})^3 \;\Rightarrow\; -\frac{dw_{\rm inj}}{dk} \sim k^{-2/3} \;\Rightarrow\; y_k \sim k^{-2/3} \quad (k \text{ large})$$

Leads to power-law number distribution, qualitatively different from scaling solution



Number distribution at $w = 25$ for $R_{1,1} = 1, 10^{-1}, \ldots, 10^{-6}$

Summary: Synthesis of Nuclear Dark Matter

- Considered DM models with large bound states of strongly-interacting constituents
- Properties of sufficiently large 'dark nuclei' may obey geometrical scaling laws — this can determine number distribution from Big Bang Dark Nucleosynthesis
- If small-small fusions are fast enough, obtain universal scaling form of number distribution — may have $A \gtrsim 10^8$
- With a bottleneck at small numbers, may build up even larger nuclei, with power-law number distribution
- In both cases, most of build-up completes within a Hubble time
- Have assumed that deviations from geometrical cross sections are eventually unimportant — not necessarily the case!

Signatures of Nuclear Dark Matter

Most model-independent consequences:

- Soft scatterings coherently enhanced by A^2
  - Number density $\propto 1/A$, so total direct detection rate $\propto A$
  - For given direct detection rate, production at colliders etc. *suppressed*
- Possibility of new momentum-dependent form factors in direct detection
- Low-energy collective excitations may allow coherently enhanced inelastic scattering
- Inelastic self-interactions between DM may lead to indirect detection signals, or modify distribution in halos / captured distribution in stars

Many other model-dependent possibilities still to be investigated

Twin Higgs

- Proposed solution to 'little hierarchy' problem — stabilising the EW scale up to collider energies, $\Lambda \sim 5 - 10\,\mathrm{TeV}$.
- SM Higgs as PNGB of approximate $SU(4)$ global symmetry, broken down to $SU(3)$:

$$\mathcal{H} = (H_A, H_B) \quad , \quad V = \lambda(|\mathcal{H}|^2 - f^2/2)^2$$

- $SU(4)$ explicitly broken by SM gauge and Yukawa couplings — but, if A, B sectors related by approximate Z_2, this gives us back accidental $SU(4)$.
- Since observed light Higgs is SM-like, need to break Z_2 so that PNGB Higgs is mostly aligned with A,

$$f^2 = v_A^2 + v_B^2 \quad , \quad v_A^2 \ll v_B^2$$

The Minimal ('Fraternal') Twin Higgs

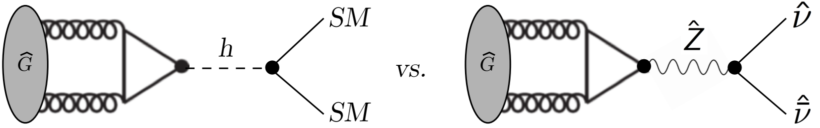
- Idea: introduce only those B-sector states we need in order to have acceptable tuning up to $\Lambda \sim \mathcal{O}(10\,\mathrm{TeV})$.
- Main contributions from SM: top, $SU(2)_L$ gauge bosons, QCD (two loops).
- B sector:
  - $\hat{t}$ with $\hat{y}_t \simeq y_t$.
  - $SU(2)'_L$ gauge group, $\hat{g}_2 \simeq g_2$ (and so $\hat{b}$ partner of $\hat{t}$).
  - $SU(3)'$ gauge group, roughly similar confinement scale.
- For anomaly cancellation, need to have $\hat{b}_R$ and $(\hat{\tau}, \hat{\nu})$ lepton doublet.
- As long as $\hat{y}_b, \hat{y}_\tau \ll y_t$, no effect on tuning.

Fraternal Twin Higgs — Nuclear DM?

- Stable states: $\hat{B} = \hat{b}\hat{b}\hat{b}$ baryons, $\hat{\tau}$, $\hat{\nu}$
- In analogy to SM, $\hat{B}$ asymmetry $\Rightarrow$ asymmetric relic $\hat{B}$ population.
- Possibility of $\hat{B}$ bounds states? Nuclear matter? Synthesis problem: de-excitation from small-number fusions.
- Radiative capture via $SU(2)'_L$ too slow. Introducing gauged $U(1)'_Y$, bound state formation of $\hat{B}\hat{B}$ suppressed compared to non-identical fermions — still appears to be too slow.
- Models with additional twin quark generation have a better chance of forming bound states.

Fraternal Twin Higgs — Cosmology

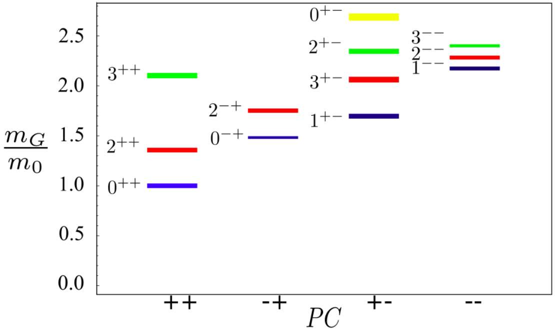
- $\hat{B}$ nucleons as viable ADM
- Analogously to Lee-Weinberg bound, $\hat{\tau}$ abundance sub-DM requires either $\hat{m}_\tau \lesssim \mathrm{eV}$, or $\hat{m}_\tau \gtrsim 70\,\mathrm{GeV}$. For $m_{\hat{\tau}} \sim 70\,\mathrm{GeV}$, have symmetric $\hat{\tau}$ DM
- Interesting effects related to $SU(3)'$ phase transition:
  - If there are light hidden sector states (e.g. $\hat{\nu}$), does twin sector entropy end up there or in SM?



