

$N_f=8$ QCD

Yasumichi Aoki [Kobayashi-Maskawa Institute(KMI), Nagoya University]

for the LatKMI collaboration

- Lattice for Beyond the Standard Model Physics @ LLNL -

April 23, 2015



LatKMI collaboration (2015)

- YA, T.Aoyama, M.Kurachi, T.Maskawa, K.Miura,



K.Nagai, H.Ohki, K.Yamawaki



名古屋大学



- E. Bennett



E. Rinaldi



- A. Shibata



T. Yamazaki



LatKMI collaboration (2015.4)

- KMI / Nagoya Univ.

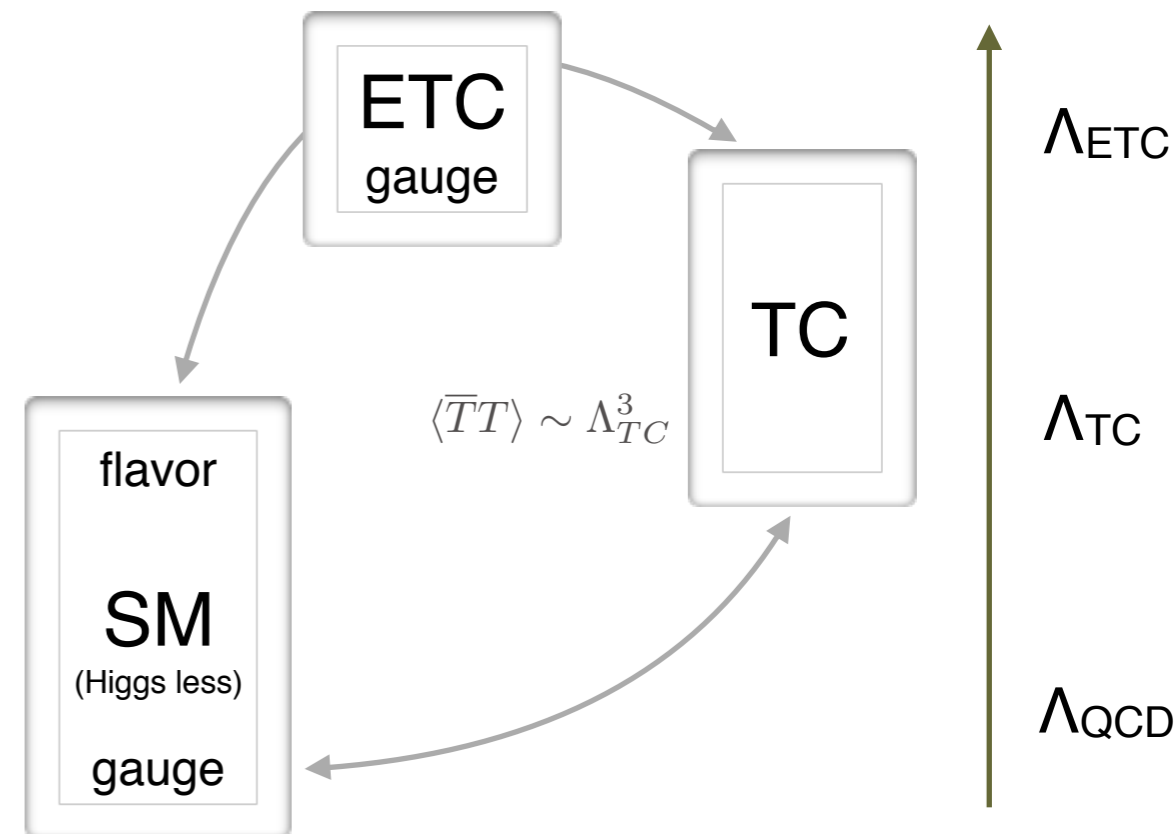


- outside



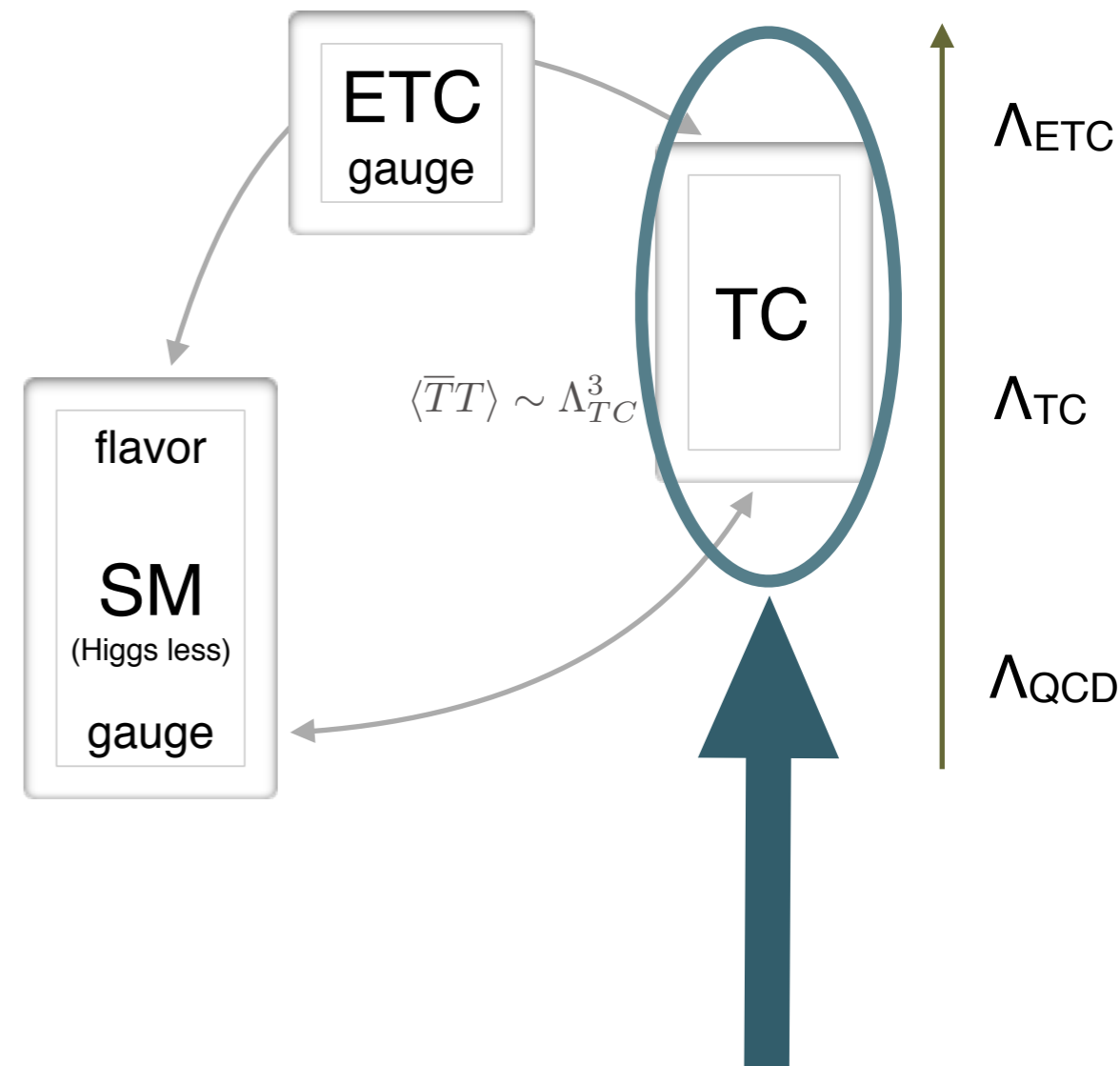
Technicolor and Extended Technicolor

- QCD like dynamics can trigger the Electroweak symmetry breaking
- Techni pion act as NG mode of Higgs
 - give mass to W and Z bosons
- SM fermion masses are given through ETC
- Tension:
 - FCNC must be suppressed
 - sizable m_f needs to be generated



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 - FCNC must be suppressed
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so far we are dealing with this only

LatKMI publications on related subjects

- LatKMI, PRD 85 (2012), “Study of the conformal hyperscaling relation through the Schwinger-Dyson equation” [non-lattice]
- LatKMI, PRD 86 (2012), “Lattice study of conformality in twelve-flavor QCD”
- LatKMI, PRD 87 (2013), “Walking signals in $N_f=8$ QCD on the lattice”
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- LatKMI, in preparation on $N_f=8$ QCD update

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Walking signals in $N_f = 8$ QCD on the lattice

Yasumichi Aoki,¹ Tatsumi Aoyama,¹ Masafumi Kurachi,¹ Toshihide Maskawa,¹ Kei-ichi Nagai,¹ Hiroshi Ohki,¹
Akihiro Shibata,² Koichi Yamawaki,¹ and Takeshi Yamazaki¹

(LatKMI Collaboration)

¹*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan*

²*Computing Research Center, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan*

(Received 27 February 2013; published 20 May 2013)

We investigate chiral and conformal properties of the lattice QCD with eight flavors ($N_f = 8$) through meson spectrum using the highly improved staggered quark (HISQ) action. We also compare our results with those of $N_f = 12$ and $N_f = 4$ which we study on the same systematics. We find that the decay constant F_π of the pseudoscalar meson “pion” π is nonzero, with its mass M_π consistent with zero, both in the chiral limit extrapolation of the chiral perturbation theory. We also measure other quantities which we find are in accord with the π data results: The ρ meson mass is consistent with nonzero in the chiral limit, and so is the chiral condensate, with its value neatly coinciding with that from the Gell-Mann-Oakes-Renner relation in the chiral limit. Thus, our data for the $N_f = 8$ QCD are consistent with the spontaneously broken chiral symmetry. Remarkably enough, while the $N_f = 8$ data near the chiral limit are well described by the chiral perturbation theory, those for the relatively large fermion bare mass m_f away from the chiral limit actually exhibit a finite-size hyperscaling relation, suggesting a large anomalous dimension $\gamma_m \sim 1$. This implies that there exists a remnant of the infrared conformality, and suggests that a typical technicolor (“one-family model”) as modeled by the $N_f = 8$ QCD can be a walking technicolor theory having an approximate scale invariance with large anomalous dimension $\gamma_m \sim 1$.

Nf=8 QCD: candidate of Walking Technicolor

PHYSICAL REVIEW D **87**, 094511 (2013)

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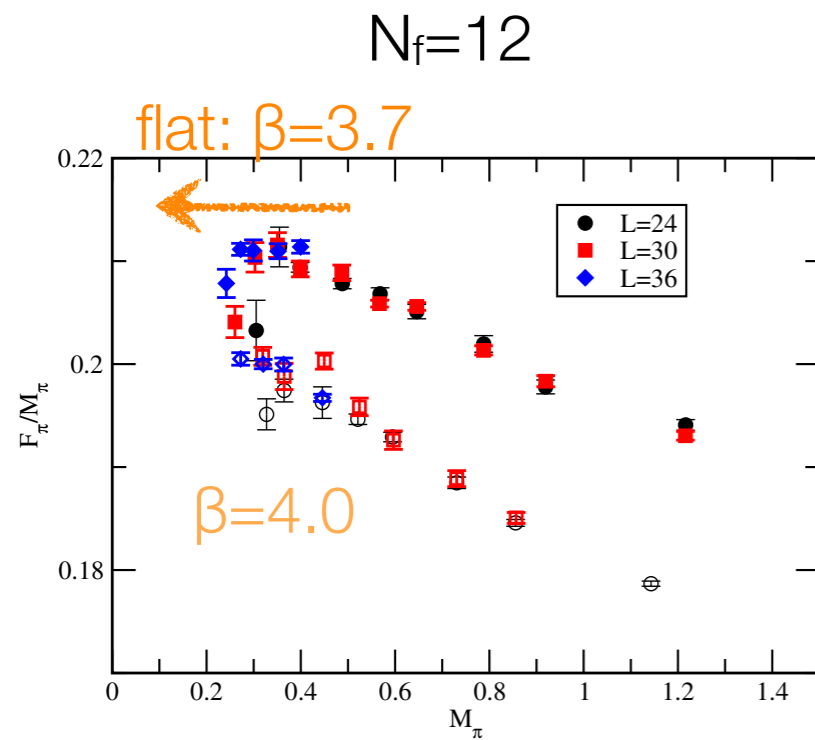
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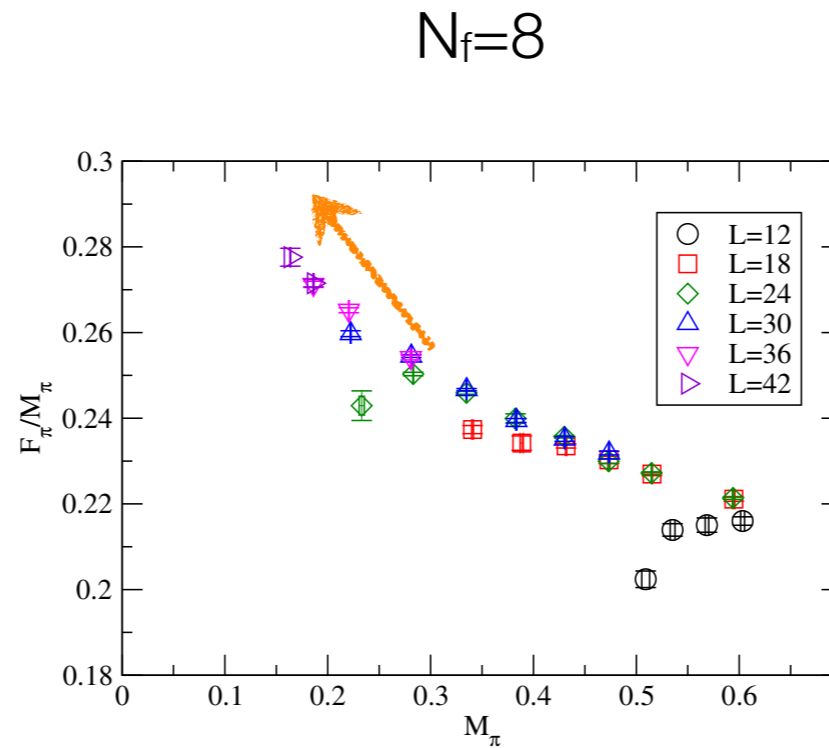
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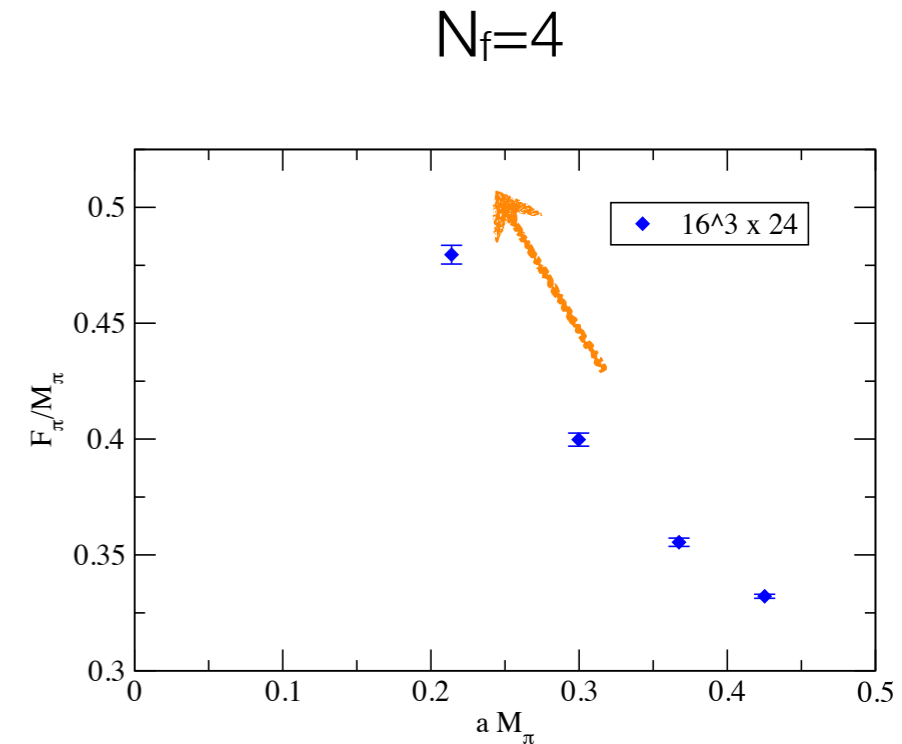
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- conformality



