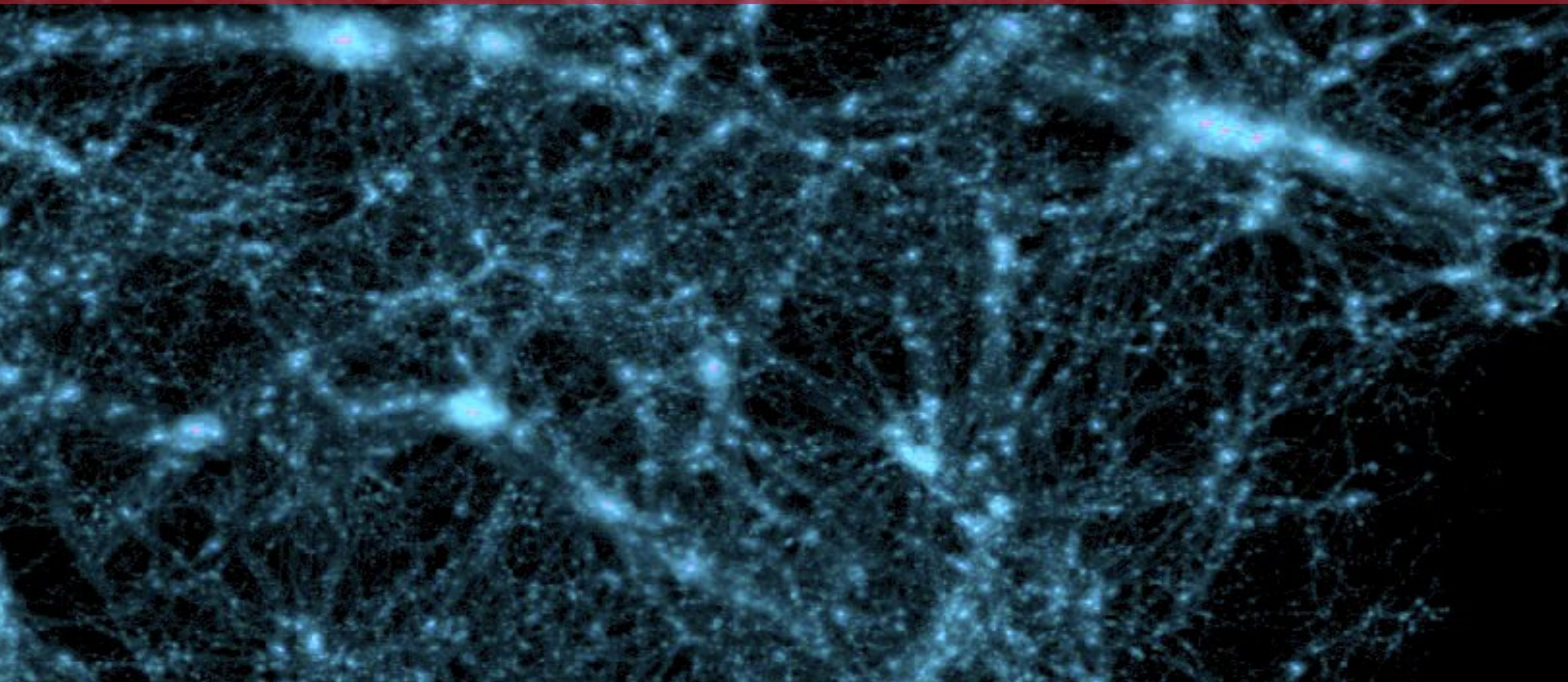
Dark Nuclei



William Detmold, MIT

Lattice for Beyond the Standard Model Physics, Livermore, April 23rd 2015

Dark nuclei

- Dark nuclear physics of QCD like theory with N_c=2 and N_f=2
- Based on two papers in collaboration with
  Matthew McCullough & *Andrew Pochinsky*
  - *Dark Nuclei I: Cosmology and Indirect Detection* —1406.2276
  - *Dark Nuclei II: Nuclear Spectroscopy* in Two-Colour QCD —1406.4116
- Motivation
  - Dark matter model building:
    - Binding energy is a new scale
    - Interesting new phenomenology
  - Understand what is "nuclear physics" in a general context
    What generic feature, what is special?

Two-colour QCD

- Two-colour QCD with two flavours of fundamental fermions
  - Numerically feasible (simpler than QCD)
  - Emergent complexity: novel phenomenological aspects
- Single hadron aspects already considered in DM context
  [Lewis et al., Neil & Buckley, Hietanen et al.]
- Also lattice investigations of quenched $N_c=4$ QCD and other theories in this context
  - Sigma terms, polarisabilities,…

Symmetries of two-colour QCD

- Global flavour symmetry $SU(2)_L \times SU(2)_R$ enlarges to SU(4)
  - Pseudo-reality of SU(2) - left and right handed quarks can be combined into multiplets

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -i\sigma_2 C \bar{u}_R^\top \\ -i\sigma_2 C \bar{d}_R^\top \end{pmatrix} \qquad \Psi \xrightarrow{SU(4)} \exp\left(i \sum_{j=1}^{15} \theta_j T_j \right) \Psi$$

- Strong interactions result in condensate that spontaneously breaks the global symmetry: SU(4)→ Sp(4)~SO(5) [Peskin 1980]
- Numerical calculations have significant explicit symmetry breaking: $m_u = m_d \sim \Lambda_{QC_2D}$

Spectrum

- Simplest colour singlets
  - "Pions": $\pi^- \sim \bar{u}\gamma_5 d$, $\pi^0 \sim \bar{u}\gamma_5 u + \bar{d}\gamma_5 d$, $\pi^+ \sim \bar{d}\gamma_5 u$ $J^P=0^-$
  - (anti-)"Nucleons": ud, $\bar{u}\bar{d}$ $J^P=0^+$

  } Degenerate SO(5) multiplet

  - "Rhos": $\rho^- \sim \bar{u}\gamma_\mu d$, $\rho^0 \sim \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$, $\rho^+ \sim \bar{d}\gamma_\mu u$ $J^P=1^-$
  - (anti-)"Deltas": $u\gamma_\mu\gamma_5 d$, $\bar{u}\gamma_\mu\gamma_5\bar{d}$ $J^P=1^+$

  } Degenerate SO(5) multiplet

  - Axial vector, scalar, tensor mesons + associated baryons
- Single hadron spectrum studied by [Hietanen et al. 1404.2794]
  - Pion multiplet are pseudoGoldstone bosons of χSB: SU(4) → Sp(4)
  - Rho stable for masses considered

Spectrum

- Colour singlets can combine
  - Two-, three-, … particle scattering states
  - "Nuclei" for sufficiently attractive interactions–<u>not</u> *a priori* obvious
- Two "pions" combine to give 25 of states: $\mathbf{5}\otimes\mathbf{5}=\mathbf{1}\oplus\mathbf{10}\oplus\mathbf{14}$
  - J=0 systems, contains B=2,1,0,-1,-2 states
- "pion"+ "rho": J=1 systems with same flavour breakdown

$$\boldsymbol{D}^\mu = \begin{pmatrix} S^\mu_+ & D^\mu_{2,0} & D^\mu_{1,0} & D^\mu_{1,-1} & D^\mu_{1,1} \\ \overline{D}^\mu_{2,0} & S^\mu_- & D^\mu_{-1,0} & D^\mu_{-1,-1} & D^\mu_{-1,1} \\ \overline{D}^\mu_{1,0} & \overline{D}^\mu_{-1,0} & S^\mu_0 & D^\mu_{0,-1} & D^\mu_{0,1} \\ \overline{D}^\mu_{1,-1} & \overline{D}^\mu_{-1,-1} & \overline{D}^\mu_{0,-1} & S^\mu_B & D^\mu_{0,2} \\ \overline{D}^\mu_{1,1} & \overline{D}^\mu_{-1,1} & \overline{D}^\mu_{0,1}+ & \overline{D}^\mu_{0,2} & S^\mu_{\overline{B}} \end{pmatrix}$$

$$\boldsymbol{D}^\mu_{\mathbf{1}} = \mathrm{Tr}(\boldsymbol{D}^\mu)\ ,$$

$$\boldsymbol{D}^\mu_{\mathbf{10}} = \frac{i}{2}\left(\boldsymbol{D}^\mu - \boldsymbol{D}^{\mu T}\right)\ ,$$

$$\boldsymbol{D}^\mu_{\mathbf{14}} = \frac{1}{2}\left(\boldsymbol{D}^\mu + \boldsymbol{D}^{\mu T}\right) - \frac{1}{5}\mathrm{Tr}(\boldsymbol{D}^\mu)\mathbb{1}_5$$

- Higher body systems: J=0,1, flavour = $\underbrace{\Box\Box\Box\cdots\Box\Box}_{n}$, n=2,…,8

Simulations

- Wilson gauge and fermion actions
- HMC using modified **`chroma`**
- 4 lattice spacings (β), 6 masses
  - Isospin symmetric
- 3 or 4 volumes per choice (β ,m_0)
- Long streams of configurations

Label	β	m_0	$L^3 \times T$	N_{traj}
A	1.8	−1.0890	$12^3 \times 72$	5,000
			$16^3 \times 72$	4,120
			$20^3 \times 72$	3,250
B	2.0	−0.9490	$12^3 \times 48$	10,000
			$16^3 \times 48$	4,000
			$20^3 \times 48$	3,840
			$24^3 \times 48$	2,930
C	2.0	−0.9200	$12^3 \times 48$	10,000
			$16^3 \times 48$	9,780
			$20^3 \times 48$	10,000
D	2.0	−0.8500	$12^3 \times 48$	9,990
			$16^3 \times 48$	5,040
			$16^3 \times 72$	5,000
			$20^3 \times 48$	5,000
			$24^3 \times 48$	5,050
E	2.1	−0.7700	$12^3 \times 72$	5,000
			$16^3 \times 72$	5,000
			$20^3 \times 72$	4,300
F	2.2	−0.6000	$12^3 \times 72$	5,000
			$16^3 \times 72$	5,000
			$20^3 \times 72$	5,000
			$24^3 \times 72$	5,070

Single hadron systems

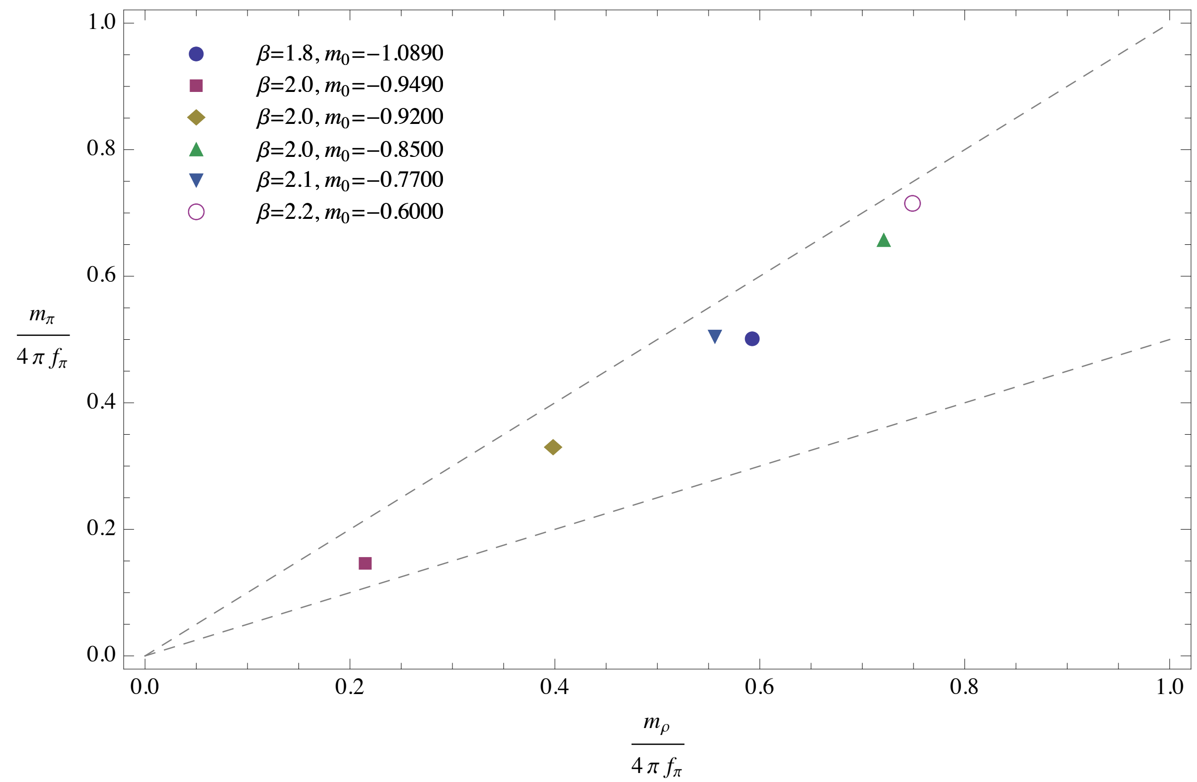
- Single hadron systems needed to set scales, characterise theory

Ensemble	β	m_0	$a\ m_q$	$a\ f_\pi$	$a\ m_\pi$	m_π/m_ρ
A	1.8	-1.0890	0.1299(1)(1)	0.259(1)(1)	0.8281(8)(5)	0.844(1)(2)
B	2.0	-0.9490	0.0280(2)(4)	0.101(3)(5)	0.347(6)(13)	0.663(9)(10)
C	2.0	-0.9200	0.0823(3)(3)	0.159(2)(4)	0.609(3)(4)	0.826(3)(4)
D	2.0	-0.8500	0.1911(3)(2)	0.2156(16)(11)	0.9151(13)(6)	0.910(2)(2)
E	2.1	-0.7700	0.1442(1)(1)	0.1582(1)(1)	0.7450(9)(7)	0.904(2)(2)
F	2.2	-0.6000	0.2277(2)(1)	0.1525(5)(7)	0.8805(7)(5)	0.951(3)(3)