$\hat{g}$ entropy $\Rightarrow \Delta N_{eff} \simeq 0.5$, $\quad \hat{b} + \hat{g}$ entropy $\Rightarrow \Delta N_{eff} \simeq 1$

$SU(3)'$ phenomenology

Dependence on glueball/meson spectrum, decay constants, transition matrix elements.



Stable glueball spectrum in pure $SU(3)$ *(Morningstar and Peardon)*

If there are no light hidden sector states, Higgs mixing portal $\Rightarrow$ possibility of (meta)-stable glueball states.

More $SU(3)'$ phenomenology

- Pure-glue case appropriate to heavy quarks — light quarks generally imply faster energy loss to hidden sector.
- Dynamics of phase transition: only heavy quarks $\Rightarrow$ first order phase transtion $\Rightarrow$ entropy production, gravitational radiation.
- Effect of CP violation in twin sector (effect on SM EDMs small) — e.g. $\hat{\theta}$ angle?
- Summary:
  - Fraternal twin Higgs provides motivated, 'minimal' example of strongly-coupled hidden sector
  - Demonstrates that assumptions about SM portal may have important consequences for cosmology of hidden sector phase transition, in some regions of parameter space

BACKUPS

Freeze-out of dissociations

- Overall forward rate for $k + (A-k) \leftrightarrow A$ is

$$\langle \sigma v \rangle_{(k,A-k)\to A} n_k n_{A-k} - \Gamma_{A\to(k,A-k)} n_A$$

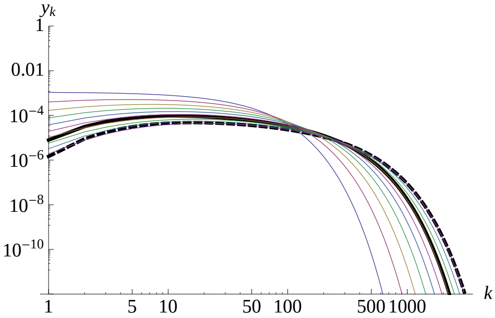
- Fusions dominate over dissociations if

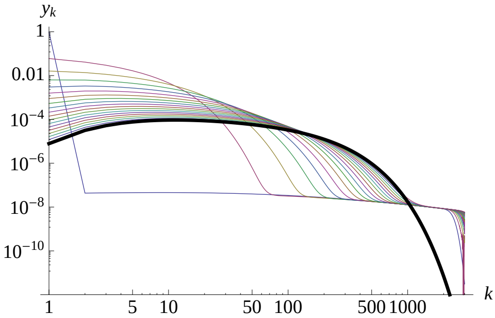
$$\frac{\langle \sigma v \rangle n_k n_{A-k}}{\Gamma n_A} \gg 1 \quad \Leftarrow \quad n_0 \Lambda^3 e^{\Delta B/T} \gg (\text{const. wrt } T)$$

- Since $n_0\Lambda^3 \ll 1$, equality is at $T \ll \Delta B$
- Go from equality to $n_0\Lambda^3 e^{\Delta B/T} \gg$ const. within small fraction of Hubble time.

Independence of initial conditions

Exponentially-falling ICs,
$y_k(0) \propto e^{-k/30}$





'Broad tail' ICs,
$y_1(0) = 0.97$

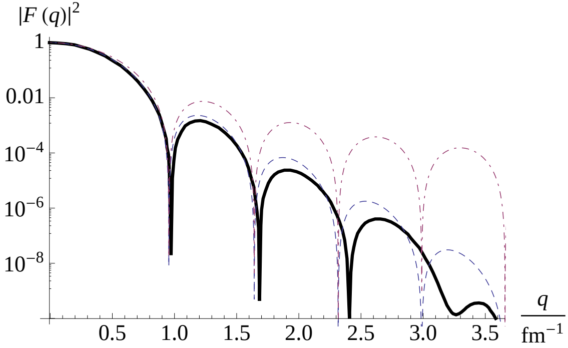
Form factors in scattering

If $R_{\mathrm{DM}} > (\Delta p)^{-1}$, probe DM form factor

Sharp boundary $\Rightarrow$ spherical Bessel function form factor

$$F(q) = \frac{qR\cos(qR) - \sin(qR)}{(qR)^3} \sim \frac{1}{(qR)^2}$$

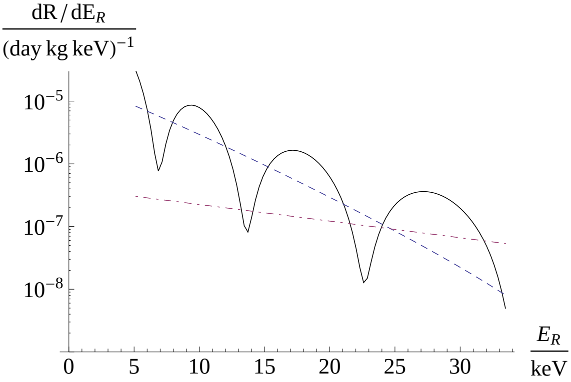
If skin depth etc of DM is smaller than SM nuclear scales, good approximation



e.g. form factor for nuclear charge distribution of ^{70}Ge.