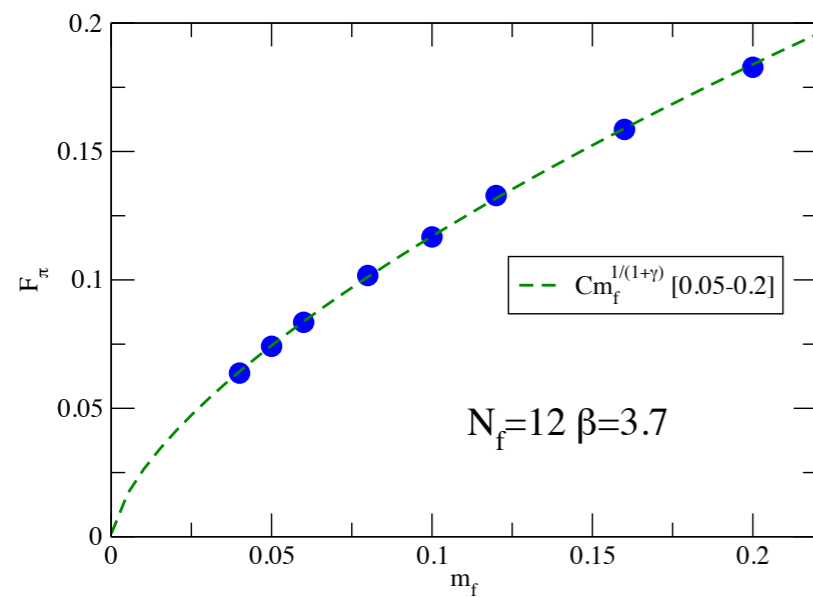
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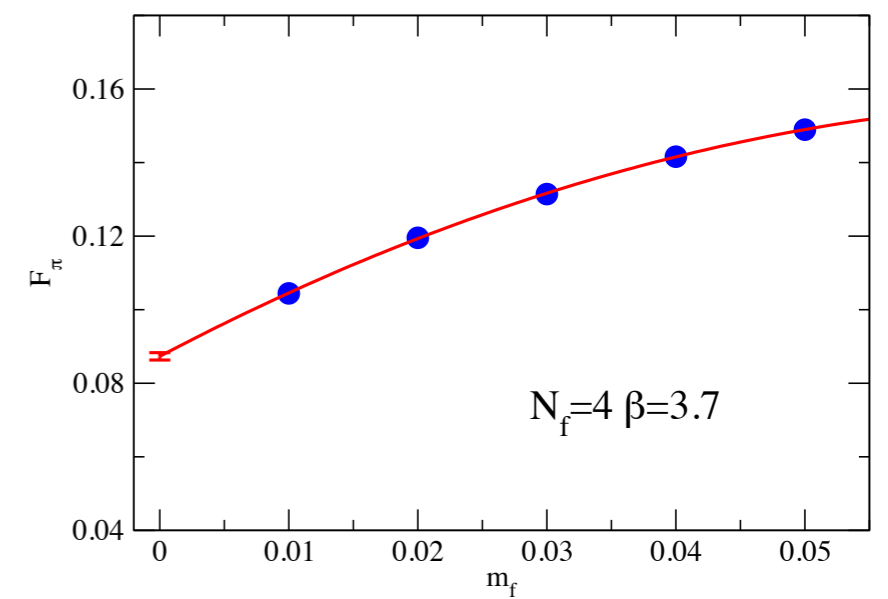
F_π vs m_f

$N_f=12$



- conformality
- $F_\pi \rightarrow C m_f^{1/(1+\gamma)}$
- $\gamma \sim 0.5$

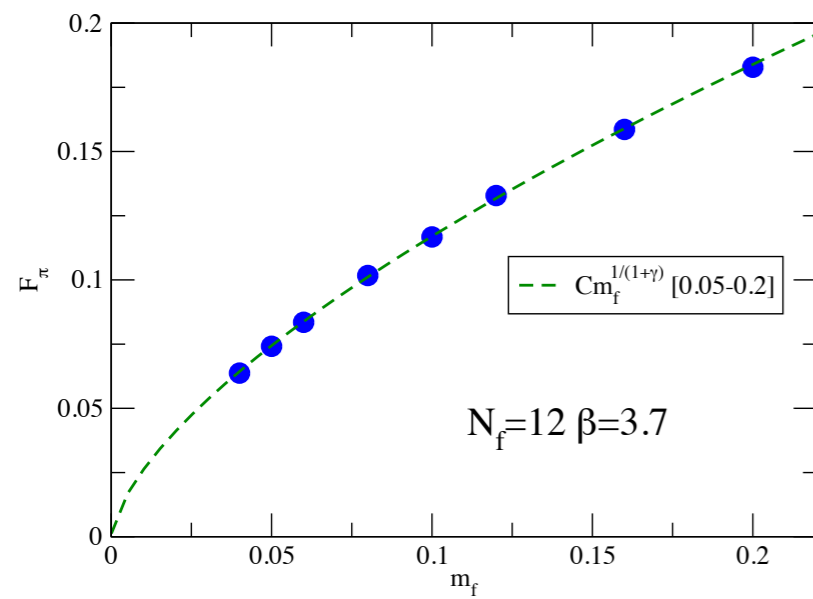
$N_f=4$



- ~~chiral symmetry~~
- $F_\pi \rightarrow F \neq 0$

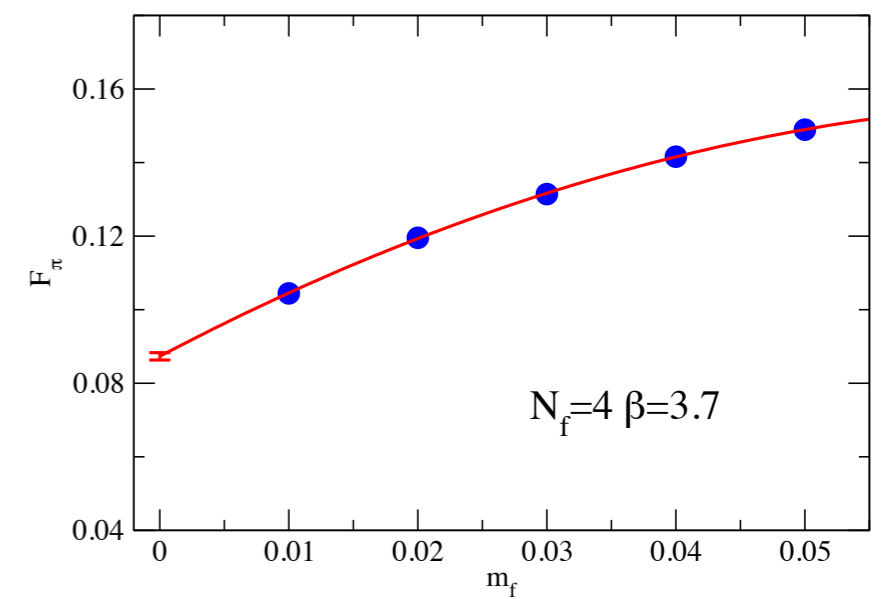
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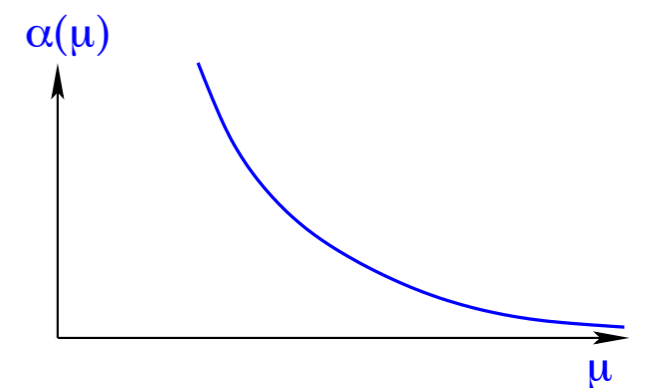


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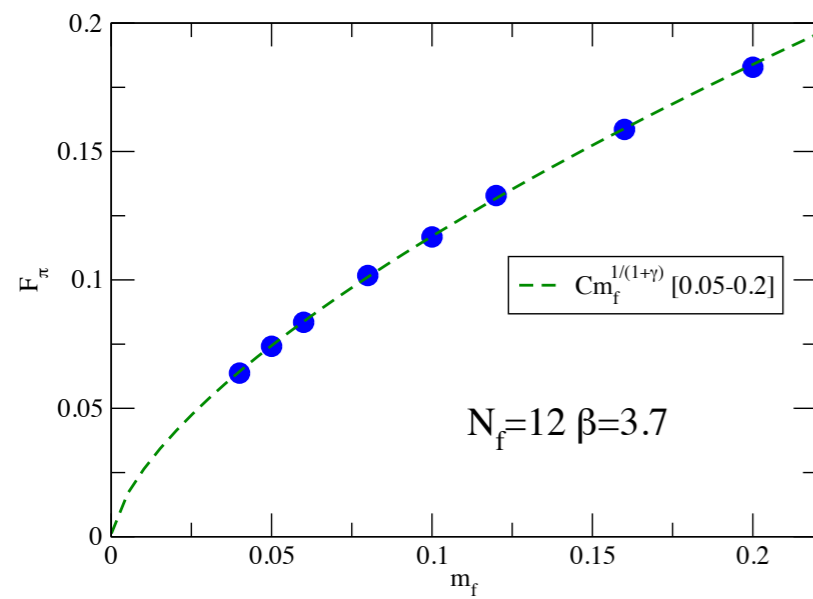


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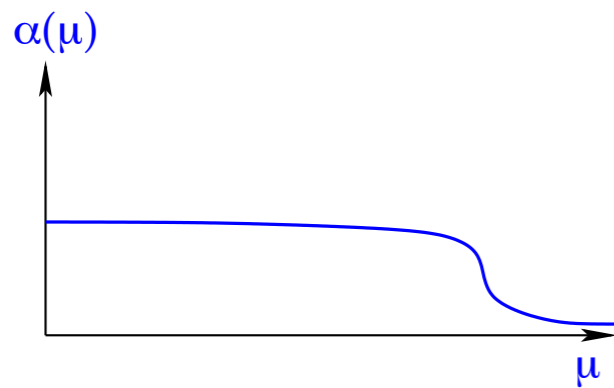


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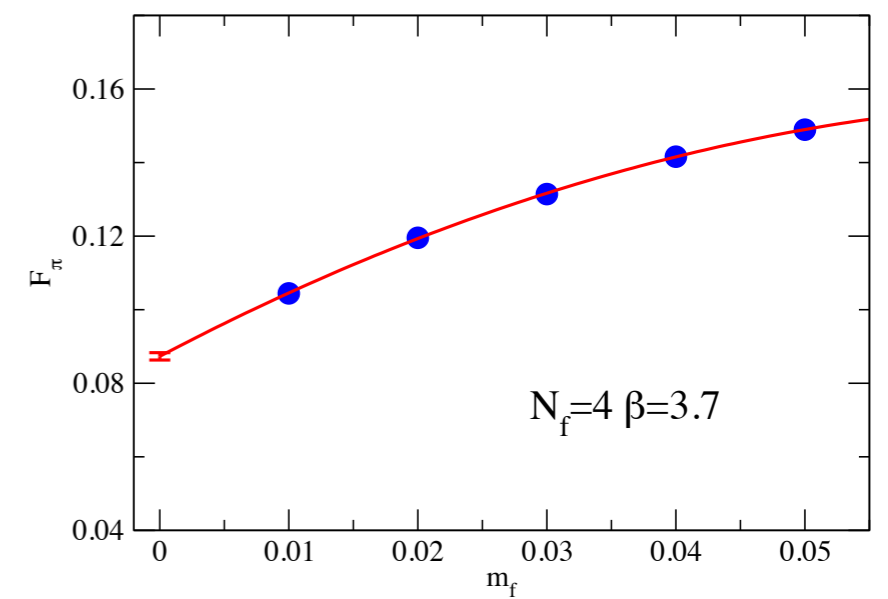
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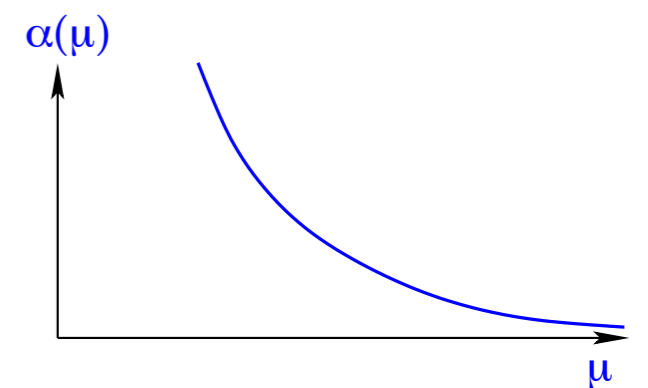
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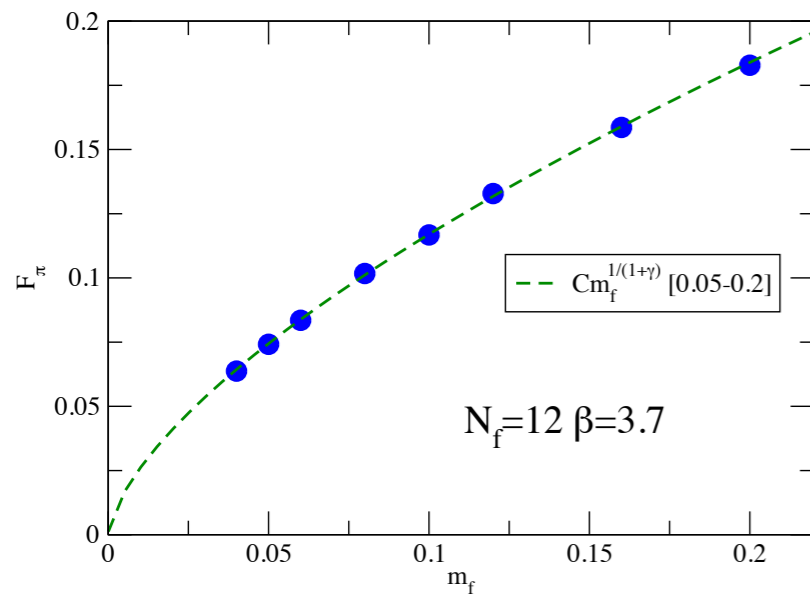