- Scale set by demanding f_π=246 GeV @ fixed m_π/m_ρ=0.9

β	$a\ [10^{-3}\ \text{fm}]$
1.8	0.35(2)
2.0	0.24(1)
2.1	0.19(1)
2.2	0.14(2)

- Single hadron volume effects are small for most ensembles



Multi-baryon spectrum

- Extract spectrum of multi-baryon states from correlators

$$C_{nN}(t) = \left\langle 0 \left| \left(\sum_{\mathbf{x}} \mathcal{O}_N^{\mathcal{P}}(\mathbf{x}, t) \right)^n \left(\mathcal{O}_N^{\mathcal{S}\dagger}(\mathbf{x}_0, t_0) \right)^n \right| 0 \right\rangle$$

$$C_{nN,\Delta}^{(i,j)}(t) = \left\langle 0 \left| \left(\sum_{\mathbf{x}} \mathcal{O}_N^{\mathcal{P}}(\mathbf{x}, t) \right)^n \sum_{\mathbf{x}} \mathcal{O}_{\Delta_j}^{\mathcal{P}}(\mathbf{x}, t) \left(\mathcal{O}_N^{\mathcal{S}\dagger}(\mathbf{x}_0, t_0) \right)^n \mathcal{O}_{\Delta_i}^{\mathcal{S}\dagger}(\mathbf{x}_0, t_0) \right| 0 \right\rangle$$

$$\mathcal{O}_{\{N,\Delta_i\},s}(\mathbf{x}, t) = \psi_u^{\top}(\mathbf{x}, t)(-i\sigma_2)C\{1, \gamma_i\gamma_5\}\psi_d(\mathbf{x}, t)$$

  - Local and smeared sources and sinks

- Need to be careful of thermal effects

$$C_{X,Y}^{s,s'}(t, T; \mathbf{0}) \stackrel{T\to\infty}{\longrightarrow} \sum_n Z_{X,s}^{(n)\dagger} Z_{Y,s'}^{(n)} e^{-E_n t}$$

- Effective mass $M(t) = \ln\left[C(t)/C(t+1)\right] \stackrel{t\to\infty}{\longrightarrow} E_0$

Multi-baryon contractions

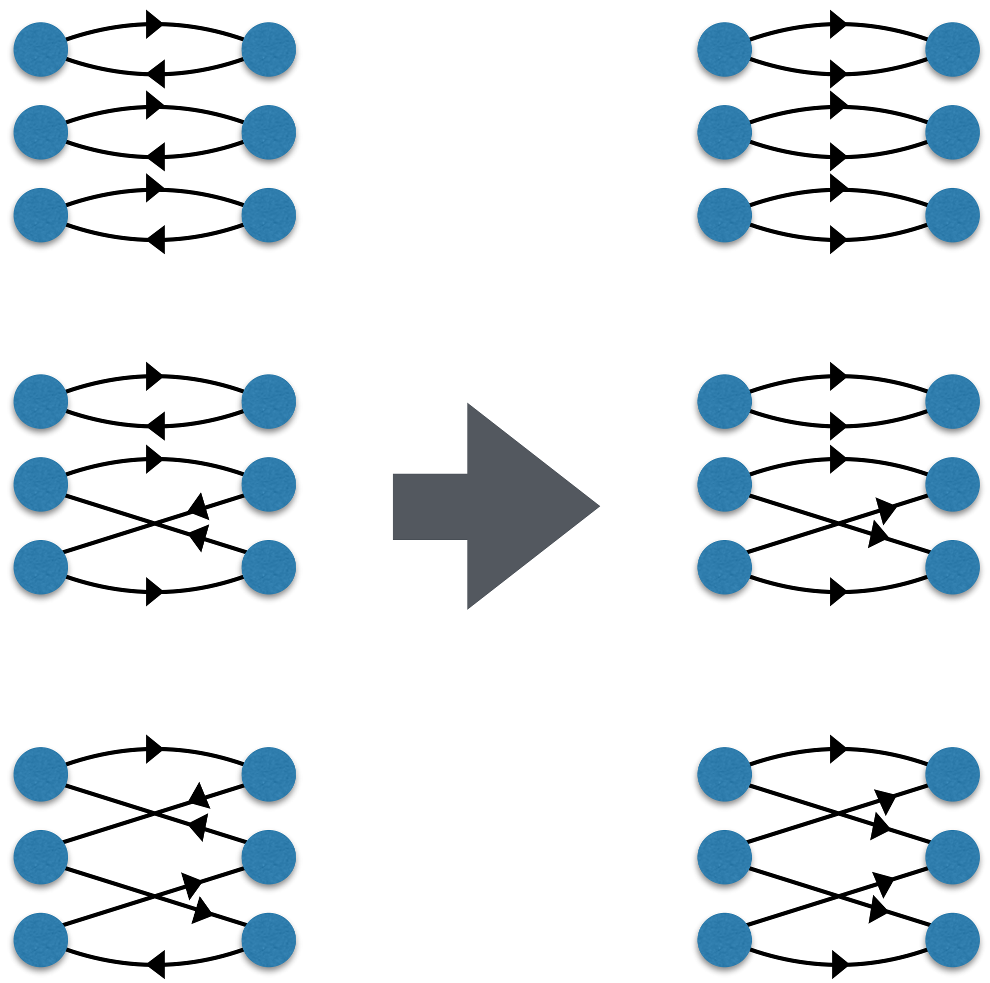
- SU(2) multi-baryon contractions equivalent to maximal isospin multi-meson contractions
  - Clear from degeneracies but explicitly

$$S(y,x) = C^\dagger(-i\sigma_2)^\dagger S(x,y)^T(-i\sigma_2)C$$
$$S(y,x) = \gamma_5 S^\dagger(x,y)\gamma_5$$

first relation specific to N_c=2

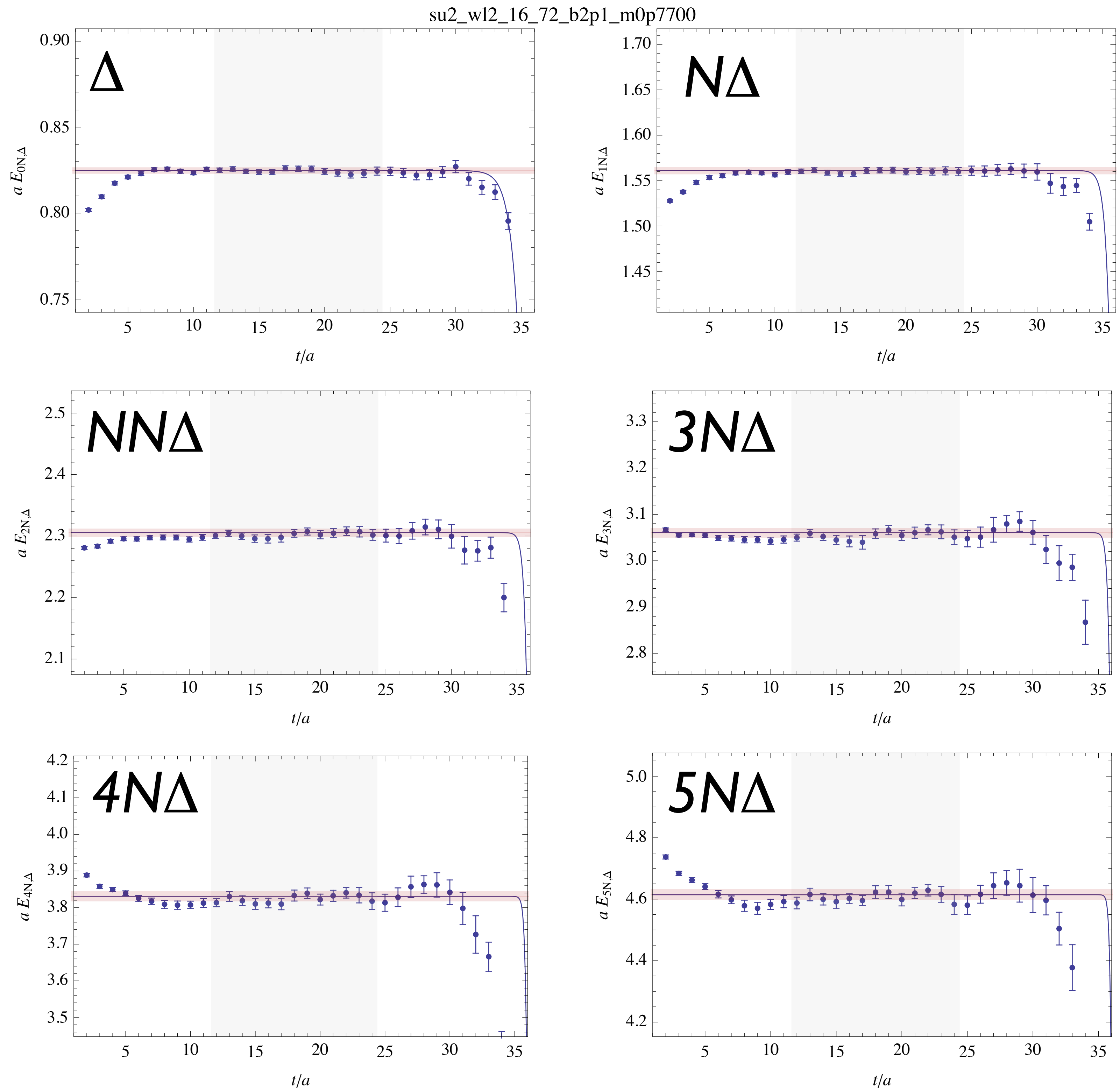
- Use algorithms from N_c=3 QCD
[WD & Savage 2011,;WD, Orginos, Shi 2012]

- (n-1)$N\Delta$ ~ mixed pion-kaon contractions
[WD & Smigielski 2011]



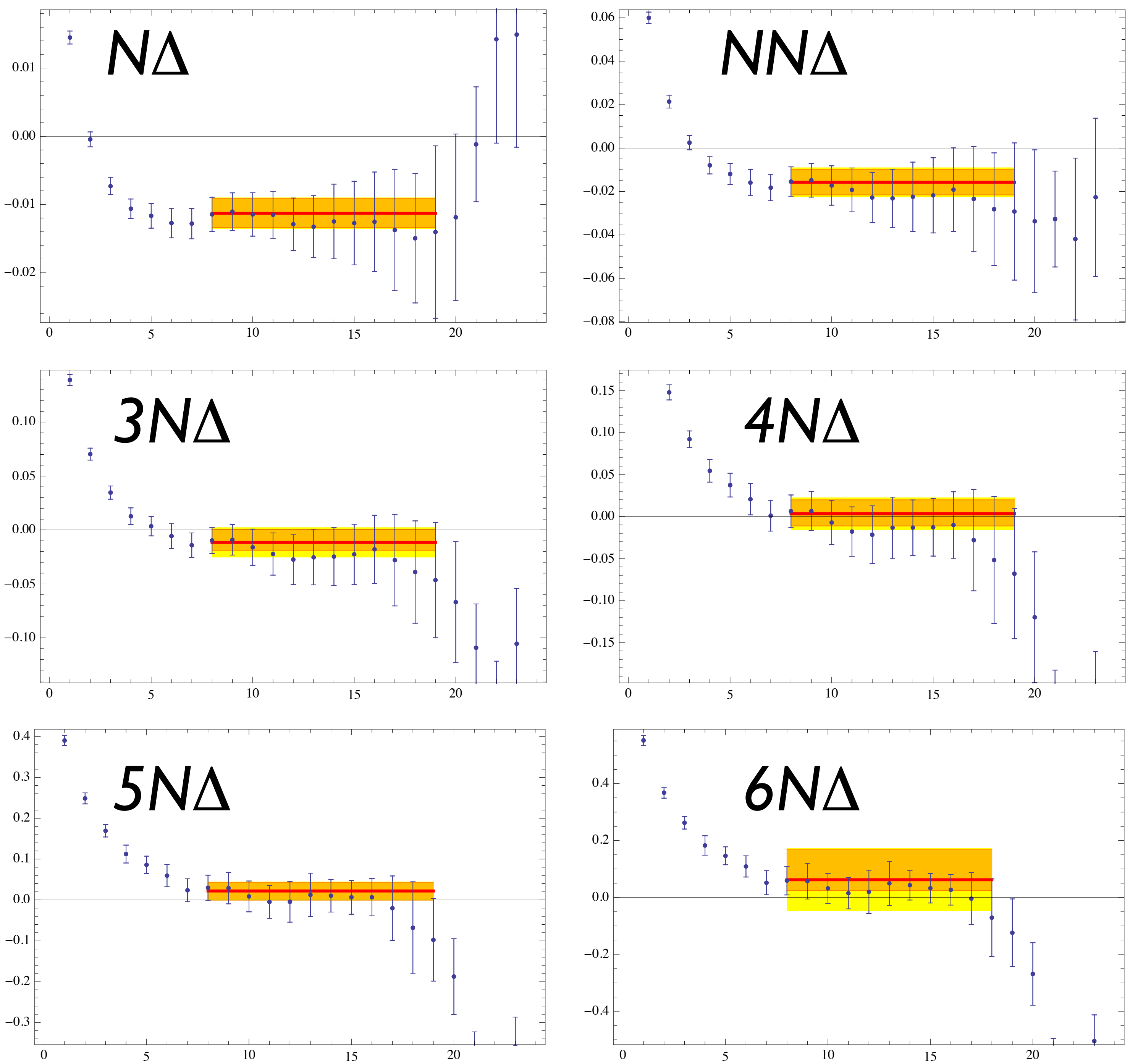
Ex:: three types of contractions for I=3 $\pi\pi\pi$ and *NNN*