Coherent enhancement

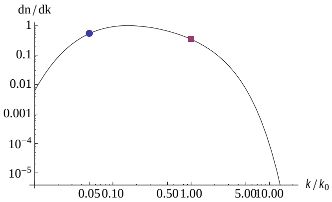
e.g. dim-6 interactions: $\sigma(q=0) \sim A^2 N^2 \frac{\mu^2}{\Lambda^4}$



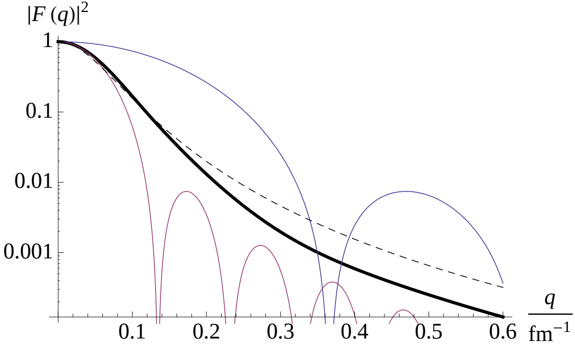
Energy recoil spectrum for state with $R = 50\,\mathrm{fm}$, $A = 3 \times 10^6$, each constituent with $M_1 = 20\,\mathrm{GeV}$, $\sigma_n = 2 \times 10^{-13}\,\mathrm{pb}$. Blue, red curves for $20\,\mathrm{GeV}$, $1\,\mathrm{TeV}$ WIMP ($\sigma_n = 10^{-9}\,\mathrm{pb}$).

Effective form factor from distribution of sizes

Distribution over radii averages out peaks and troughs



Effective form factor similar to intermediate-mass mediator



Dependence on DM velocity distribution

Differential event rate:

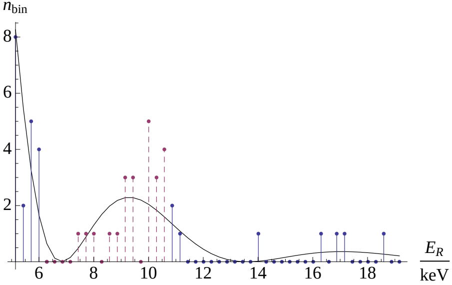
$$\frac{dR}{dE_R} \propto \left(\int_{|v|>v_{\min}} d^3v \frac{f(v)}{v} \right) F_N(q)^2 F_D(q)^2$$

with

$$v_{\min} \propto \sqrt{E_R}$$

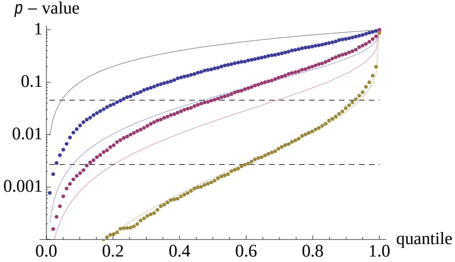
Consequence: ignoring $F_N(q), F_D(q)$, energy recoil spectrum is non-increasing with E_R.

Rising energy recoil spectrum



Samples from recoil spectrum,
$R_{\rm DM} = 50\,{\rm fm}$

p-value CDFs for 30, 50, 100 events



Astrophysical consequences

- Self-interaction cross section & DM halo constraints?

$$\frac{\sigma_{AA}}{m_A} \simeq \frac{0.05\,\mathrm{barn}}{\mathrm{GeV}} A^{-1/3} \left(\frac{1\,\mathrm{GeV}}{M_1}\right)^{1/3} \left(\frac{1\,\mathrm{GeV\,fm^{-3}}}{\rho_b}\right)^{2/3}$$

  Cross sections saturate at geometrical value, so can be safe from elastic-scattering constraints

- Proportion of DM mass density released by fusions:

$$\langle \sigma v \rangle n_A t_{\mathrm{gal}} \frac{\Delta BE}{M_A} \sim 10^{-3} A^{-2/3} \frac{\rho_{\mathrm{DM}}}{0.3\,\mathrm{GeV\,cm^{-3}}}$$

  For comparison, annihilating symmetric DM has

$$\langle \sigma v \rangle n_X t_{\mathrm{gal}} \sim 3 \times 10^{-8} \left(\frac{100\,\mathrm{MeV}}{m_X}\right) \left(\frac{\langle \sigma v \rangle_X}{\mathrm{pb}}\right)$$

  Possibility of detectable annihilation-type signal from fusions: depends on SM injection channels etc.