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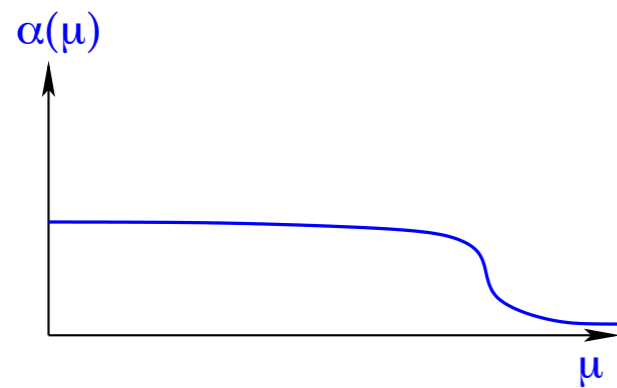


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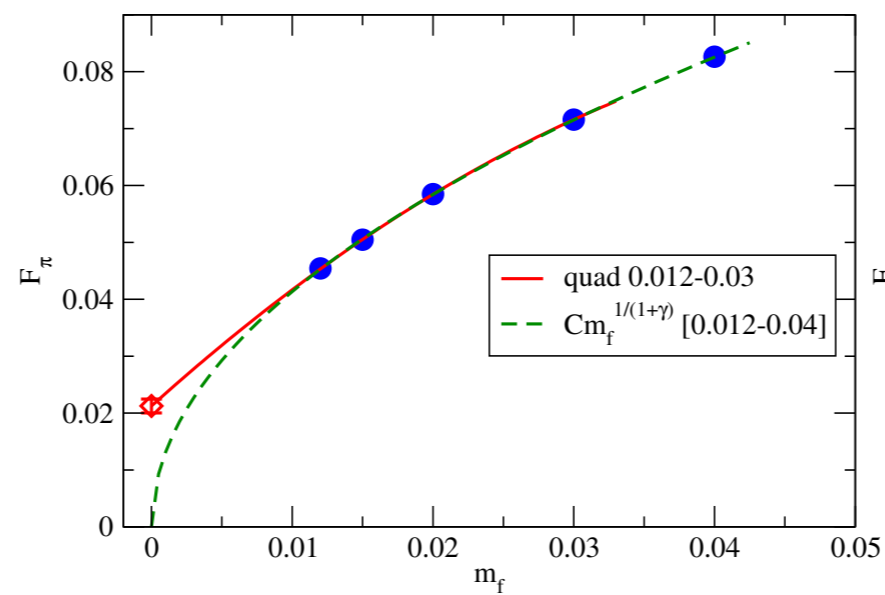
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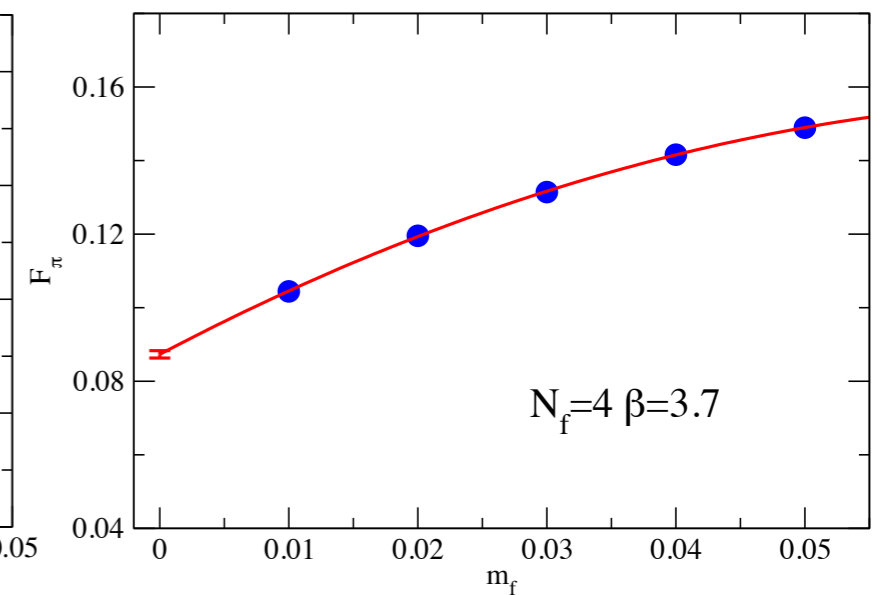


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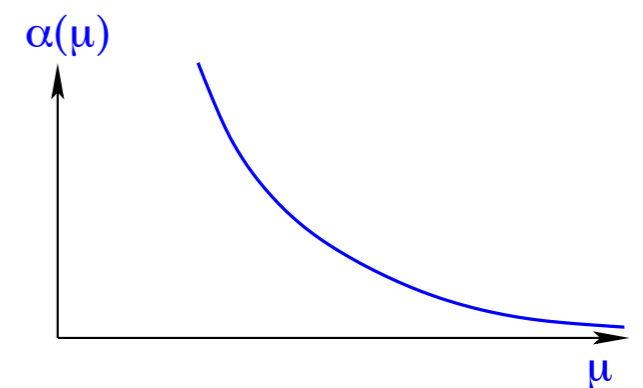


- ~~chiral symmetry~~
- $F_\pi \rightarrow F \neq 0$ $m_f \rightarrow 0$
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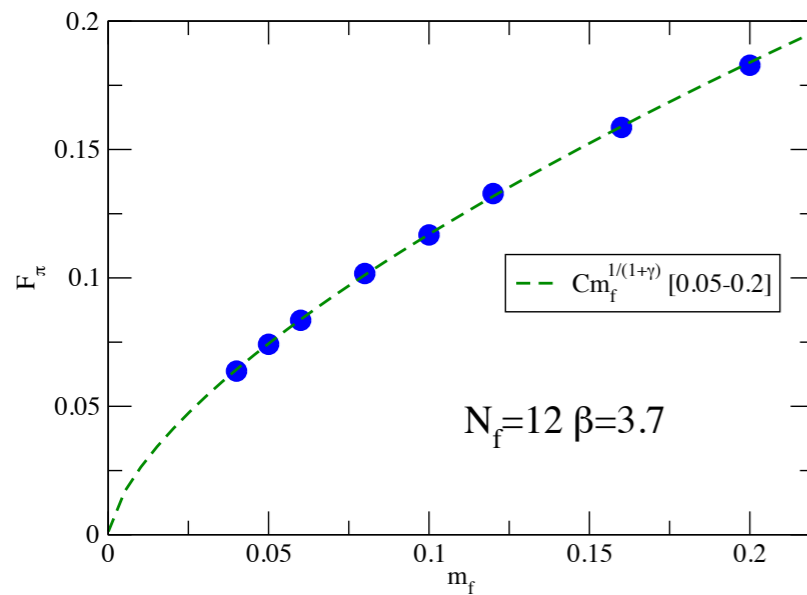


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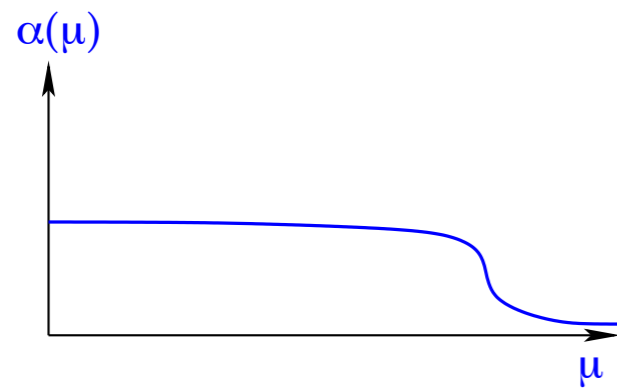


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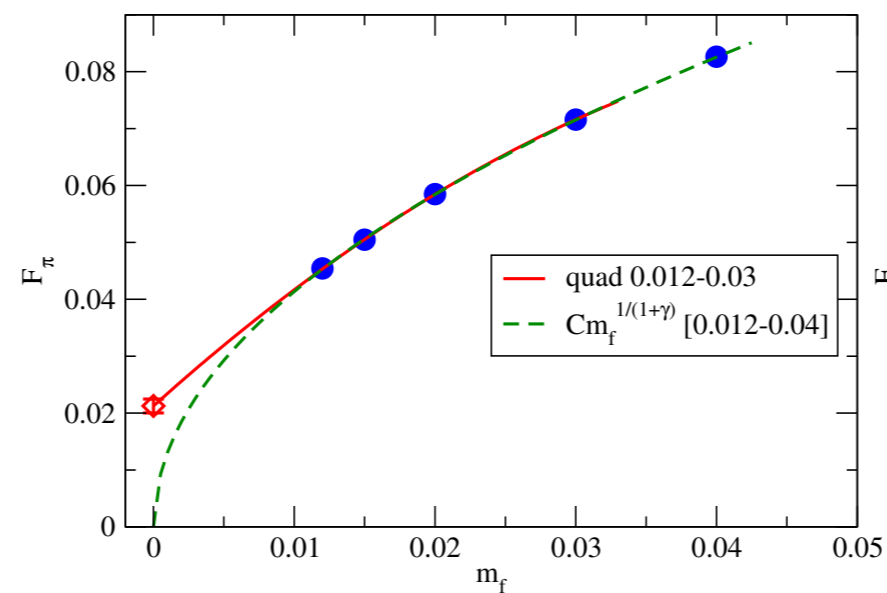
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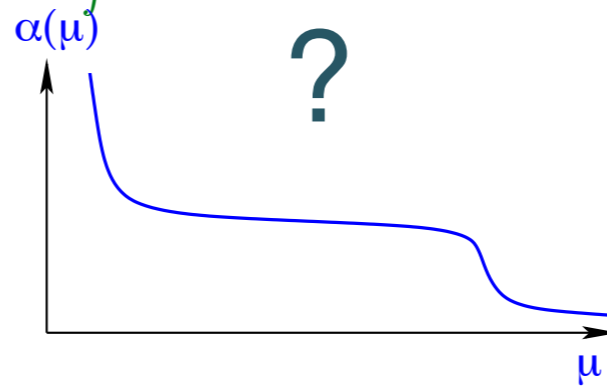
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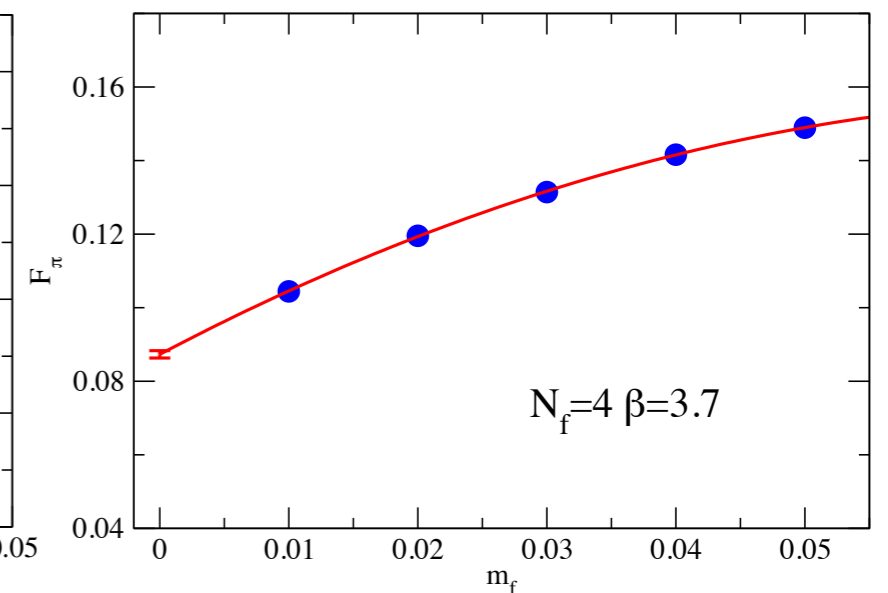
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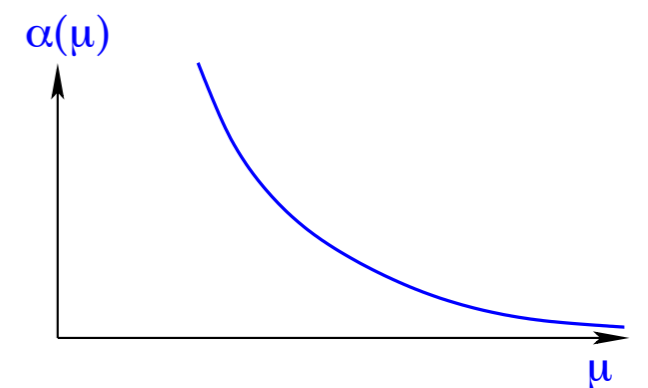
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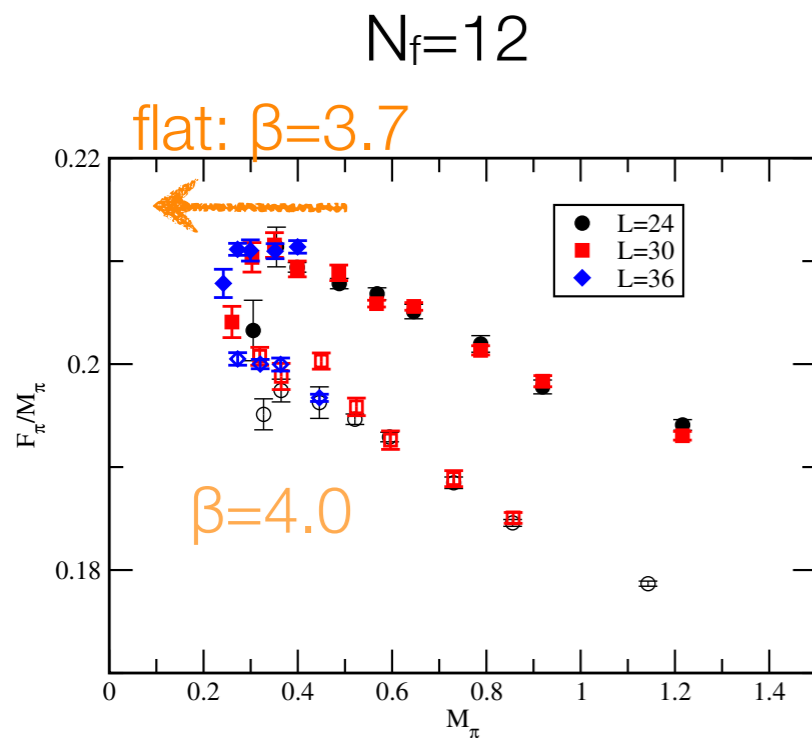
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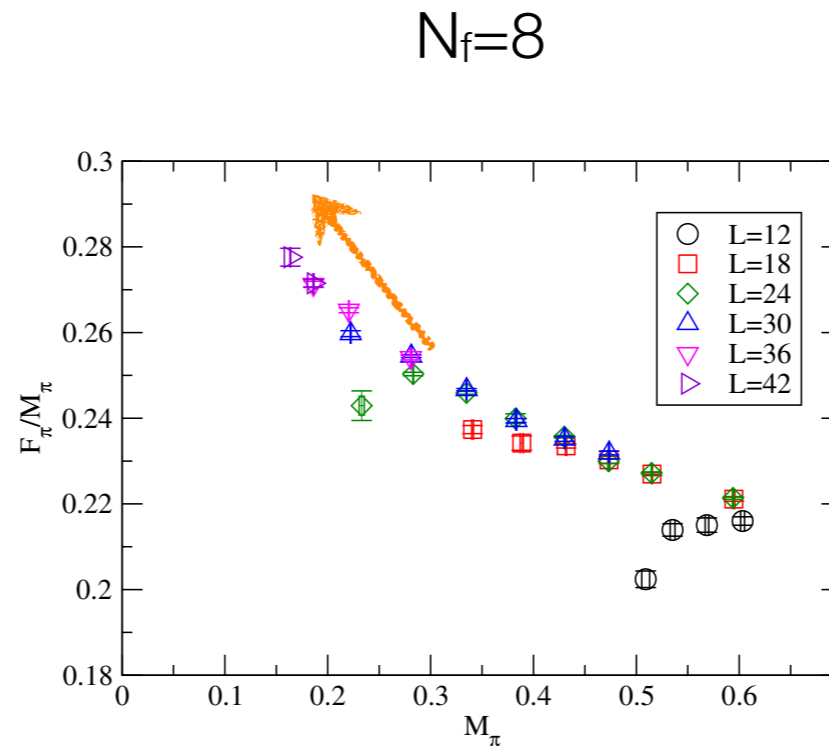
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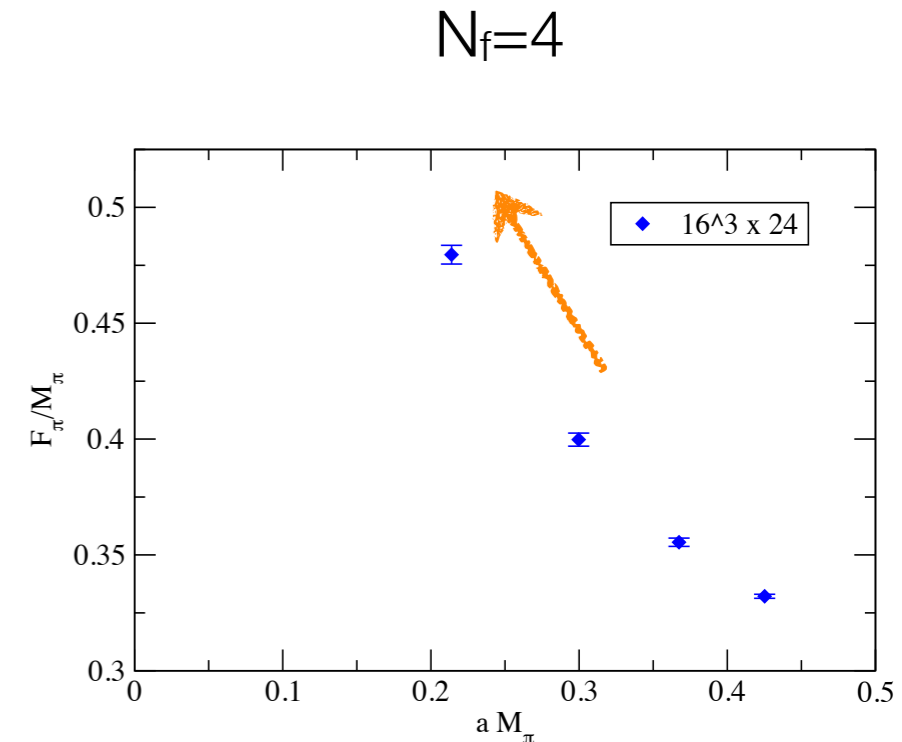
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even with this case,