Example effective mass plots



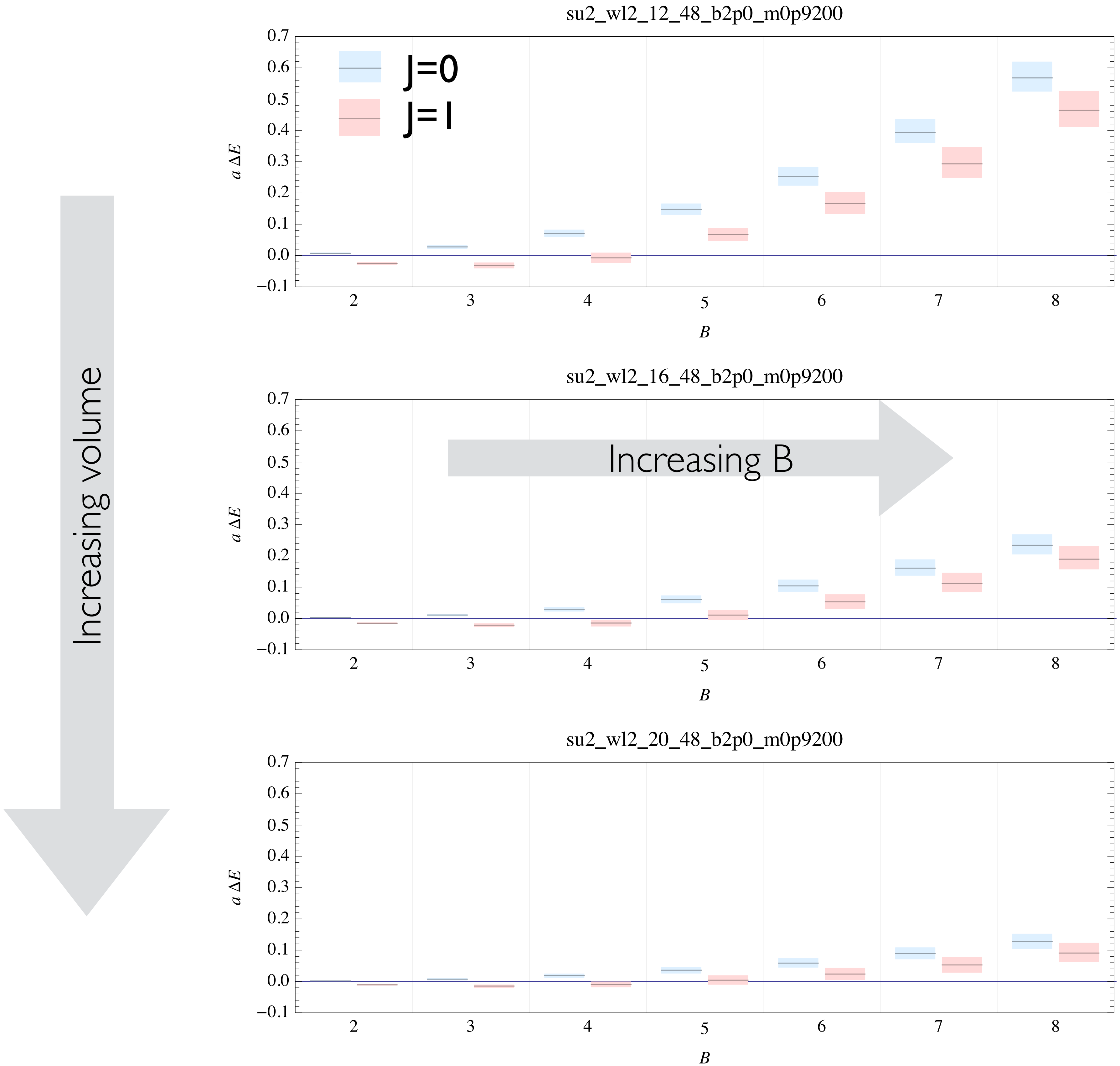
Example effective mass shift plots

$$R_{nN}(t) \equiv \frac{C_{nN}(t)}{[C_{1N}(t)]^n}, \qquad R_{nN,\Delta}(t) \equiv \frac{\sum_i C^{(i,i)}_{nN,\Delta}(t)}{[C_{1N}(t)]^n \sum_i C^{(i,i)}_{0N,\Delta}(t)}$$



su2_wl2_20_48_b2p0_m0p9200

Energy shifts for different volumes



To be bound or not to be bound…

- Bound/scattering state hypotheses (Lüscher):

$$H_1: \quad \Delta E_{\text{bound}}(L) = -\Delta E_\infty \left[1 + C\,\frac{e^{-\kappa L}}{L}\right],$$

$$H_2: \quad \Delta E_{\text{scatter}}(L) = \frac{2\pi A}{\mu L^3}\binom{n}{2}\left[1 - \left(\frac{A}{\pi L}\right)\mathcal{I} + \left(\frac{A}{\pi L}\right)^2\left[\mathcal{I}^2 + (2n-5)\mathcal{J}\right]\right] + \frac{B}{L^6}$$

- Assess support for each hypothesis using the Bayes factor

$$K = \frac{P(D|H_1)}{P(D|H_2)} = \frac{\int P(D|H_1, p_1)P(p_1|H_1)dp_1}{\int P(D|H_2, p_2)P(p_2|H_2)dp_2}$$

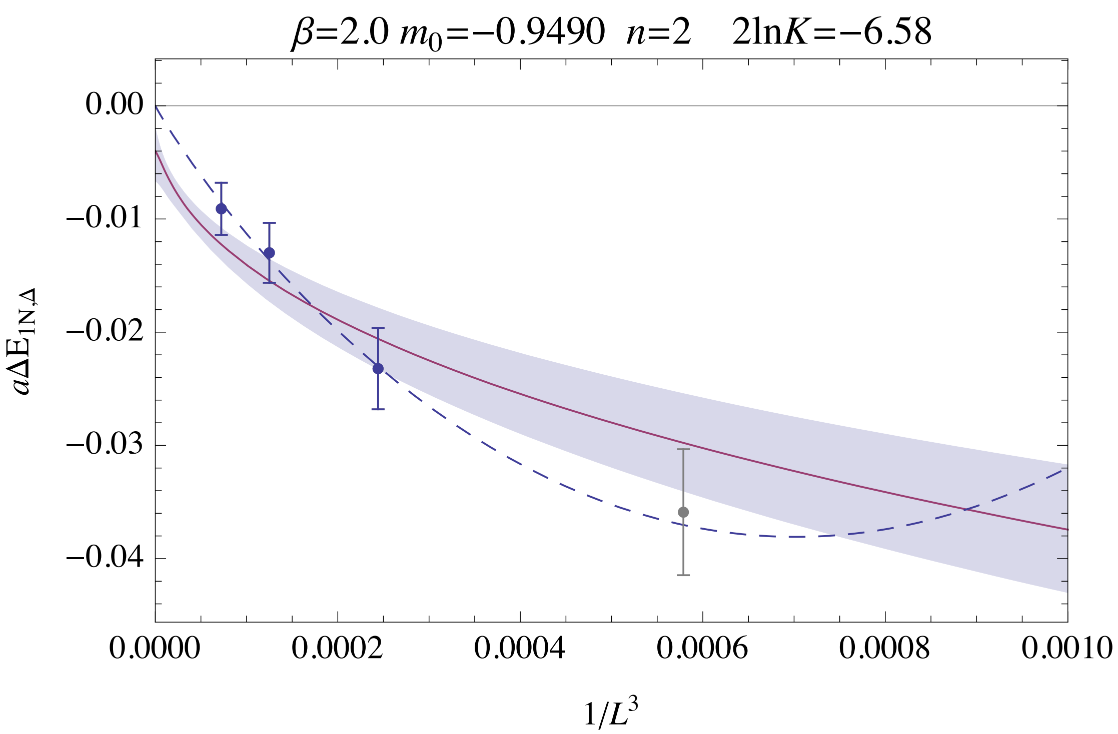
where $\log P(D|H_i, p_i) = -\frac{1}{2}\sum_{j=1}^{N}\frac{[d_j - H_i(x_j; p_i)]^2}{\sigma_j^2}$

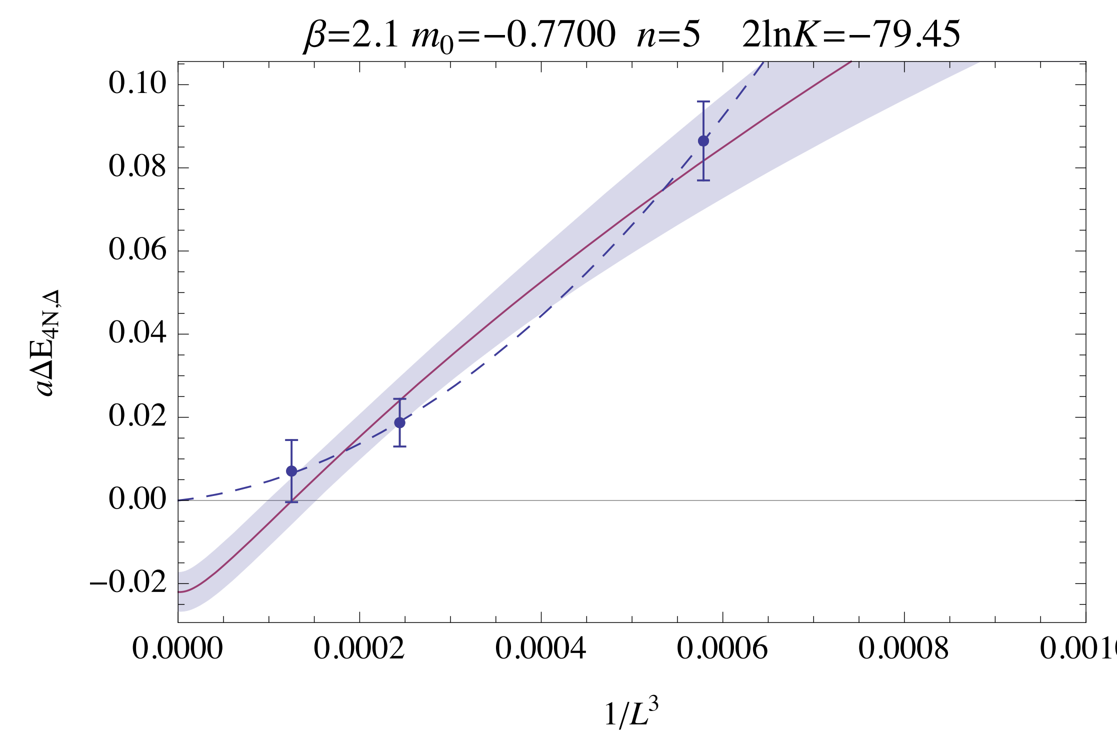
and $P(p_i|H_i)$ are broad prior distributions for convergence

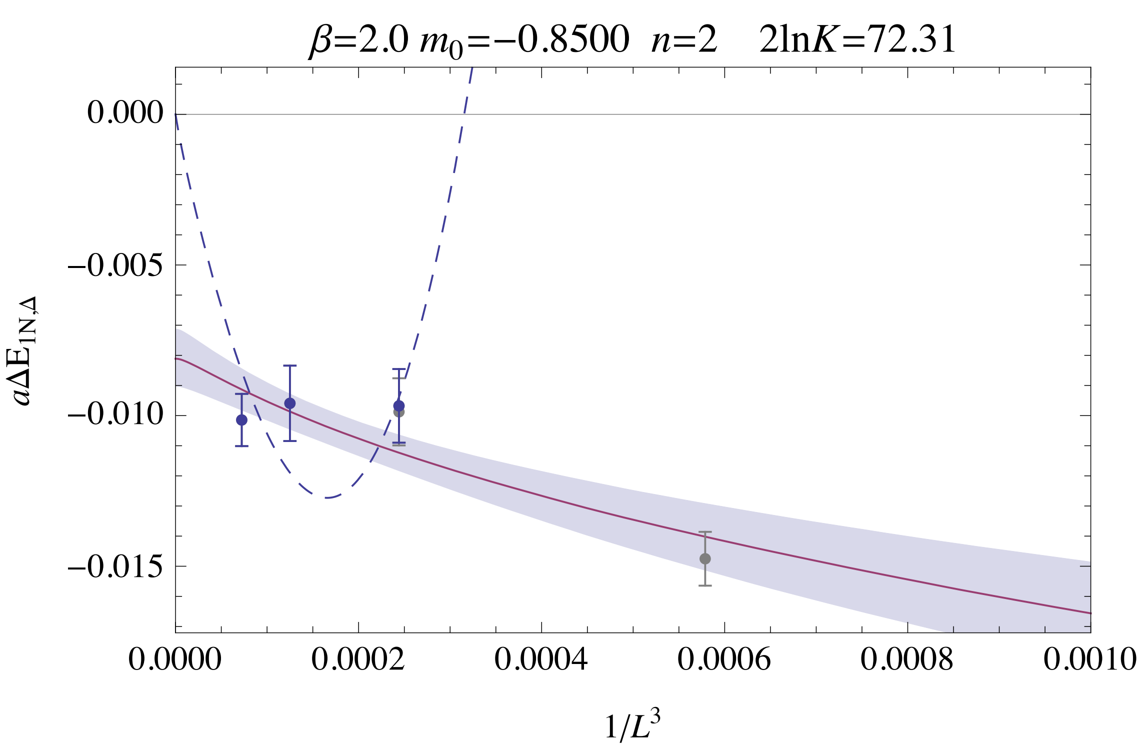
- If 2 *ln[K]* > 6 : "strong evidence" of preference for H_1 over H_2 then ask what are the bounds on the binding energy