if the $F_\pi/M_\pi(M_\pi \rightarrow 0) < \infty$, constant with conformal

$N_f=8$: conformal vs chiral symm. br.

- various quantities are tested:
 - F_π , M_π , M_ρ , M_N , M_{N^*} , M_{a0} , M_{a1} , M_{b1} , chiral condensate
 - Finite size scaling (conformal) with some types of mass correction
- just confirm what found for F_π scaling
- results can be interpreted either way: conformal or chiral symm. br.
- but, scaling exponents between quantities are not completely consistent
- this property maybe a clue of walking technicolor theory
- further getting close to chiral limit will unfold the nature of this theory

In case of S_xSB:

spectrum at chiral limit



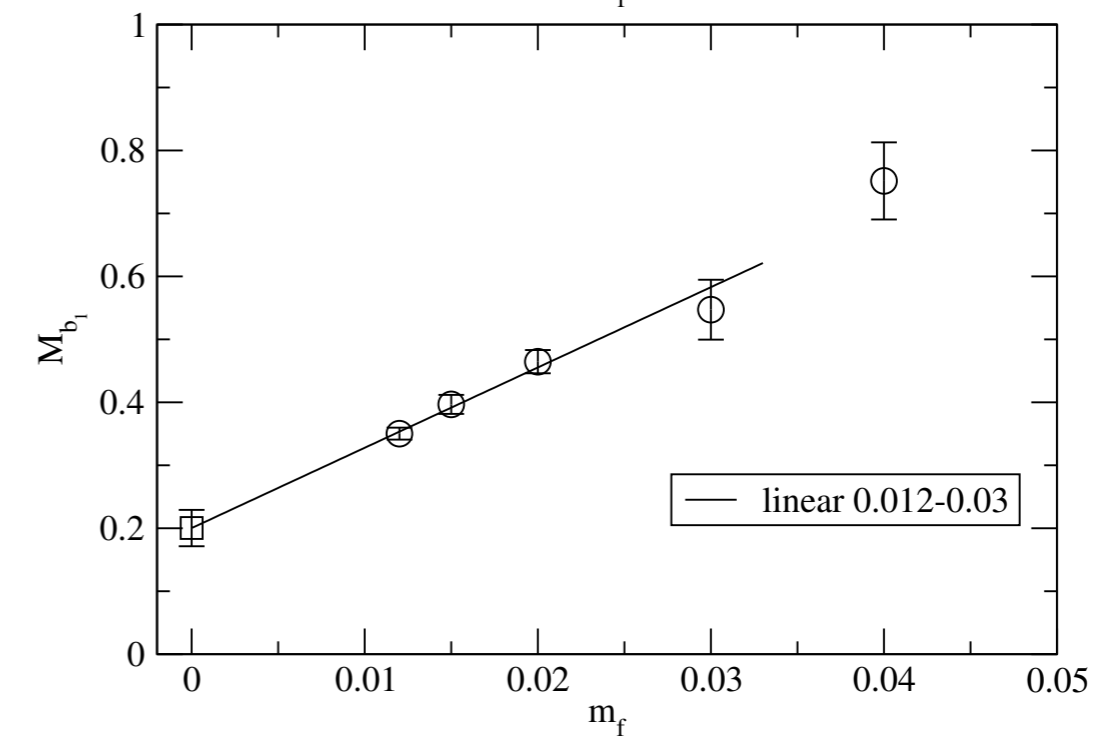
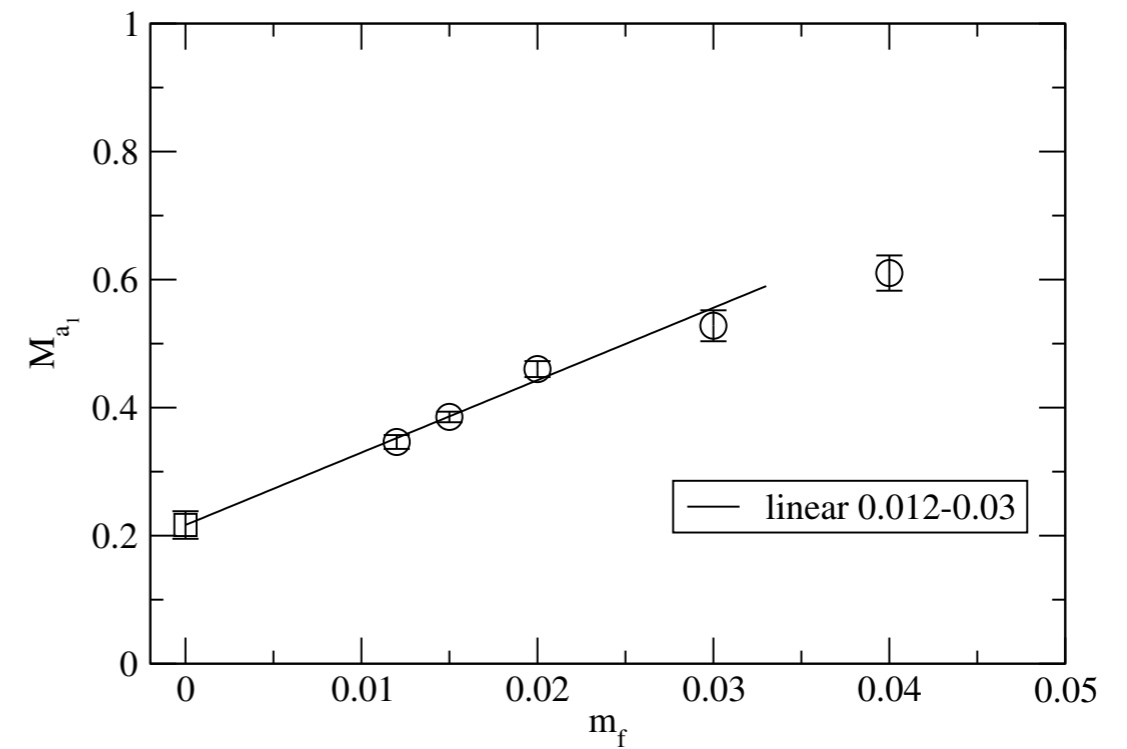
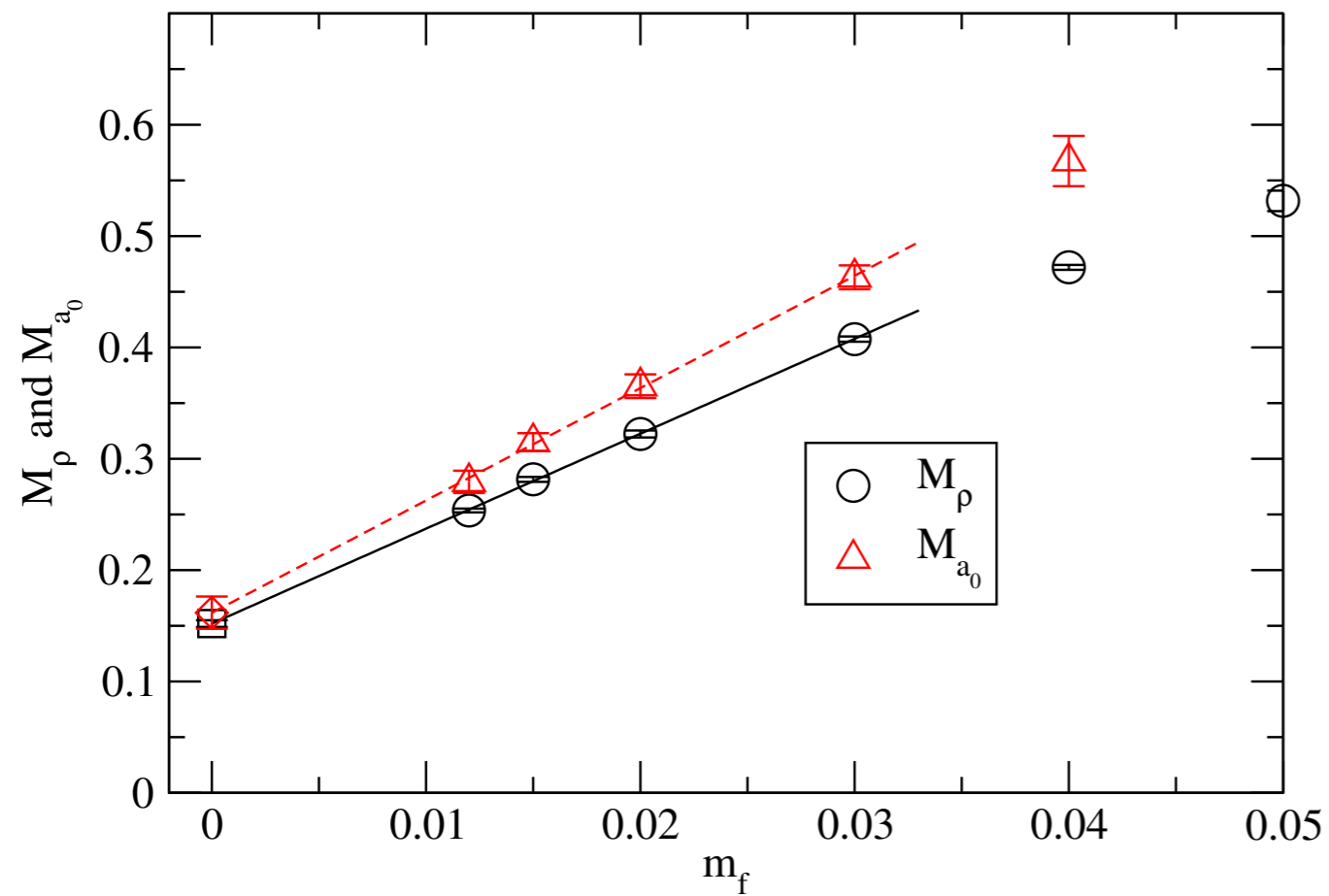
(with model structure)



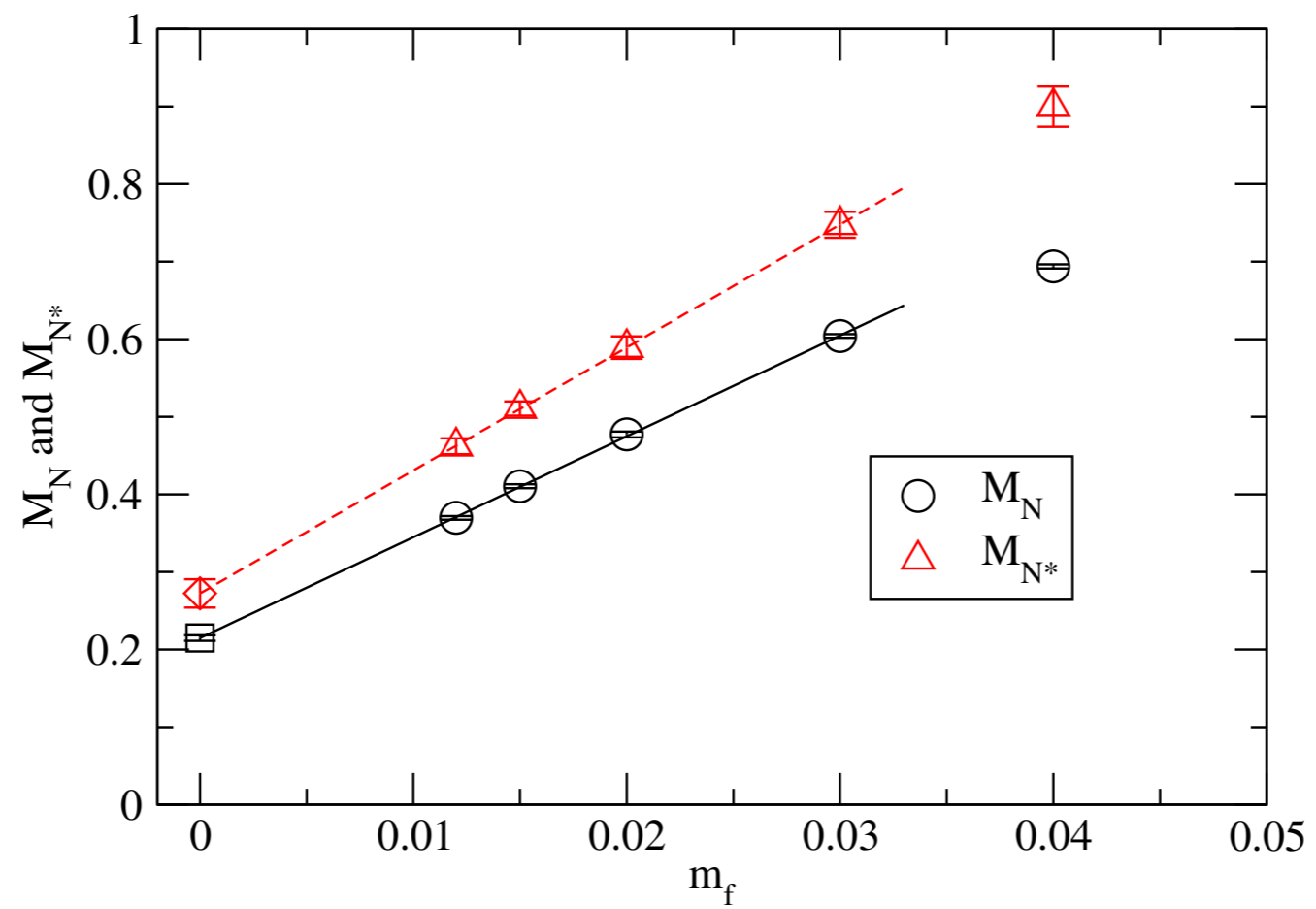
prediction to compare with experiment

following is “preliminary” results

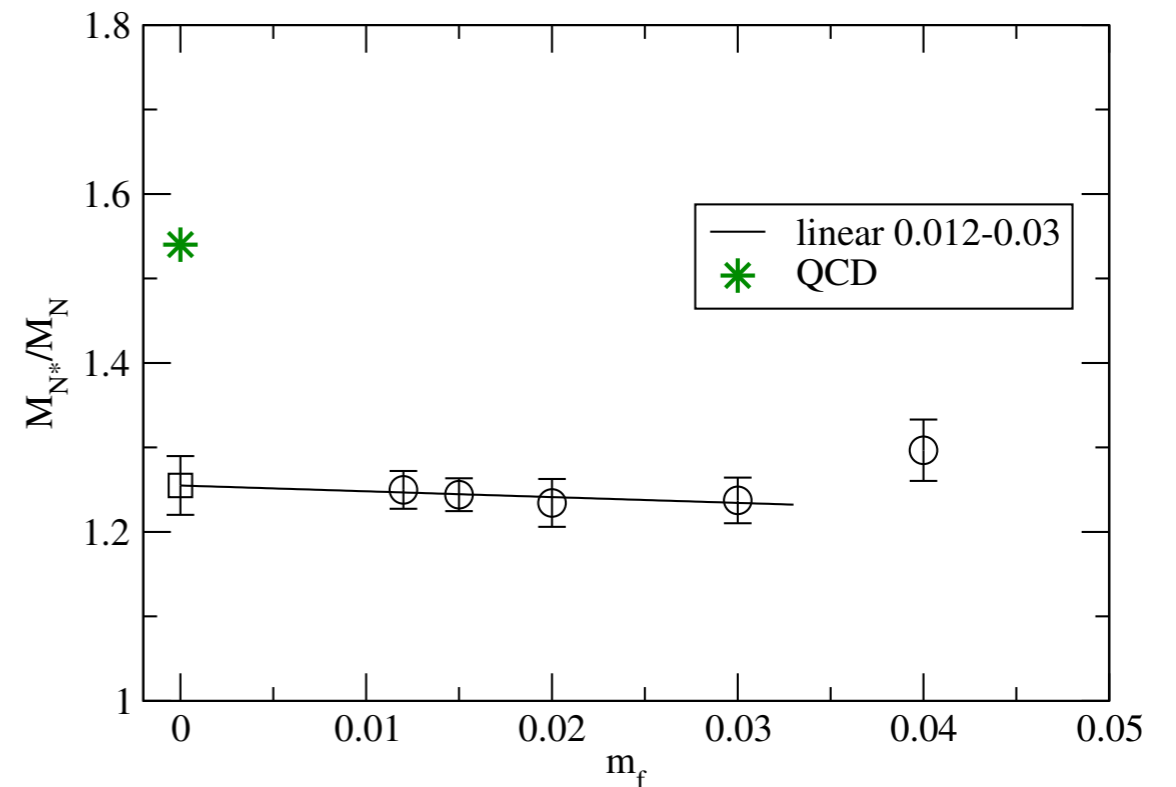
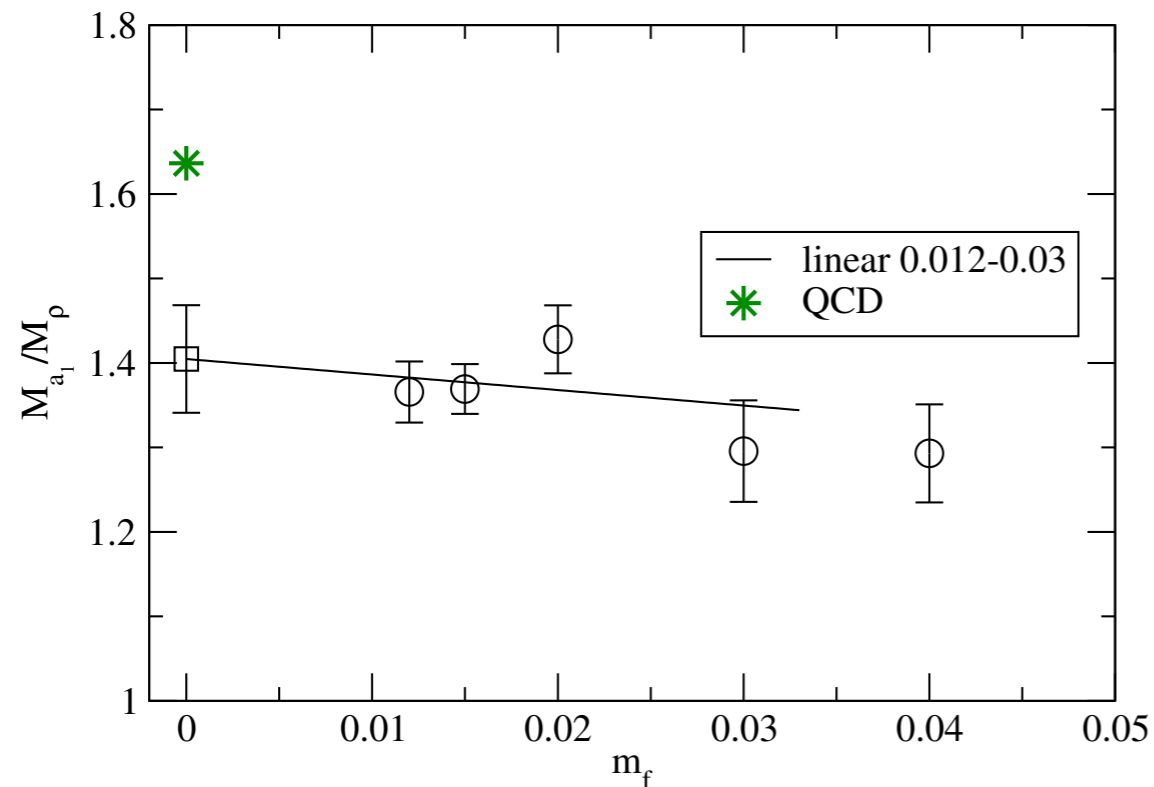
mesons: ρ , a_0 , a_1 , b_1



baryons: N , N^*



mass ratio compared with real-life QCD



- moving toward “parity doubling” from smaller N_f to $N_f=8$
 - consistent with LSD collab. with domain-wall fermions

N_f=8 composite spectrum

$$\frac{M_\rho}{F/\sqrt{2}} = 10.1(0.6)^{(+5.0)}_{(-2.5)}$$

$$\sqrt{N_d}F/\sqrt{2} = 246 \text{ GeV}$$

- NLO chiral log correction for F: dominant systematic error
- N_d depends on the model
- e.g. one family model: N_d=4 → M_ρ ~ 1.2 TeV

ρ	a_0	a_1	b_1	N	N^*
10.1(0.6) ^(+5.0) _(-2.5)	10.8(1.1) ^(+5.3) _(-2.7)	14.4(1.7) ^(+7.1) _(-3.6)	13.3(2.1) ^(+6.6) _(-3.3)	14.3(0.9) ^(+7.0) _(-3.5)	18.1(1.6) ^(+8.9) _(-4.5)

TABLE X. Ratios of $\sqrt{2}M_H/F$ with $H = \rho, a_0, a_1, b_1, N$, and N^* . The first and second errors are statistical and systematic errors.

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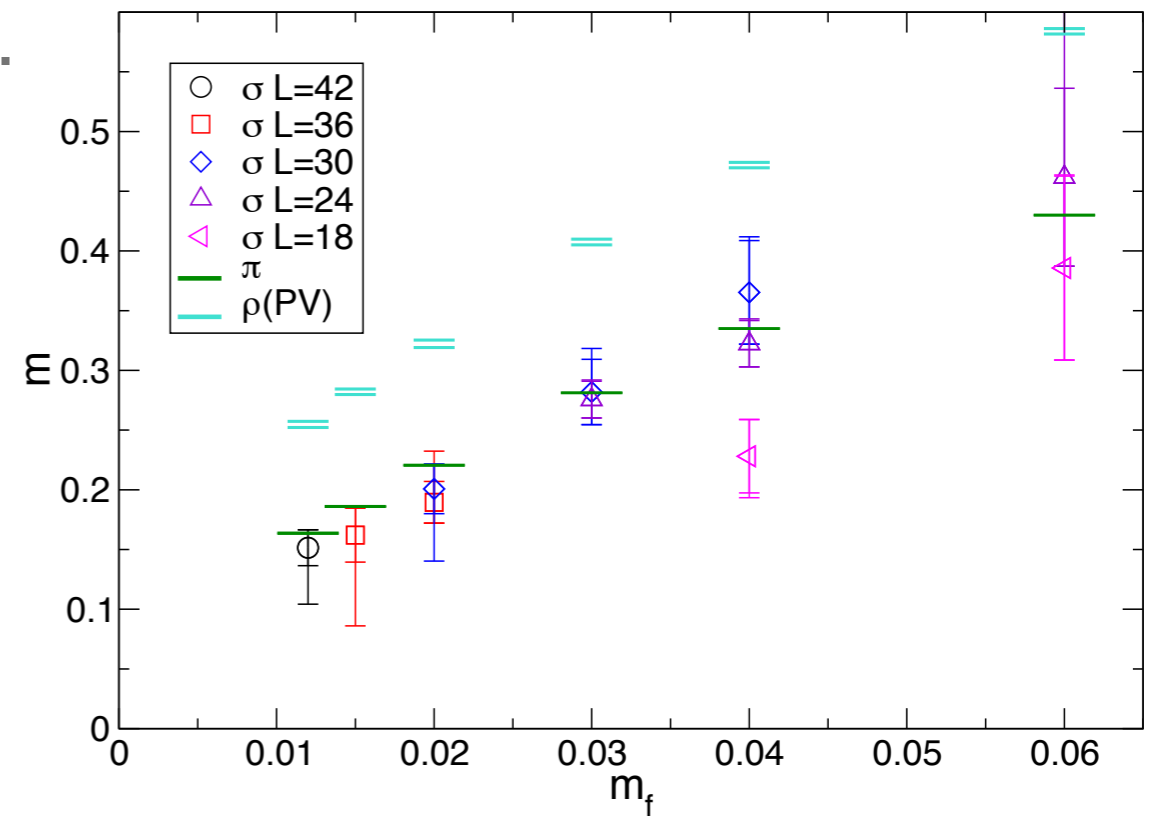
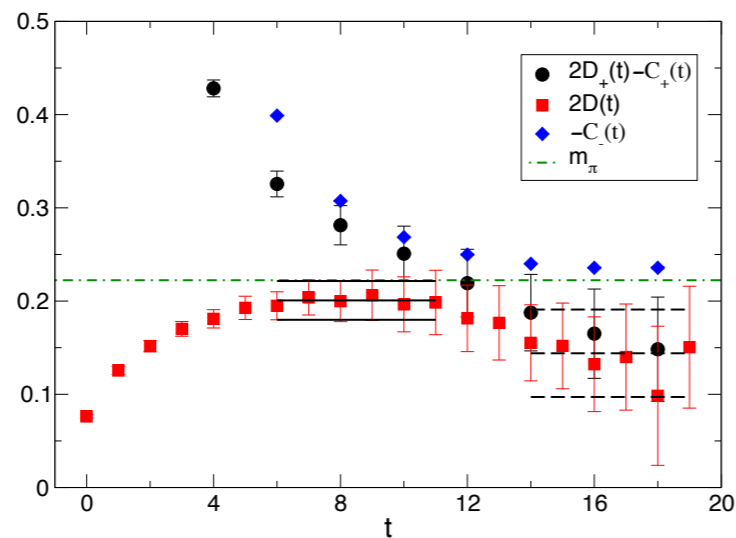
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flavor singlet scalar : Higgs channel

- flavor singlet: known as a difficult problem in lattice QCD
- variance reduction method & high stat.
- σ as light as π
 - ➔ similar to $N_f=12$ [LatKMI 2013]
- clearly lighter than ρ
 - ➔ far from heavy quark limit



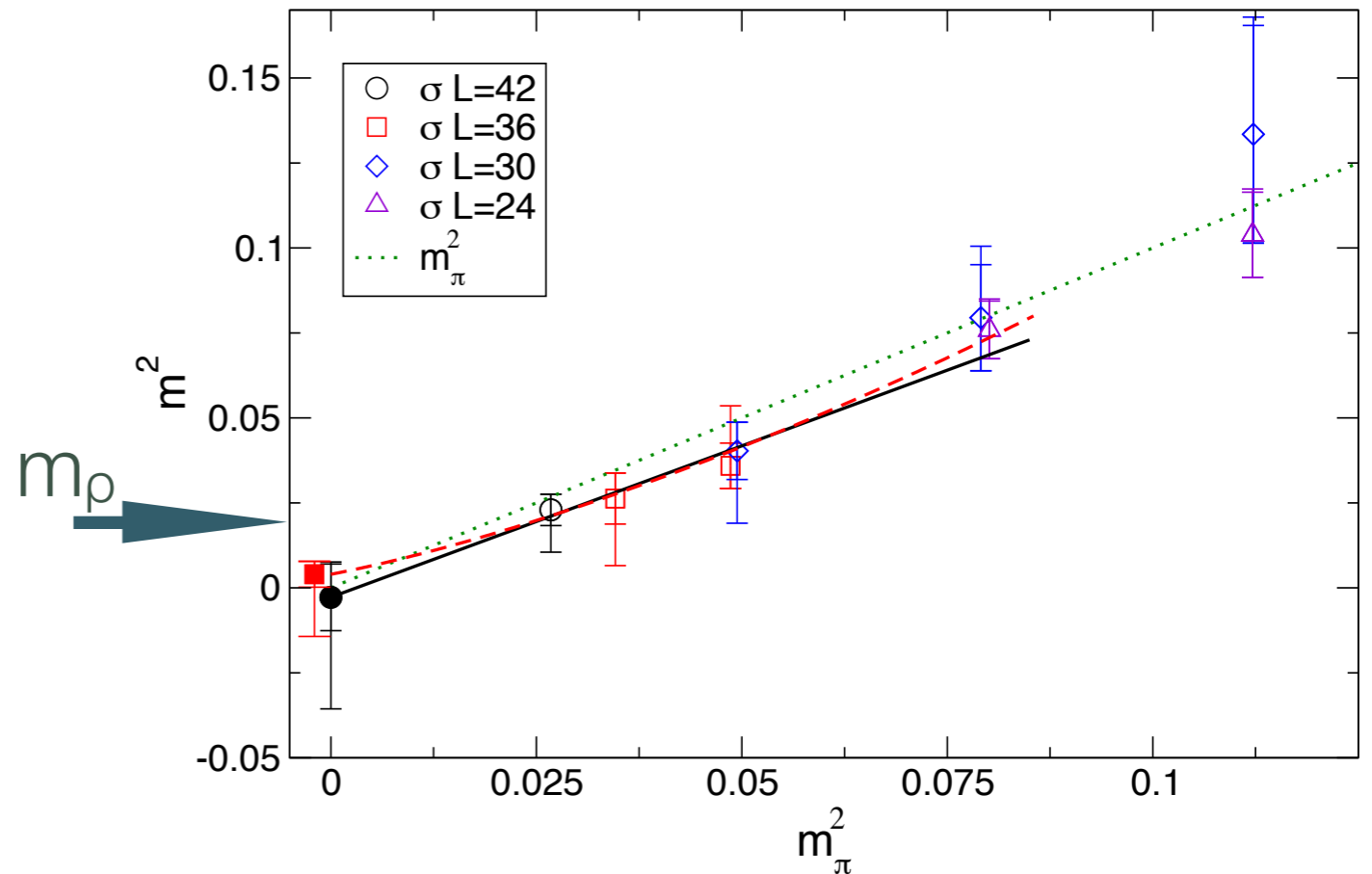
- (update from PRD 2014 LatKMI)