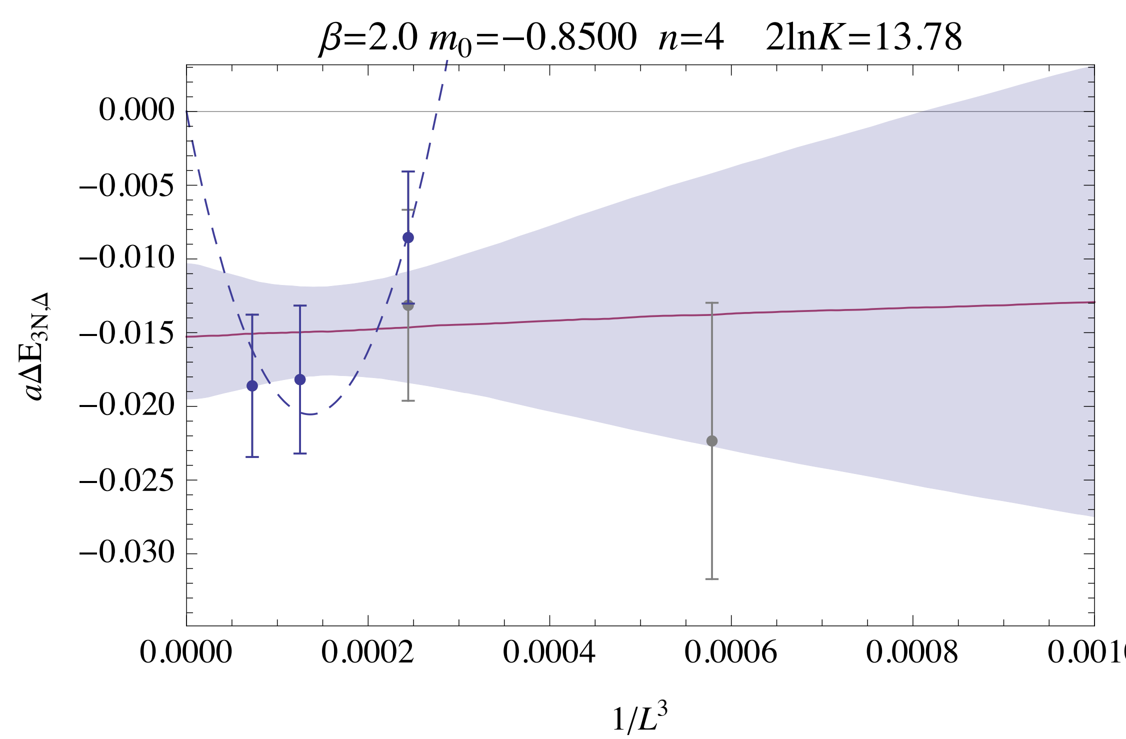
Kass and Raftery (1995). Journal of the American Statistical Association 90 (430): 791.

Infinite volume extrapolations



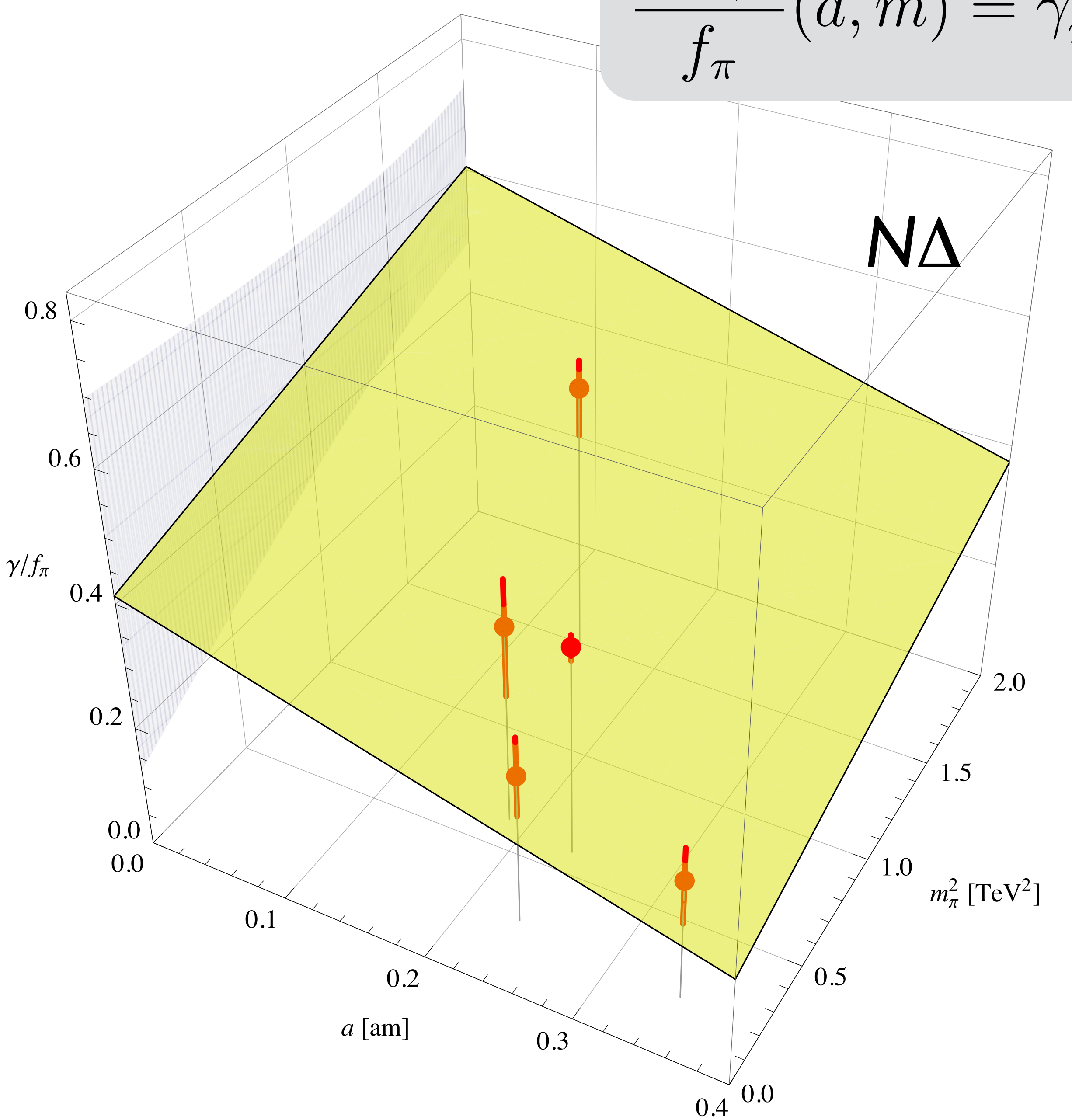


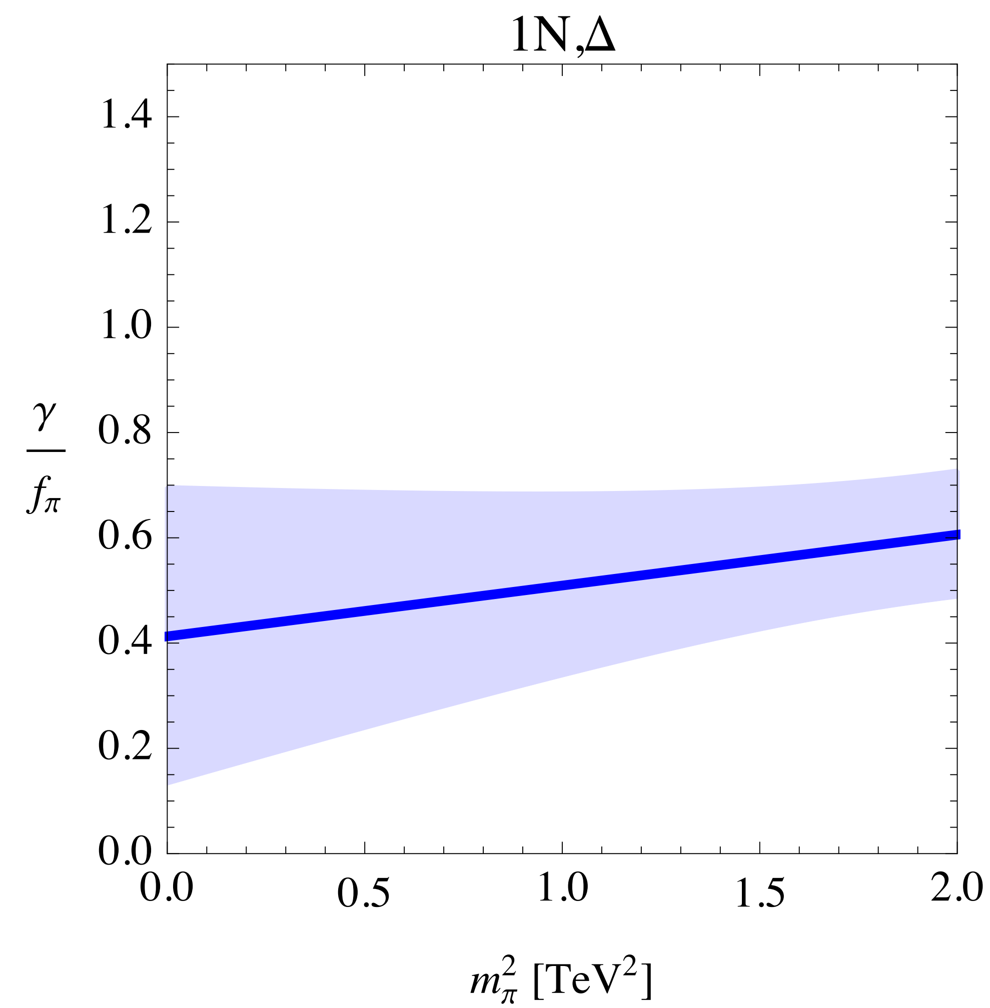




- Simple continuum limit extrapolation of binding momentum, γ

$$\frac{\gamma_{nN,\Delta}}{f_\pi}(a,m) = \gamma^{(0)}_{nN,\Delta} + a\ \delta^{(a)}_n + m_\pi{}^2 \delta^{(m)}_n$$