FIG. 16. Effective scalar mass m_σ from correlators with the projection for $L = 30$, $m_f = 0.02$.

trial chiral extrapolation for $N_f=8$ SU(3) m_σ

[LatKMI NEW: update from PRD2014]

- though it is too far, so far
- 2 ways:
 - naive linear $m_\sigma = c_0 + c_1 m_f$
 - dilaton ChPT $m_\sigma^2 = d_0 + d_1 m_\pi^2$ (Matsuzaki-Yamawaki 2013)
- differ only at higher order
- possibility to have $\sim 125\text{GeV}$ Higgs
 - $F/\sqrt{2} = 123$ GeV one-family model
- lighter mass data needed!



$$c_0 = 0.063(30)(+4-142)$$

$$d_0 = -0.0028(98)(+36-313)$$

$$\text{c.f. } m_\sigma = F/\sqrt{2} \rightarrow c_0 = 0.014 \text{ || } d_0 = 0.0002$$

$$d_1 = 0.89(26)(+75-12)$$

$$\text{c.f. } d_1 \sim 1 \text{ (holographic: } F_\sigma \sim \sqrt{N_f} F)$$

[Matsuzaki & Yamawaki 2012]

Other low energy constant: on-going project

- Vector / Axial vector decay constant \leftrightarrow by-product of S parameter meas.
- dilaton decay constant
 - mainly by Ohki



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If you are interested,
please contact / discuss with him

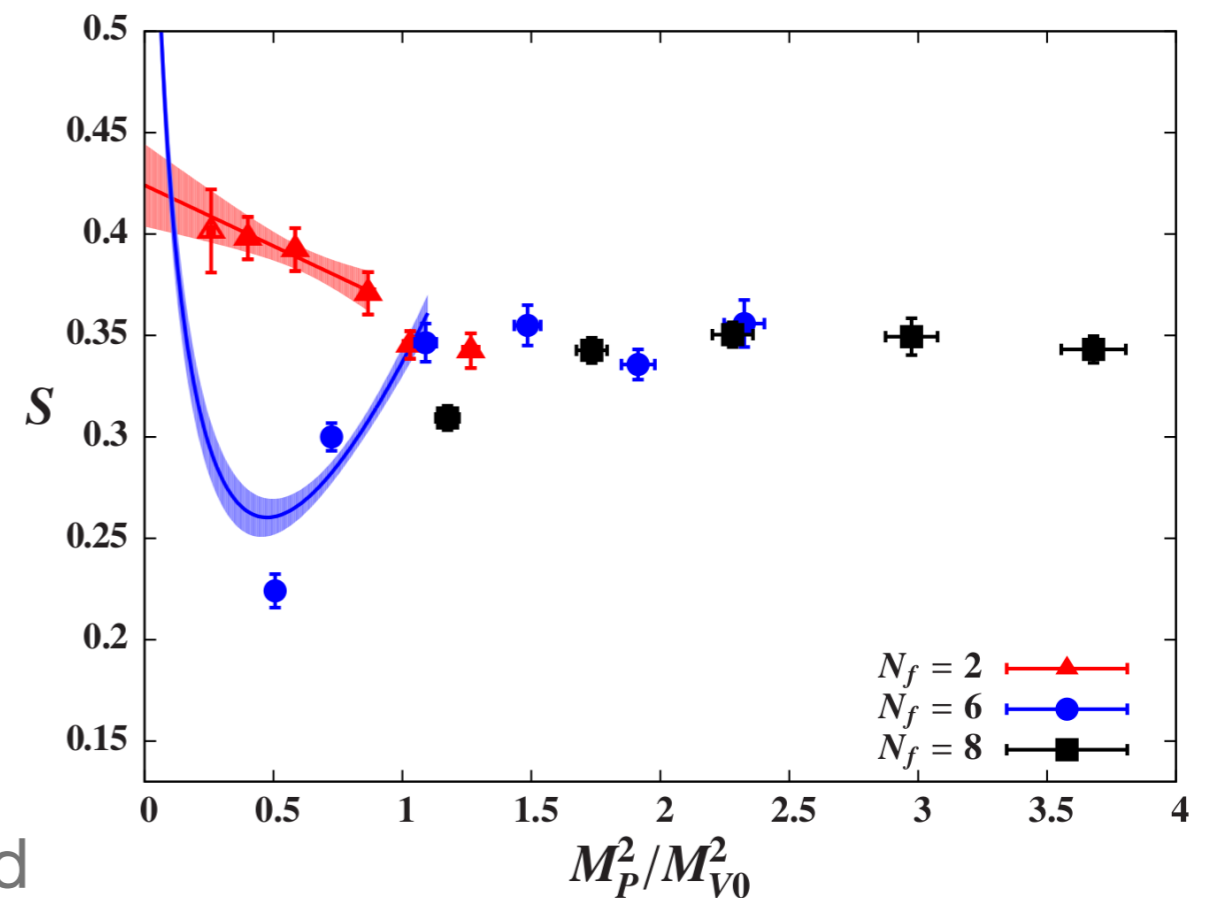
S parameter: calculation method

- calculated through vacuum polarization function of flavor non-singlet currents
- lattice calculation suffers from power divergence without exact chiral symm.
- so far, overlap and domain-wall fermion methods are reported
- Overlap
 - $N_f=2$ SU(3) [JLQCD Shintani et al PRL 2008]
- Domain-wall
 - $N_f=2+1$ SU(3) [RBC/UKQCD Boyle et al PRD 2010]
- Staggered (utilizing exact non-singlet symmetry due to multiple fields)
 - $N_f=4n$ system setup on HISQ [Aoki (LatKMI) Lattice 2013]

S parameter of QCD with N_f fundamental fermions

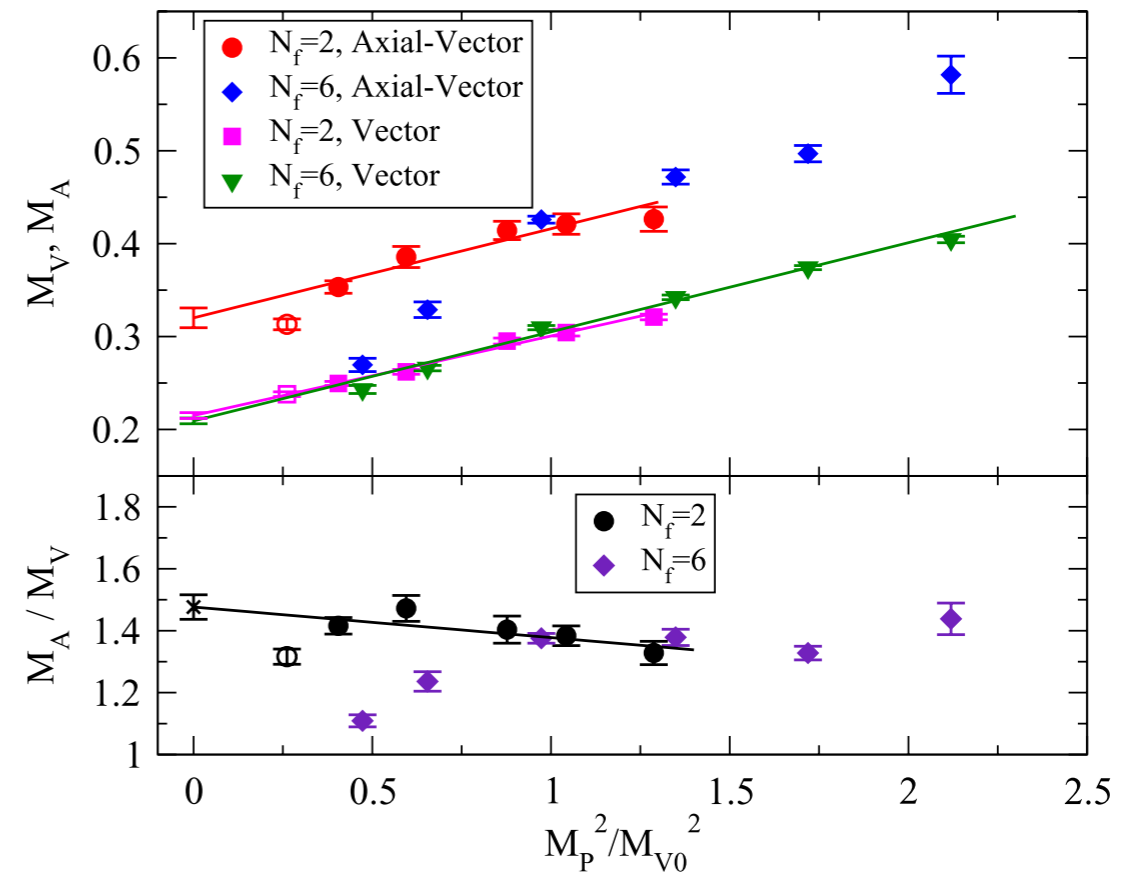
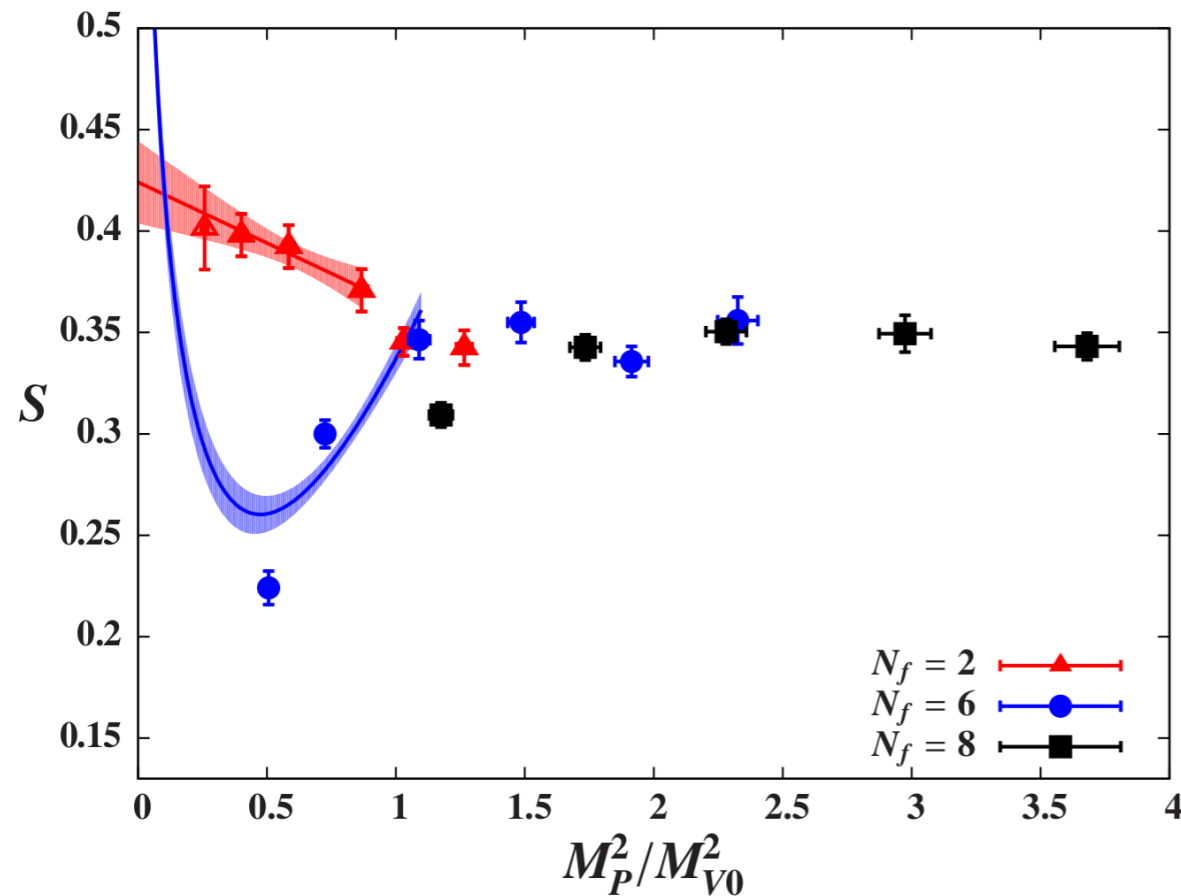
[LSD, PRL 2011 & PRD 2014]

- domain wall fermions with $N_f=2,6,8$
- one doublet has EW charge \rightarrow
- $N_f=6$
 - decreases as m_f enters chiral regime
 - turns up after chiral log sets in
 - low value of S possible for unabsorbed massive pions
- $N_f=8$
 - similar trend as $N_f=6$, but not conclusive



S parameter of $N_f=6$ and parity doubling

[LSD, PRL 2011 & PRD 2014]

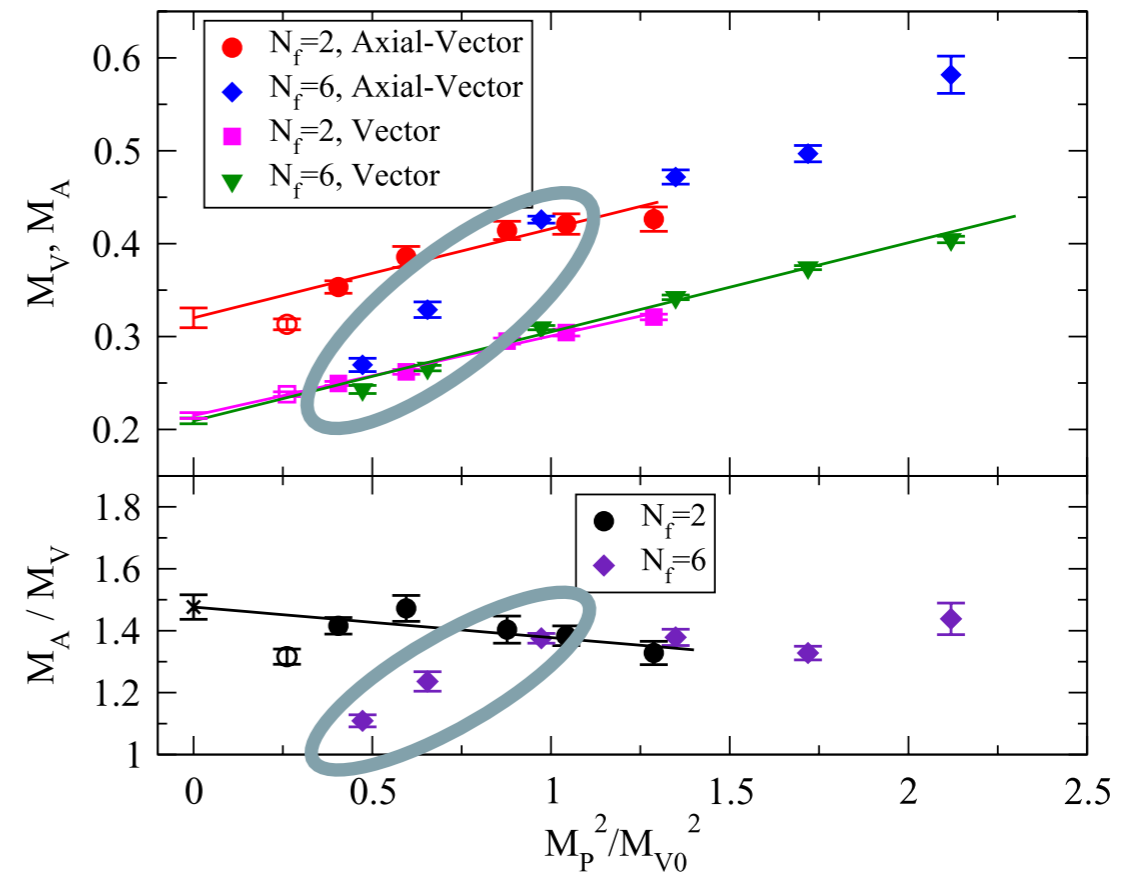
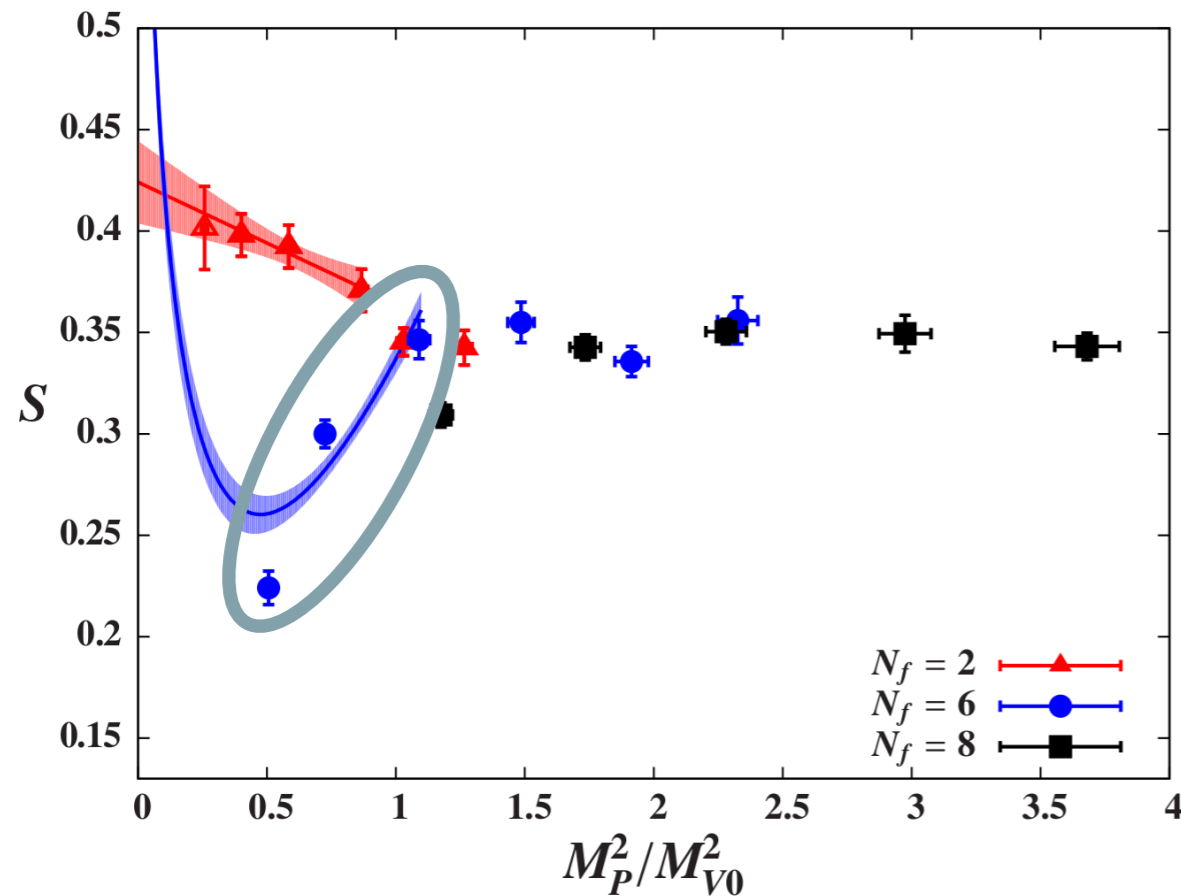


$$\begin{aligned}
 S &= 4\pi(N_f/2)[\Pi'_{VV}(0) - \Pi'_{AA}(0)] - \Delta S_{\text{SM}} \\
 &= \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \left\{ (N_f/2)[R_V(s) - R_A(s)] \right. \\
 &\quad \left. - \frac{1}{4} \left[1 - \left(1 - \frac{m_H^2}{s} \right)^3 \theta(s - m_H^2) \right] \right\},
 \end{aligned}$$

- parity doubling observed in the spectrum is consistent with the decrease

S parameter of $N_f=6$ and parity doubling

[LSD, PRL 2011 & PRD 2014]



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- parity doubling observed in the spectrum is consistent with the decrease

currents and Ward-Takahashi identities: HISQ

- obtained through local variation on the action under the exact symmetries
- one link and Naik term (Asqtad, HISQ) \rightarrow one link and three link terms

- $\mathcal{V}_\mu^{(1,2)}(x) = V_\mu^{(1,2)}(x) + \text{three link terms}$

- $\Delta_\mu \langle \mathcal{V}_\mu^{(1,2)}(x) O(0) \rangle = \langle \delta_V O(0) \rangle$

- $\mathcal{A}_\mu^{(1,2)}(x) = A_\mu^{(1,2)}(x) + \text{three link terms}$

- $\Delta_\mu \langle \mathcal{A}_\mu^{(1,2)}(x) O(0) \rangle = 2m_f \langle P^{(1,2)} O(0) \rangle + \langle \delta_A O(0) \rangle$

current-current correlation function for VPF: HISQ

- Conserved - OneLink correlators

$$\langle \mathcal{V}_\mu^{(1,2)}(x) V_\nu^{(2,1)}(0) \rangle$$

$$\Delta_\mu \langle \mathcal{V}_\mu^{(1,2)}(x) V_\nu^{(2,1)}(0) \rangle = (\delta_{x,0} - \delta_{x,0+\hat{\nu}}) \langle \tilde{V}_\nu^{(1,2)}(0) \rangle$$

$$(\langle \mathcal{V}_\mu(x) V_\nu(0) \rangle^{\text{sub}} = \langle \mathcal{V}_\mu(x) V_\nu(0) \rangle - \delta_{\mu,\nu} \delta_{x,0} \langle \tilde{V}_\nu(0) \rangle \rightarrow \text{HVPF})$$

$$\langle \mathcal{A}_\mu^{(1,2)}(x) A_\nu^{(2,1)}(0) \rangle$$

$$\Delta_\mu \langle \mathcal{A}_\mu^{(1,2)}(x) A_\nu^{(2,1)}(0) \rangle = 2m_f \langle P^{(1,2)}(x) A_\nu^{(2,1)}(0) \rangle + (\delta_{x,0} - \delta_{x,0+\hat{\nu}}) \langle \tilde{V}_\nu^{(1,2)}(0) \rangle$$

- OneLink - OneLink used for sanity check of renormalization

$$\langle V_\mu^{(1,2)}(x) V_\nu^{(2,1)}(0) \rangle \qquad \langle A_\mu^{(1,2)}(x) A_\nu^{(2,1)}(0) \rangle$$

renormalization

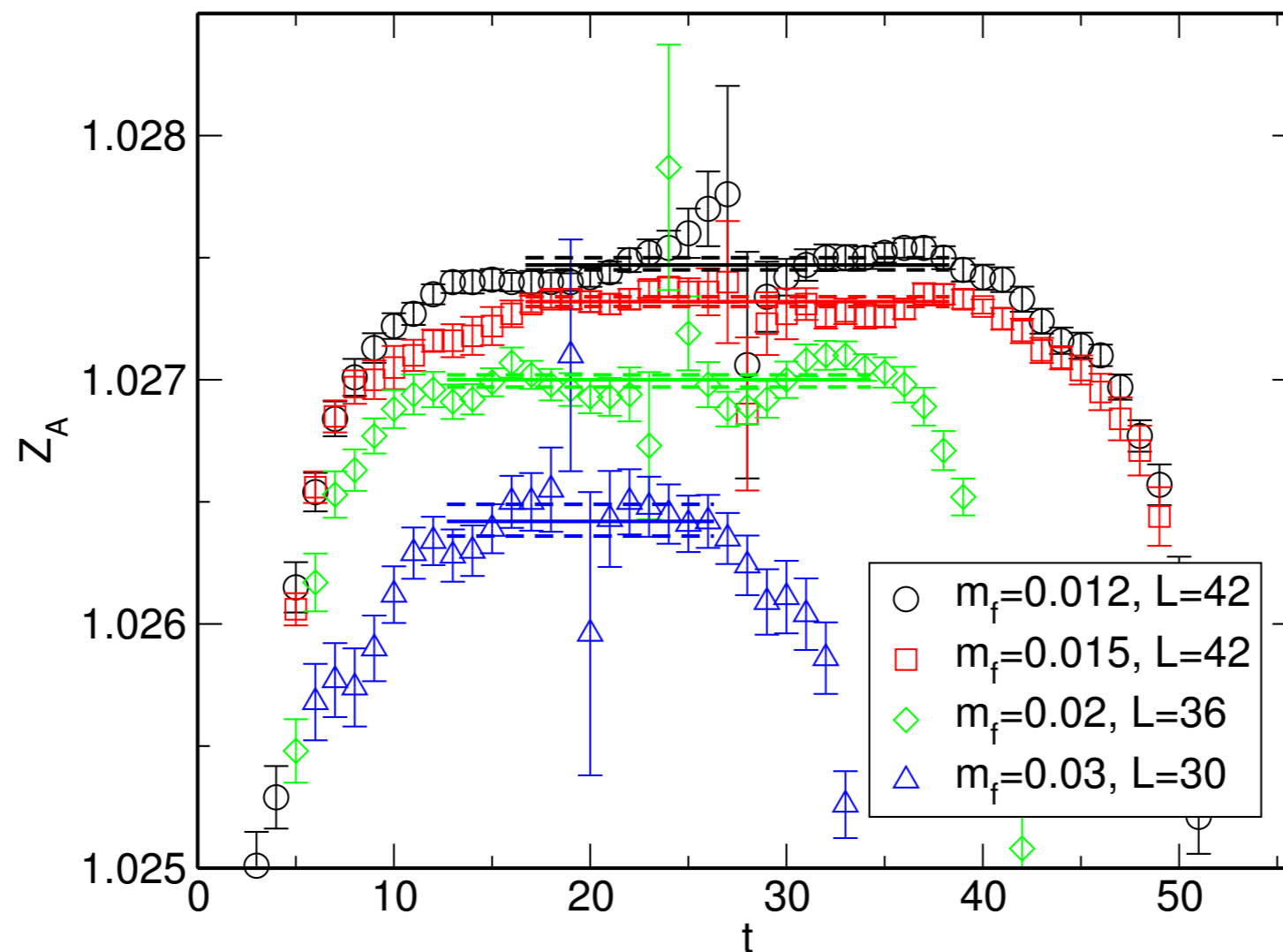
- sink: conserved, src: 1 link: src op needs renormalization

$$\langle \mathcal{A}_\mu^{(1,2)}(x) A_\nu^{(2,1)}(0) \rangle$$

- $\mathcal{A}_\mu = Z_A A_\mu$ $Z_A = \frac{\langle \mathcal{A}_4(t) P(0) \rangle}{\langle A_4(t) P(0) \rangle}$
- $\mathcal{V}_\mu = Z_V V_\mu$
- $Z_V = Z_A$ for $m_f \rightarrow 0$ (chiral symmetry)

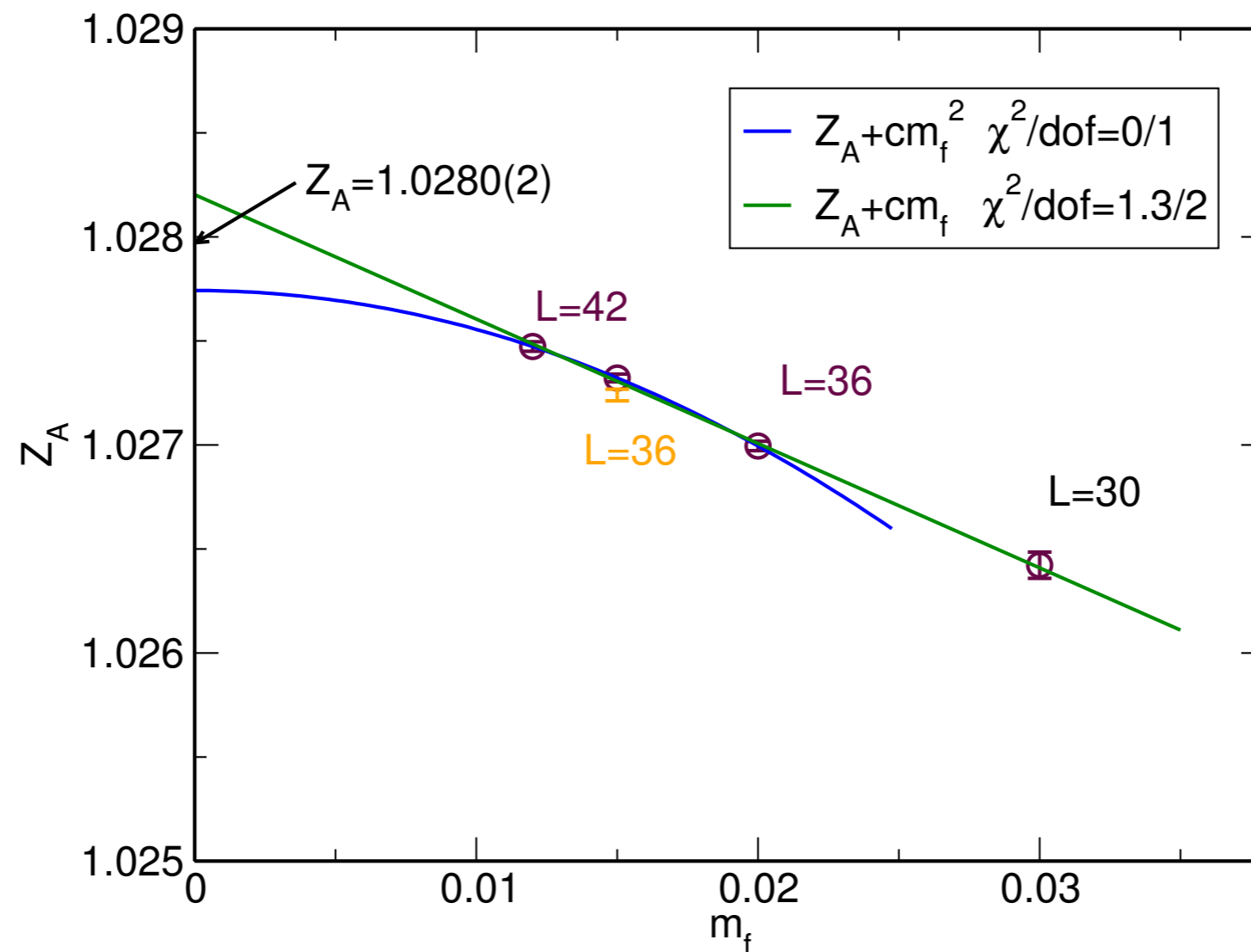
non-perturbative calculation of Z_A

- $N_f=8$, HISQ, $\beta=3.8$, $L^3 \times T$ ($L/T=4/3$)
- choice of link in one link current: HISQ w/o Naik (so to mimic conserved)