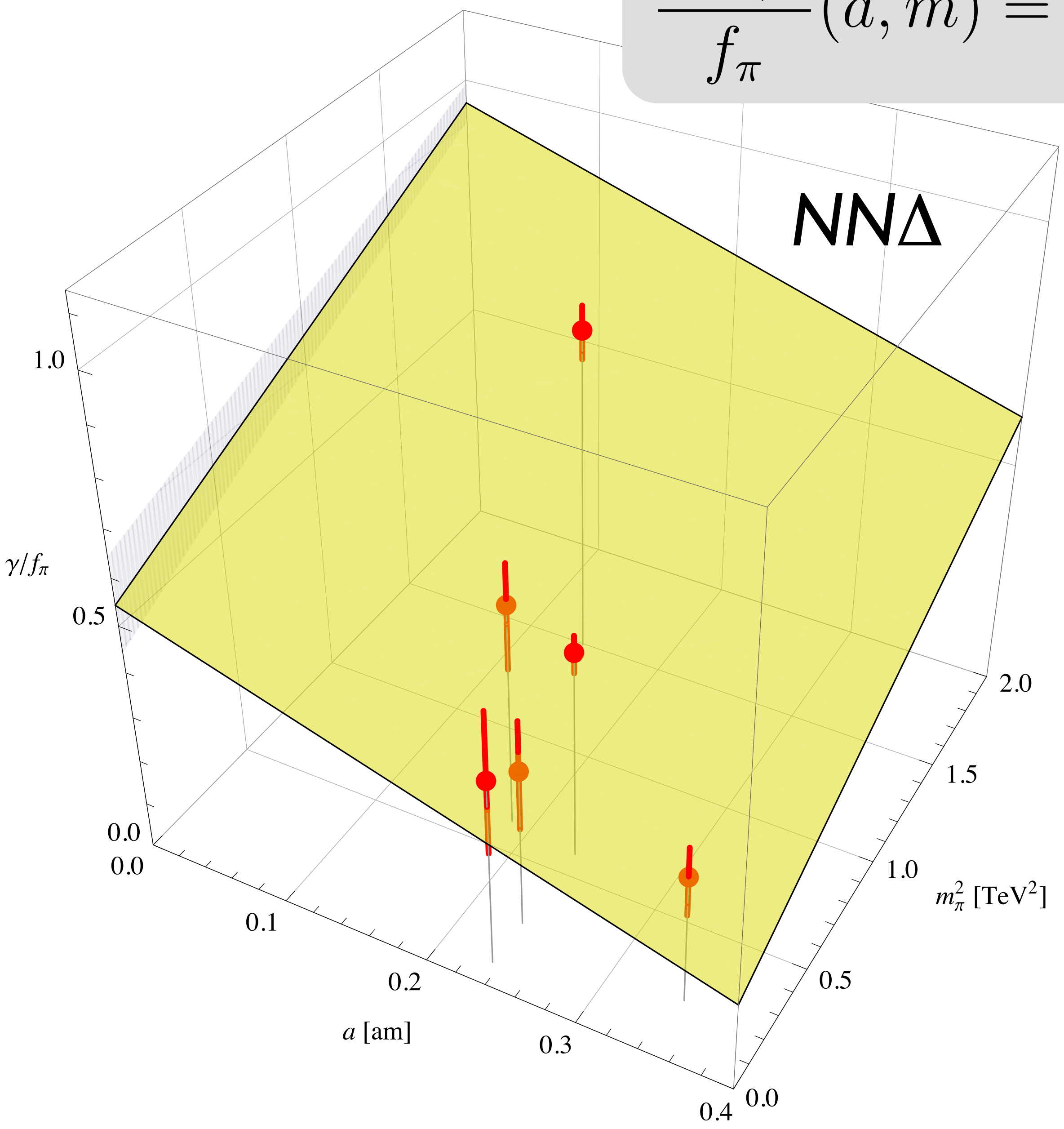


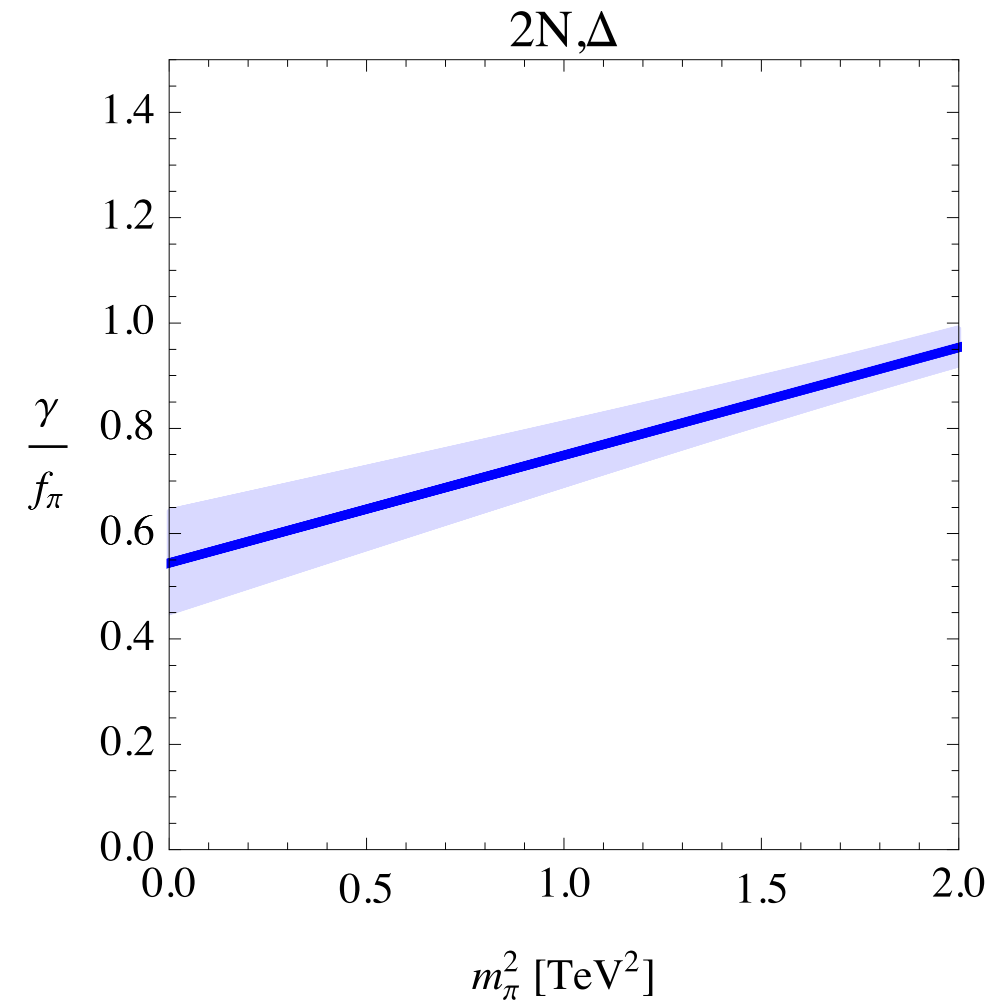


NB: physical scale set by demanding f_π=246 GeV (arbitrary)

- Simple continuum limit extrapolation of binding momentum, γ

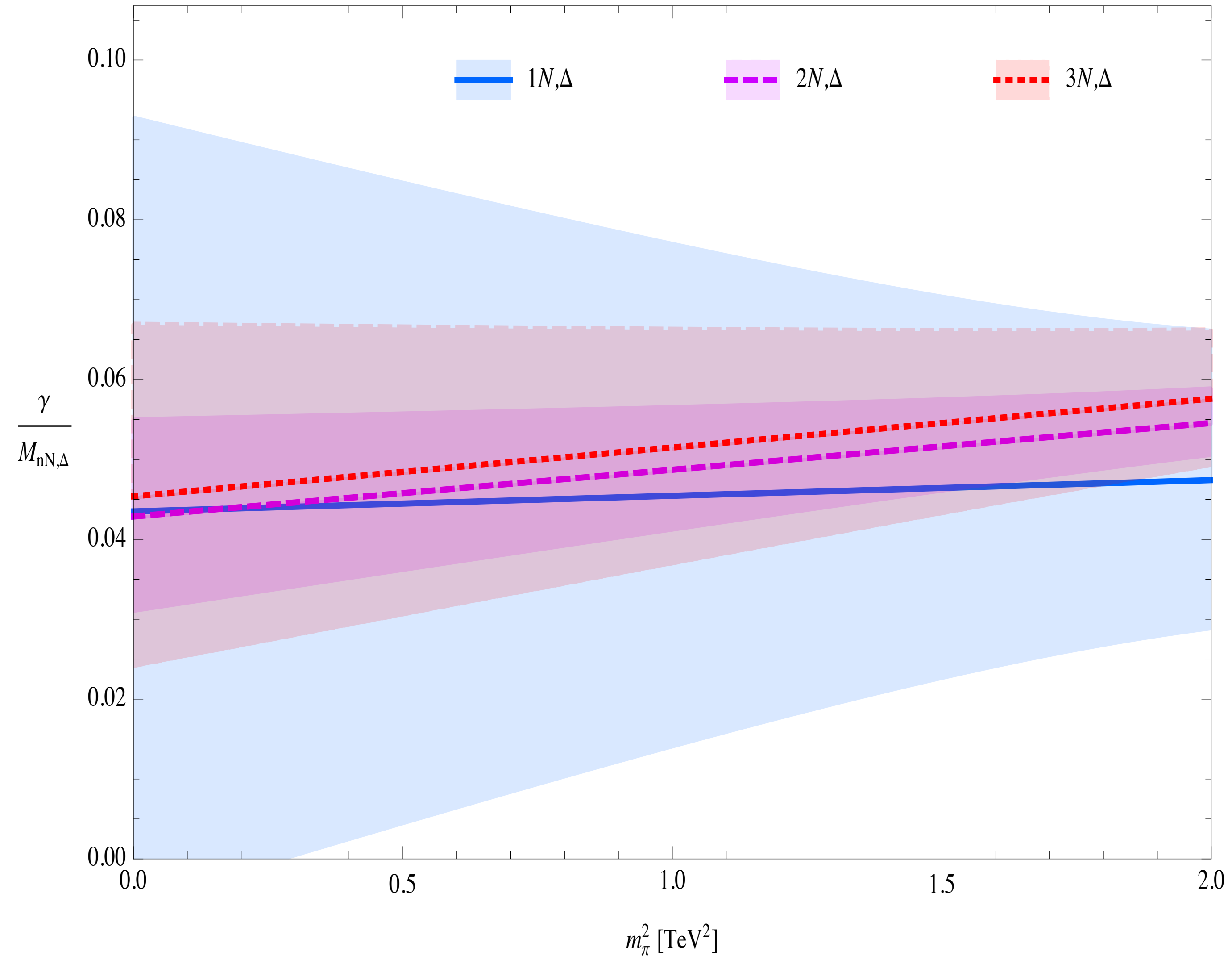
$$\frac{\gamma_{nN,\Delta}}{f_\pi}(a,m) = \gamma^{(0)}_{nN,\Delta} + a\,\delta^{(a)}_n + {m_\pi}^2\delta^{(m)}_n$$



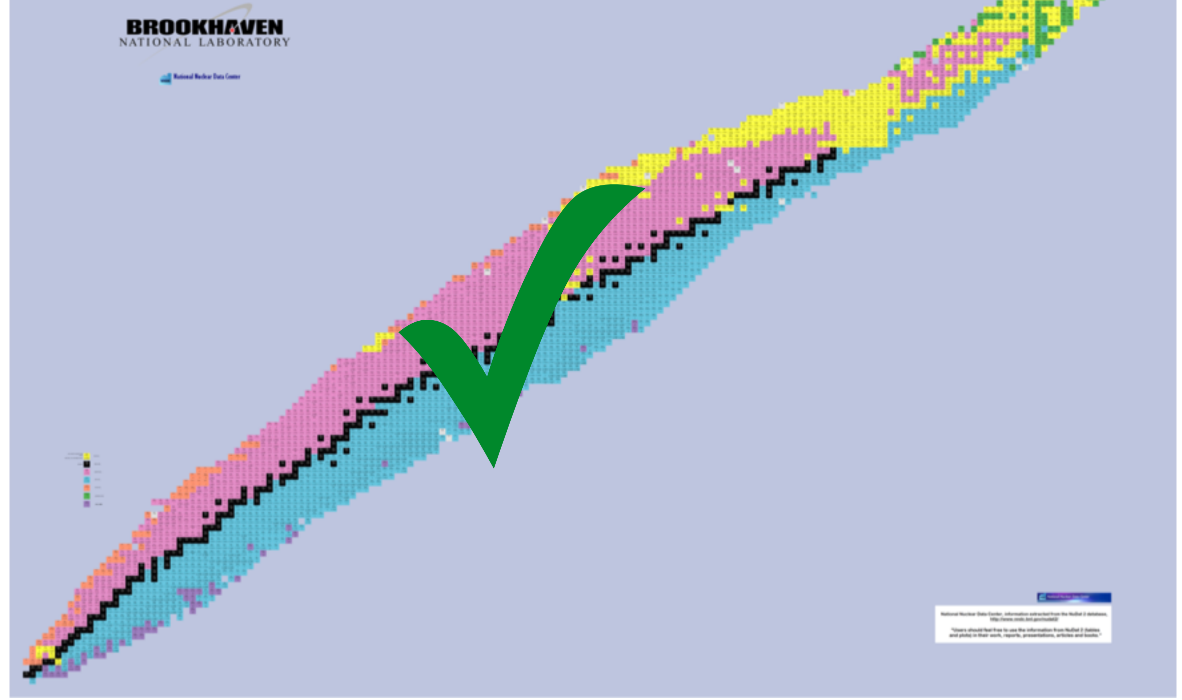


Dark nuclei

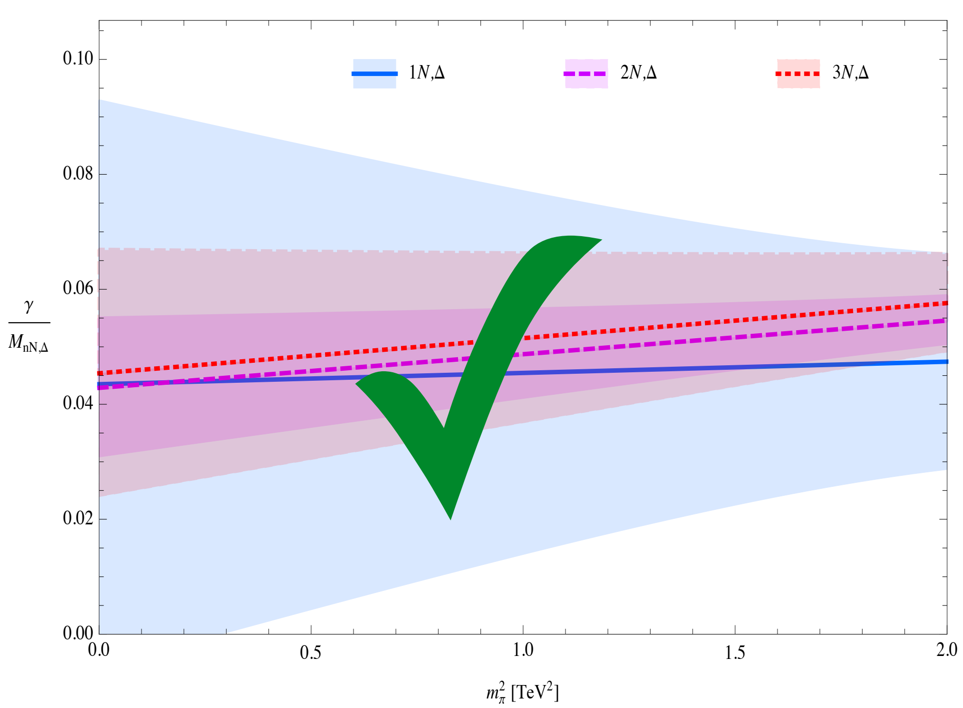
- J=0 nuclei: very likely unbound (all positively shifted)
- J=1, strong evidence for bound states for B=2,3, 4(?) B=5,..,8 seem unbound
  - Bindings decrease with quark mass and increase towards continuum
  - Strength of binding is significant w.r.t. mass
- Nuclear states with other quantum #s may also be bound



The ubiquity of nuclei?

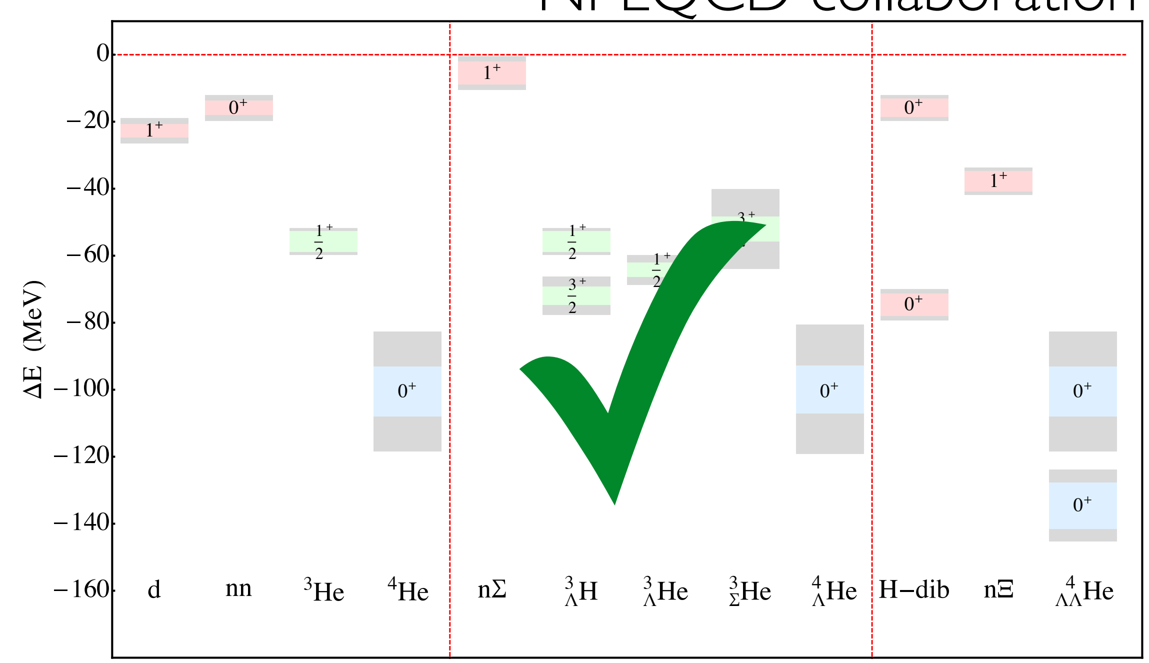


$N_c=3$, m_π = 140 MeV



$N_c=2$, m_π = 1 TeV

NPLQCD collaboration



$N_c=3$, m_π = 400–800 MeV

The ubiquity of nuclei?

- Appears nuclei are rather generic and not an accident of parameters
- What are nuclei? e.g. shell model vs quark blobs
  - More detailed studies necessary
- How generic are layers of effective degrees of freedom?
  - nucleons → nuclei → alpha clusters ….

Dark matter model building

- Extend strongly-interacting dark sector to talk minimally to SM
  - Simple extension: add scalar particle that kinematically mixes with Higgs

$$\mathcal{L} = \mathcal{L}_{\text{strong}} - \frac{\lambda}{4}\left(v_D - H_D^2\right)^2 - \left(\kappa H_D (u_R^\dagger u_L + d_L^\dagger d_R) + h.c.\right) + \delta\, H_D^2 |H|^2$$

  - Dark Higgs vev gives quark masses
  - Kinematic mixing controlled by δ: must be small ~10^{-3}

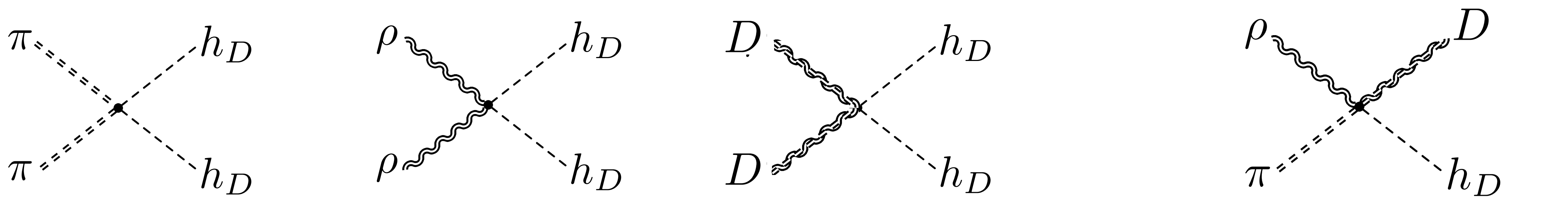
Field	Spin	$SU(2)_L$	$SU(2)_R$	$SU(2)_{QCD}$
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$1/2$	$\Box$	1	$\Box$
$\begin{pmatrix} u_R \\ d_R \end{pmatrix}$	$1/2$	1	$\Box$	$\bar{\Box}$
H_D	0	1	1	1
A_μ^a	1	1	1	adj

Dark matter model building

- Hadronic theory: construct based on broken global symmetries SU(4)→ Sp(4)~SO(5)
  - Consider only pions, rhos, "deuterons" (LQCD calculations provide motivation to consider deuterons)
  - Ignore compositeness!
  - Ignore larger baryon number states
- Interactions

$$\mathcal{L}_{\rm Int} = A_\pi h_D \left((\pi^0)^2/2 + \pi^+\pi^- + \pi^B \pi^{\overline{B}} \right) + A_\rho h_D \left((\rho^0)^2/2 + \rho^+\rho^- + \rho^B \rho^{\overline{B}} \right)$$

$$\mathcal{L}_{\rho\pi D} \sim \boldsymbol{\pi}^\dagger (\bar{\lambda}_{\mathbf{1}} \boldsymbol{D}^\mu_{\mathbf{1}} + \bar{\lambda}_{\mathbf{10}} \boldsymbol{D}^\mu_{\mathbf{10}} + \bar{\lambda}_{\mathbf{14}} \boldsymbol{D}^\mu_{\mathbf{14}}) \boldsymbol{\rho}_\mu$$



Annihilation

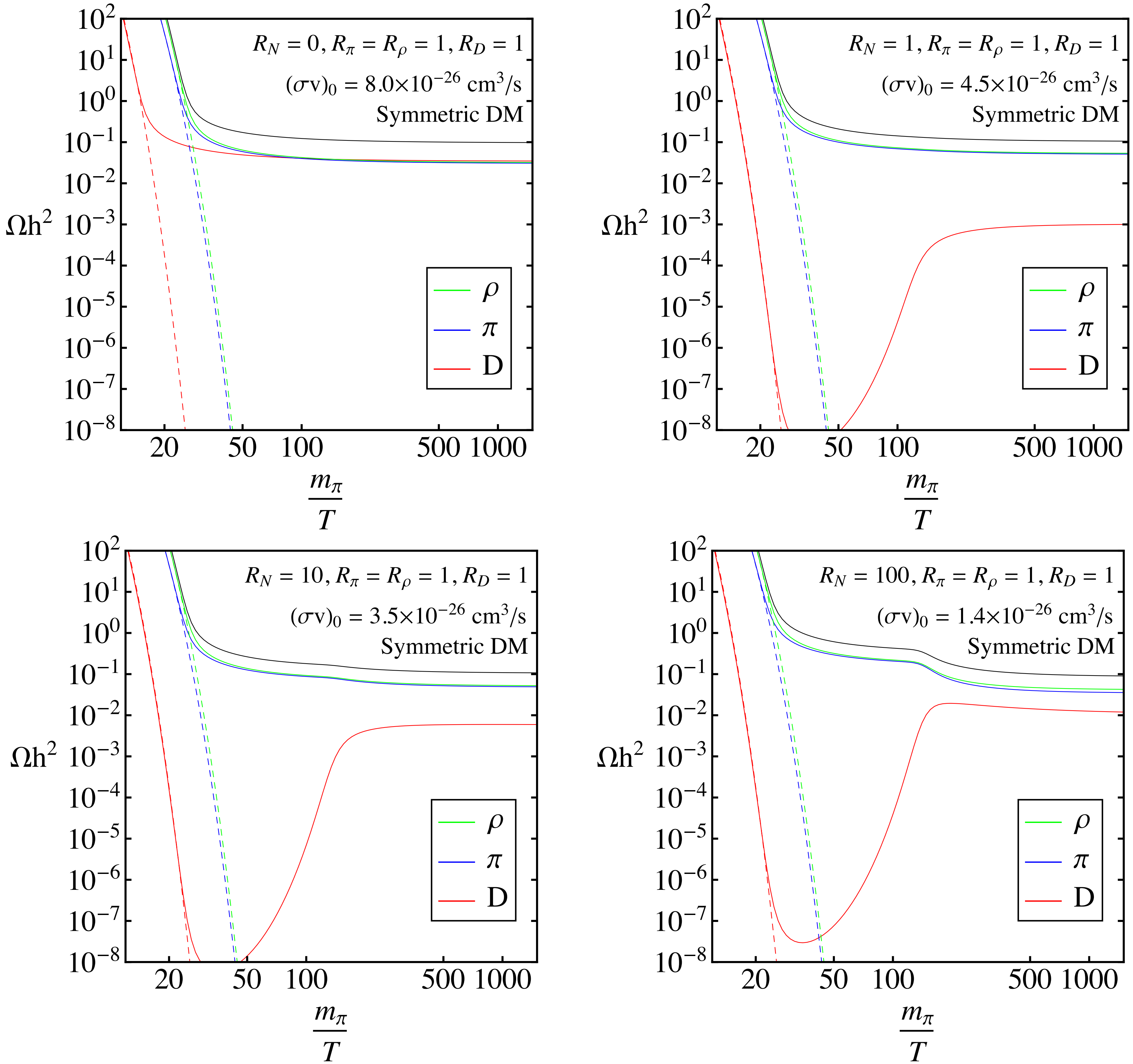
Nucleosynthesis

Cosmology

- Dark nucleosynthesis, dark capture processes modify early universe cosmology (both symmetric & asymmetric scenarios)
- Estimate cross sections and solve Boltzmann equations for co-moving number densities

$$\begin{aligned}
\frac{dY_{\pi^B}}{dx} &= -\lambda\Bigg[R_\pi\left(Y_{\pi^B}Y_{\pi^{\overline{B}}} - Y^{eq}_{\pi^B}Y^{eq}_{\pi^{\overline{B}}}\right) + R_N\left(Y_{\pi^B}Y_{D^{\overline{B}}} - \frac{Y_{\rho^{\overline{B}}}}{Y^{eq}_{\rho^{\overline{B}}}}Y^{eq}_{\pi^B}Y^{eq}_{D^{\overline{B}}}\right) \\
&\quad -R_N\left(Y_{\rho^{\overline{B}}}Y_{D^B} - \frac{Y_{\pi^B}}{Y^{eq}_{\pi^B}}Y^{eq}_{\rho^{\overline{B}}}Y^{eq}_{D^B}\right) + R_N f(x)\left(Y_{\pi^B}Y_{\rho^B} - \frac{Y_{D^B}}{Y^{eq}_{D^B}}Y^{eq}_{\pi^B}Y^{eq}_{\rho^B}\right)\Bigg] \\
\frac{dY_{\rho^B}}{dx} &= -\lambda\Bigg[R_\rho\left(Y_{\rho^B}Y_{\rho^{\overline{B}}} - Y^{eq}_{\rho^B}Y^{eq}_{\rho^{\overline{B}}}\right) + R_N\left(Y_{\rho^B}Y_{D^{\overline{B}}} - \frac{Y_{\pi^{\overline{B}}}}{Y^{eq}_{\pi^{\overline{B}}}}Y^{eq}_{\rho^B}Y^{eq}_{D^{\overline{B}}}\right) \\
&\quad -R_N\left(Y_{\pi^{\overline{B}}}Y_{D^B} - \frac{Y_{\rho^B}}{Y^{eq}_{\rho^B}}Y^{eq}_{\pi^{\overline{B}}}Y^{eq}_{D^B}\right) + R_N f(x)\left(Y_{\pi^B}Y_{\rho^B} - \frac{Y_{D^B}}{Y^{eq}_{D^B}}Y^{eq}_{\pi^B}Y^{eq}_{\rho^B}\right)\Bigg] \\
\frac{dY_{D^B}}{dx} &= -\lambda\Bigg[R_D\left(Y_{D^B}Y_{D^{\overline{B}}} - Y^{eq}_{D^B}Y^{eq}_{D^{\overline{B}}}\right) - R_N f(x)\left(Y_{\pi^B}Y_{\rho^B} - \frac{Y_{D^B}}{Y^{eq}_{D^B}}Y^{eq}_{\pi^B}Y^{eq}_{\rho^B}\right) \\
&\quad +R_N\left(\left(Y_{\pi^{\overline{B}}} + Y_{\rho^{\overline{B}}}\right)Y_{D^B} - \left(\frac{Y_{\rho^B}}{Y^{eq}_{\rho^B}}Y^{eq}_{\pi^{\overline{B}}} + \frac{Y_{\pi^B}}{Y^{eq}_{\pi^B}}Y^{eq}_{\rho^{\overline{B}}}\right)Y^{eq}_{D^B}\right)\Bigg]
\end{aligned}$$

- Range of possible compositions of relic density


$R_N = 0, R_\pi = R_\rho = 1, R_D = 1$
$(\sigma v)_0 = 8.0\times10^{-26}$ cm^3/s
Symmetric DM
$R_N = 1, R_\pi = R_\rho = 1, R_D = 1$
$(\sigma v)_0 = 4.5\times10^{-26}$ cm^3/s
Symmetric DM
$R_N = 10, R_\pi = R_\rho = 1, R_D = 1$
$(\sigma v)_0 = 3.5\times10^{-26}$ cm^3/s
Symmetric DM
$R_N = 100, R_\pi = R_\rho = 1, R_D = 1$
$(\sigma v)_0 = 1.4\times10^{-26}$ cm^3/s
Symmetric DM
ρ
π
D
Ωh^2
10^{2} 10^{1} 10^{0} 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8}
20 50 100 500 1000
$\frac{m_\pi}{T}$

Indirect detection signals

- Presence of nuclear binding energies: new scale for phenomenology is significantly different than Λ_{QC_2D}

Signature	Collider	Direct Detection	Annihilation	Nucleosynthesis	Capture
Sym-DM	$M, 2M$	$M, 2M$	$M, 2M$	$B_D \ll M$	M
Asym-DM	$M, 2M$	$M, 2M$	—	$B_D \ll M$	—

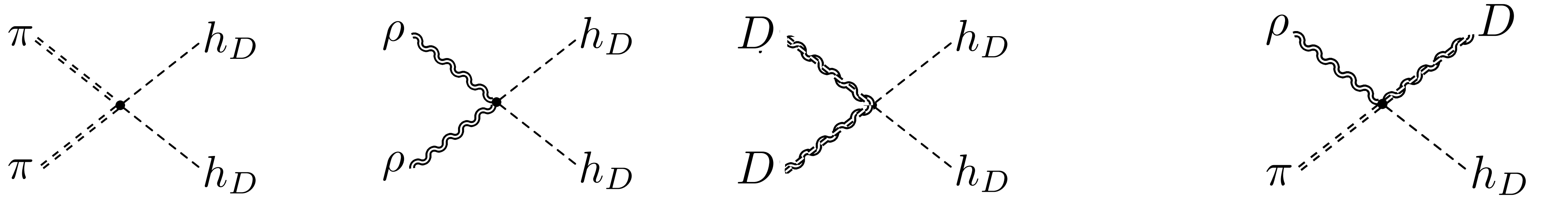
- For symmetric DM, additional scale/process may lead to signals at multiple different energy scales with identical spatial morphology
- For asymmetric DM scenarios (only dark baryon number carrying states remain) nucleosynthesis allows indirect detection signals

Dark matter model building

- Extend strongly-interacting dark sector to talk minimally to SM
  - Simple extension: add scalar particle that kinematically mixes with Higgs

$$\mathcal{L} = \mathcal{L}_{\text{strong}} - \frac{\lambda}{4}\left(v_D - H_D^2\right)^2 - \left(\kappa H_D (u_R^\dagger u_L + d_L^\dagger d_R) + h.c.\right) + \delta\, H_D^2 |H|^2$$

    - Dark Higgs vev gives quark masses
    - Kinematic mixing controlled by δ: must be small $\sim 10^{-3}$
- Hadronic theory: consider only pions, rhos, "deuterons" (LQCD calculations provide motivation to consider deuterons)
- Interactions

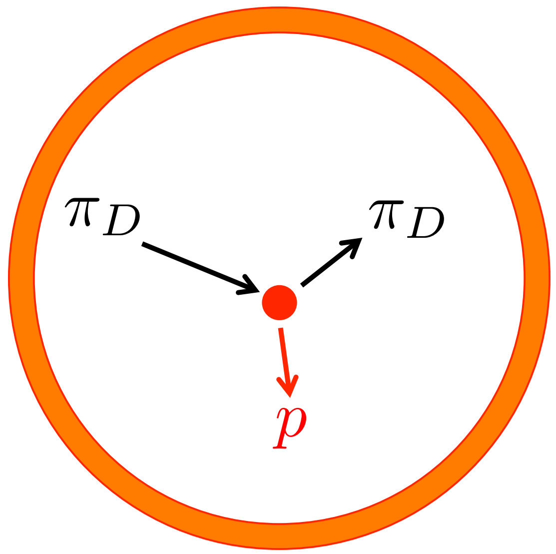


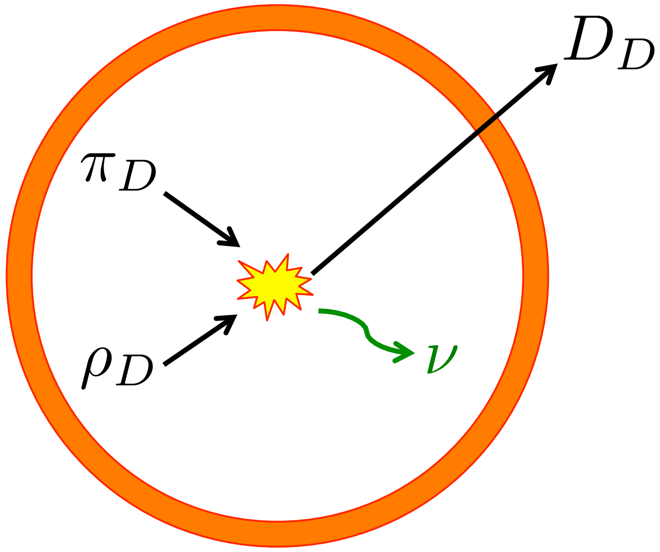
Annihilation

Nucleosynthesis

Compact Objects

- Significant modifications to physics of astrophysical bodies
  - Dark matter gravitationally captured after scattering on visible matter
  - Helioseismology and neutron star lifetimes strongly modified – strongly constrains asymmetric DM models
- Very rich phenomenology!
  - Liberation of binding energy may allow ejection of dark matter
- Star develops a co-located dark nuclear process site





Summary

- Two-flavour, two-colour QCD has a complex spectrum exhibiting the analogues of nuclei
  - Ubiquity of nuclei
- Composite dark matter is a natural scenario to consider
  - Nuclear binding provides a scale that is small relative to the QCD scale in a natural way
  - Predicts a range of different phenomenology that beyond what is possible in simpler models

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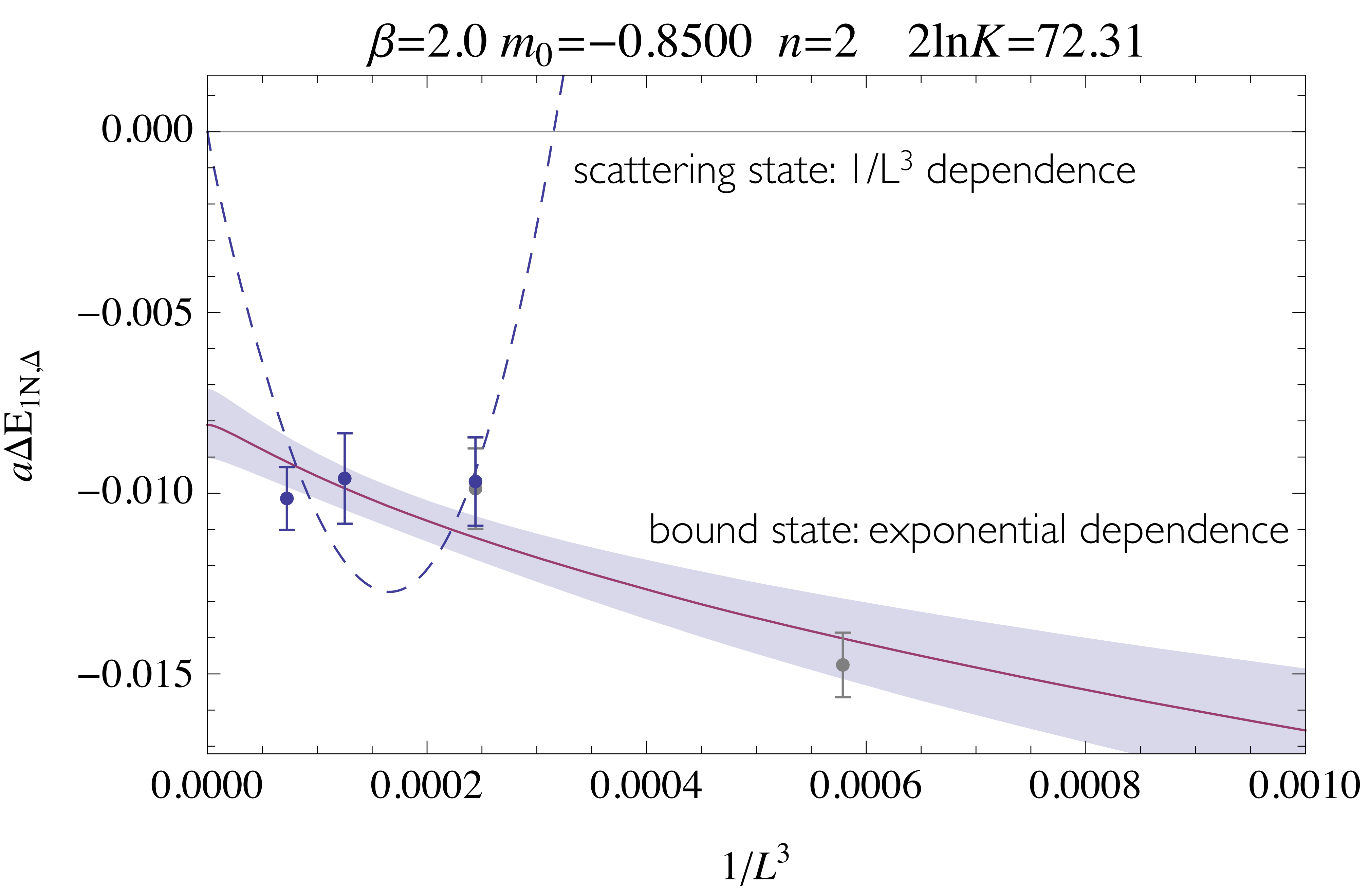
Infinite volume extrapolations



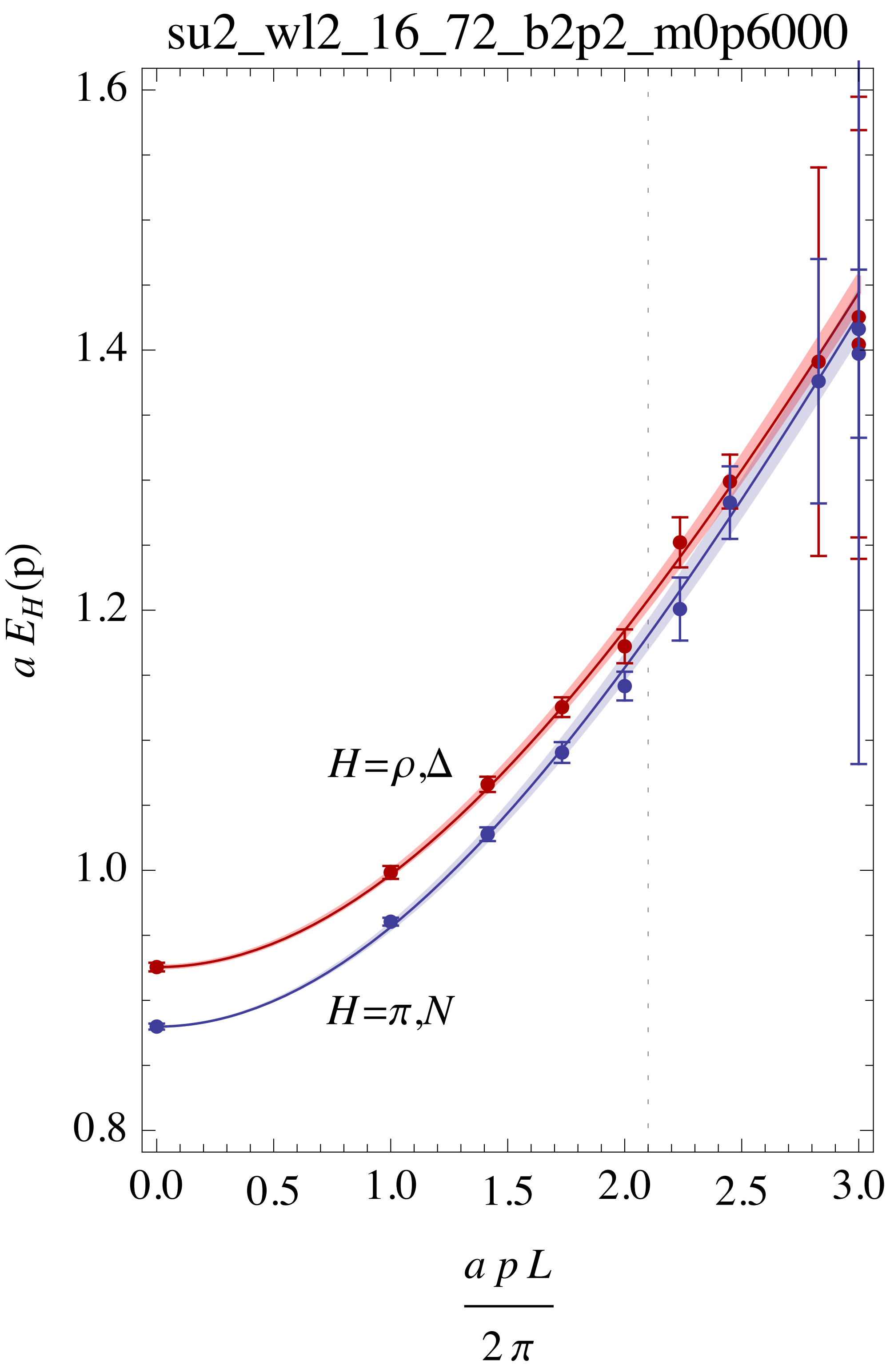
Dispersion relation

- Consider boosted hadrons
- Lattice artifacts break relativity

$$E_H(p) = \sqrt{M_H^2 + c_H^2 p^2}$$

- Speed of light ≠ 1

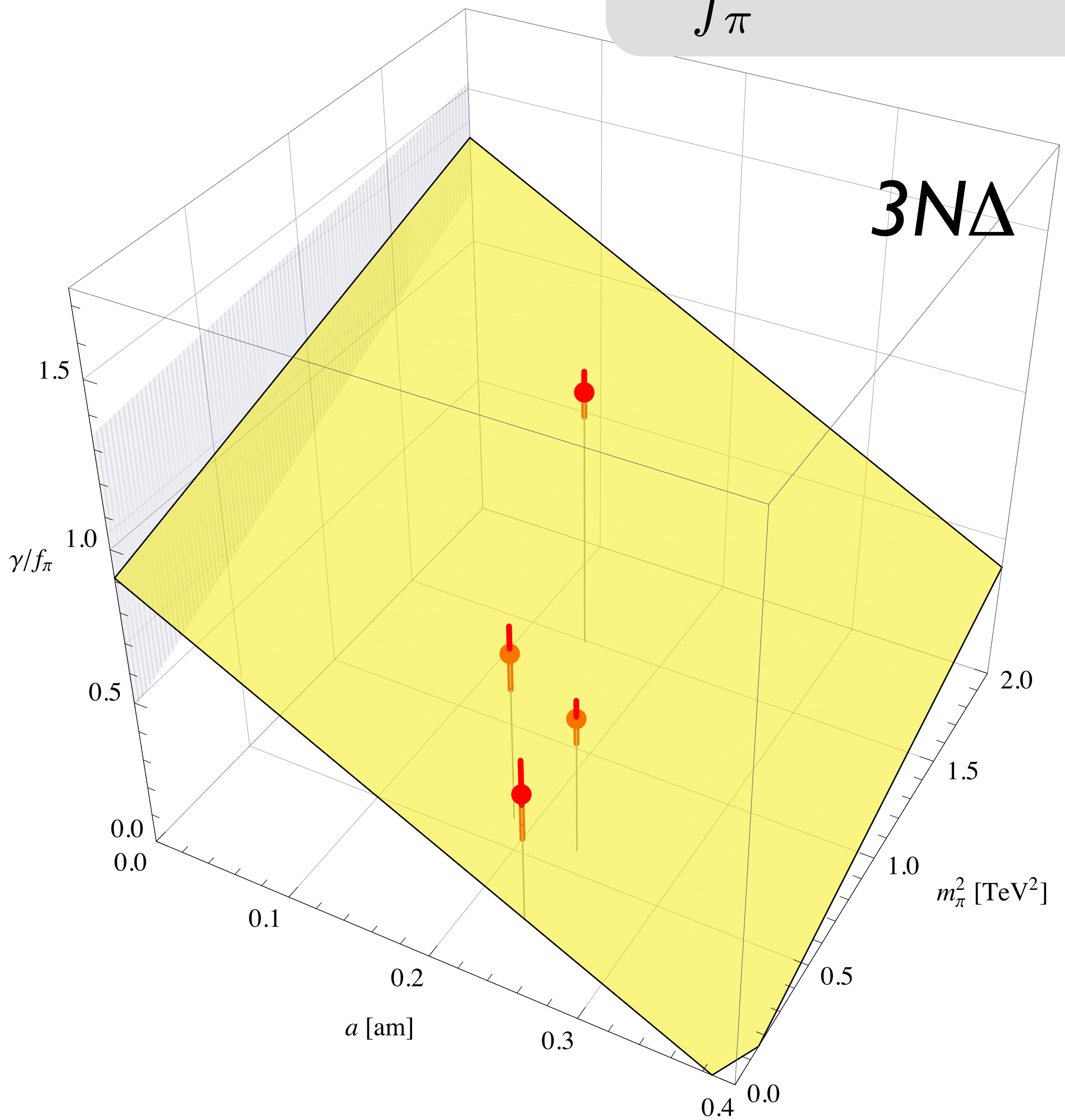
Ensemble	c_π	c_ρ
A	0.93(1)	0.87(4)
B	0.92(5)	0.97(5)
C	0.99(2)	0.94(1)
D	0.94(2)	0.92(3)
E	0.95(1)	0.93(3)
F	0.96(2)	0.94(1)

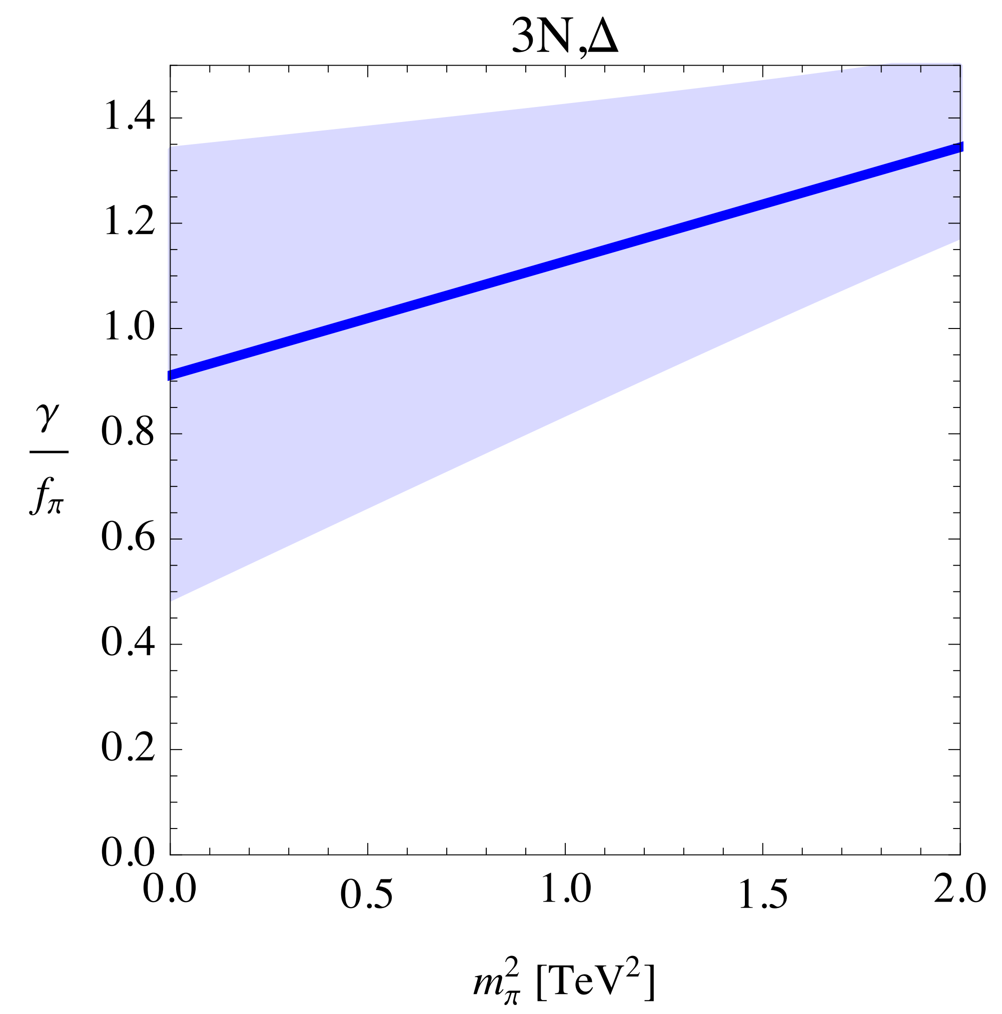


Continuum extrapolations

- Simple continuum limit extrapolation of binding momentum, γ

$$\frac{\gamma_{nN,\Delta}}{f_\pi}(a,m) = \gamma^{(0)}_{nN,\Delta} + a\ \delta^{(a)}_n + m_\pi{}^2\delta^{(m)}_n$$

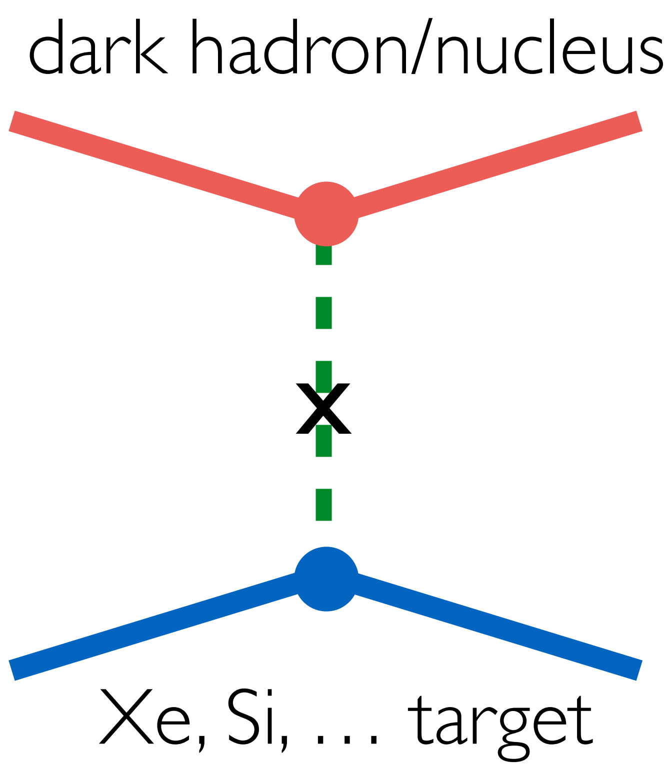




Sigma terms

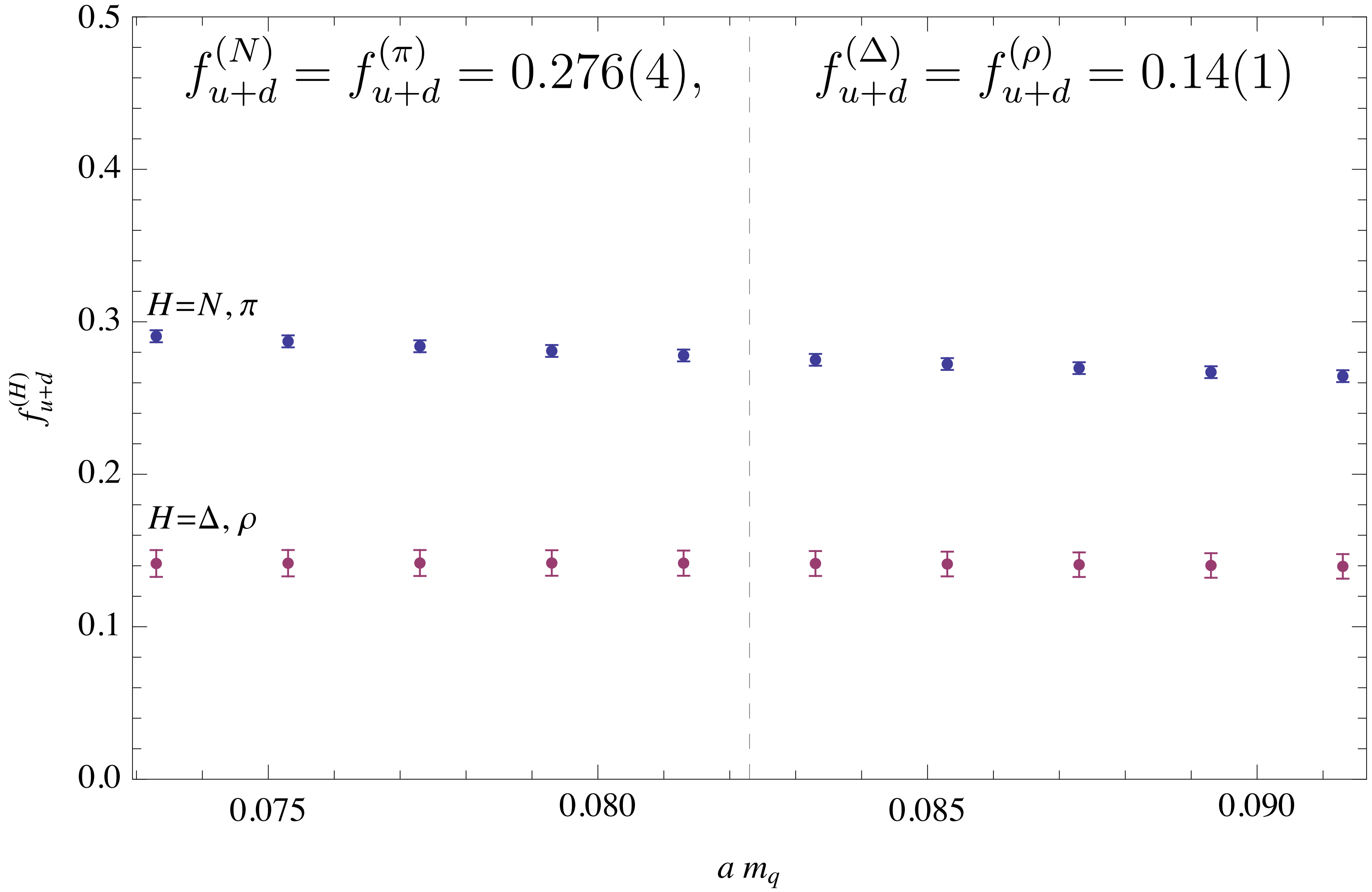
- DM may interact via scalar exchange (Higgs portal)
  - Depends on QCD <u>nuclear</u> sigma terms
  - For composite dark matter, depends on sigma terms of dark hadrons/nuclei
- Nuclear sigma terms
  - Particle physicists: A x proton sigma term
  - Reality: non-trivial deviations from 2-body physics Calculate from QCD/EFT [Beane *et al.* 2013]
- Dark sigma terms: accessible from Feynman-Hellman theorem

$$f_q^{(H)} = \frac{m_q}{M_H} \frac{\partial M_H}{\partial m_q}$$



Sigma terms

- Estimate via partially-quenched calculation for single hadrons



- Surprisingly naive dimensional analysis works

Correlations

- Different normalisation choices change results a little because of correlations