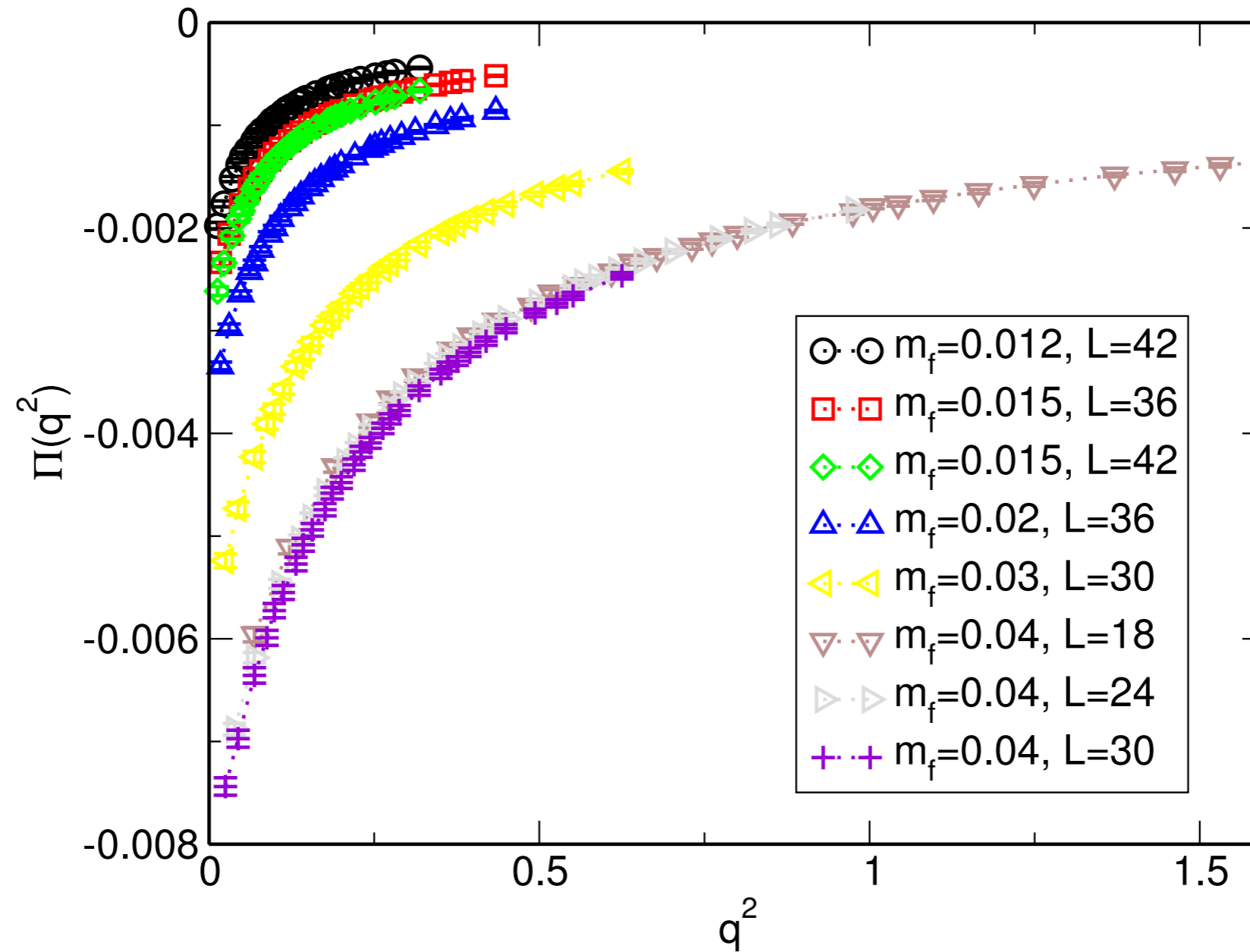


non-perturbative calculation of Z_A

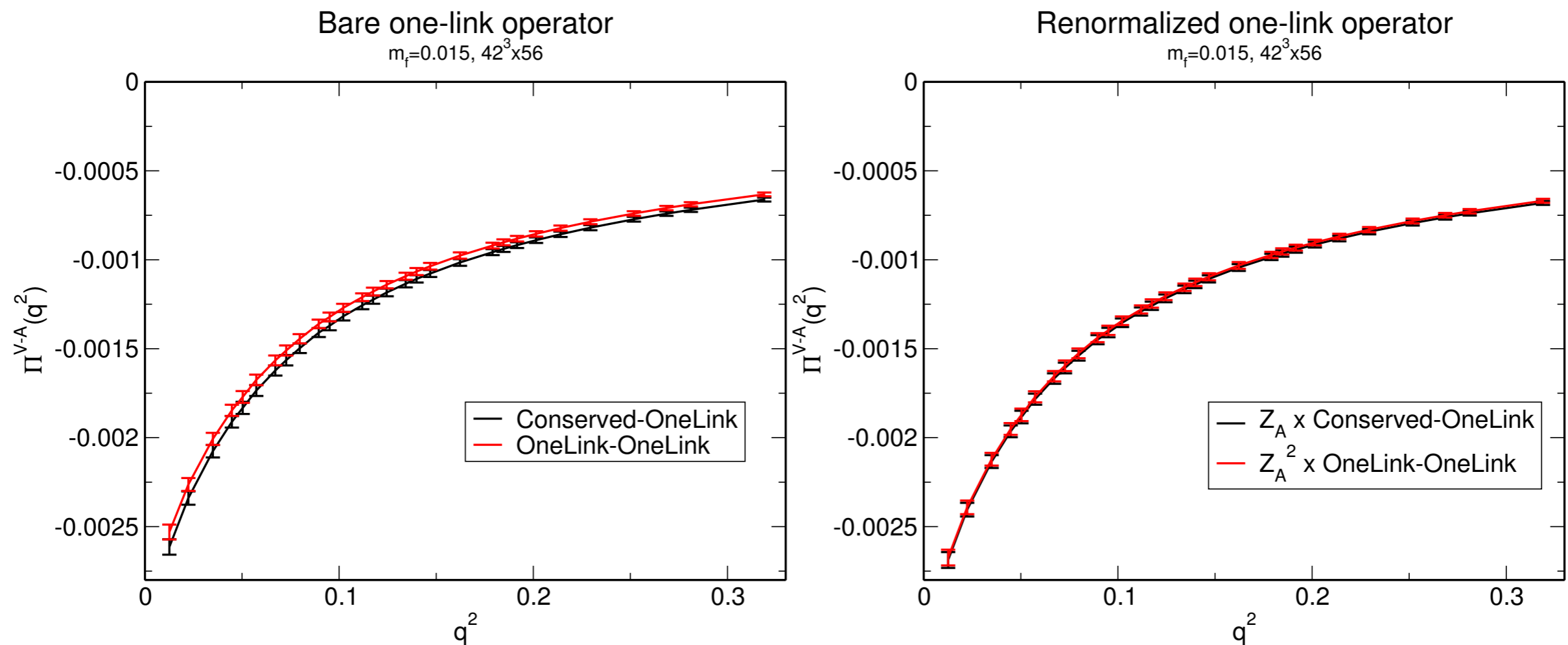
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- choice of link in one link current: HISQ w/o Naik (so to mimic conserved)



V-A vacuum polarization function (transverse)

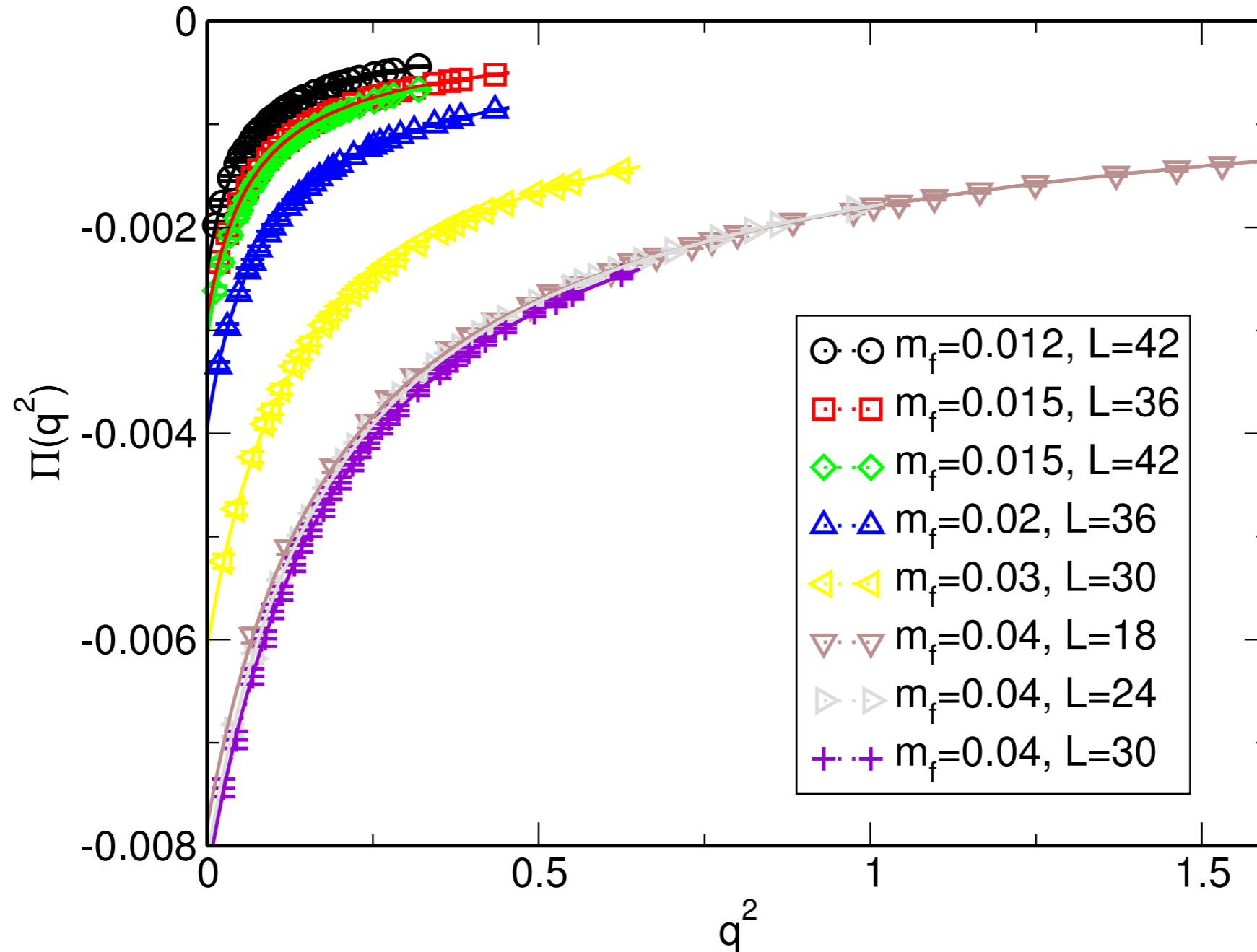


sanity check of non-perturbative renormalization

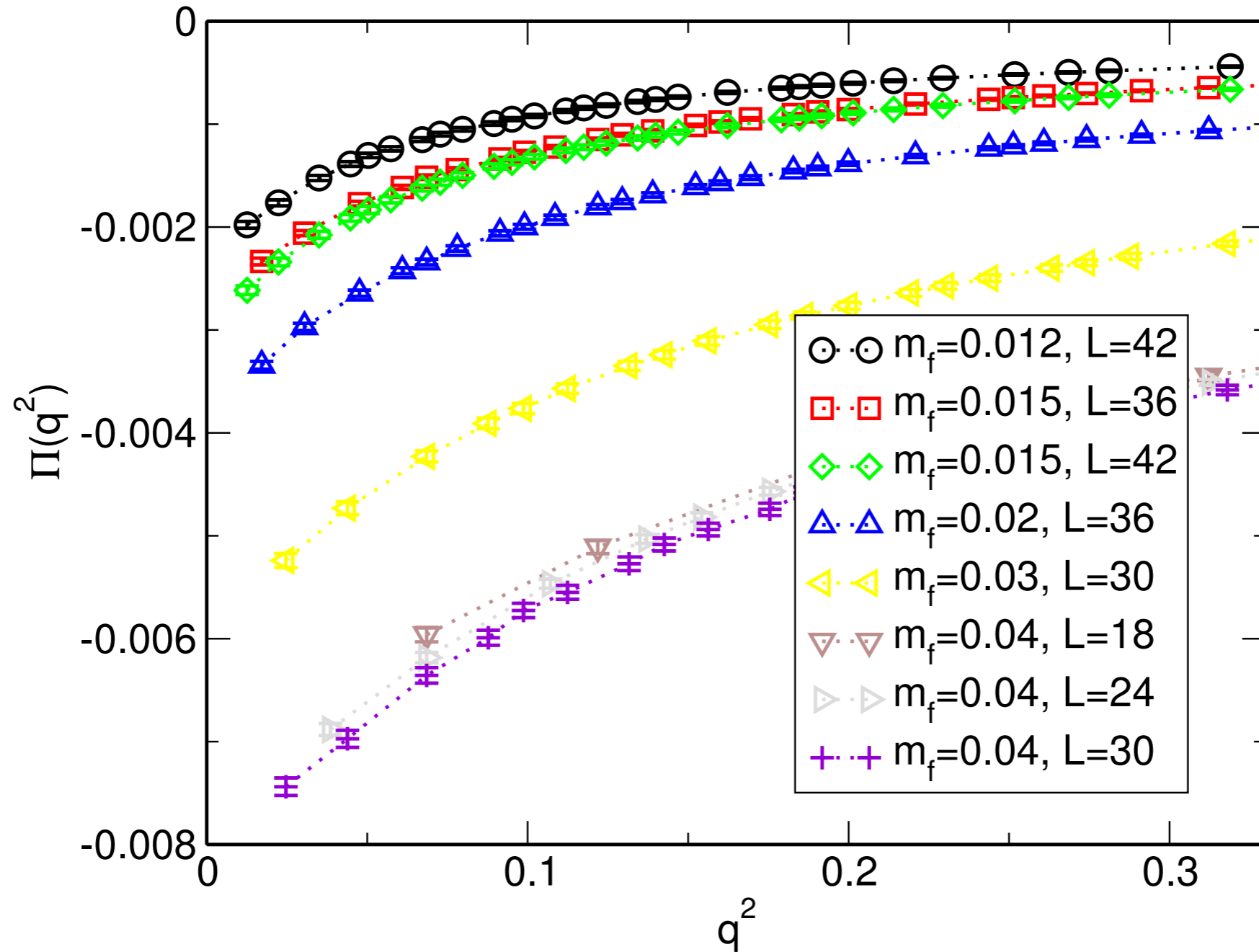


- non-perturbative renormalization is working, especially well at low q^2

V-A vacuum polarization function: Pade(1,2) fit

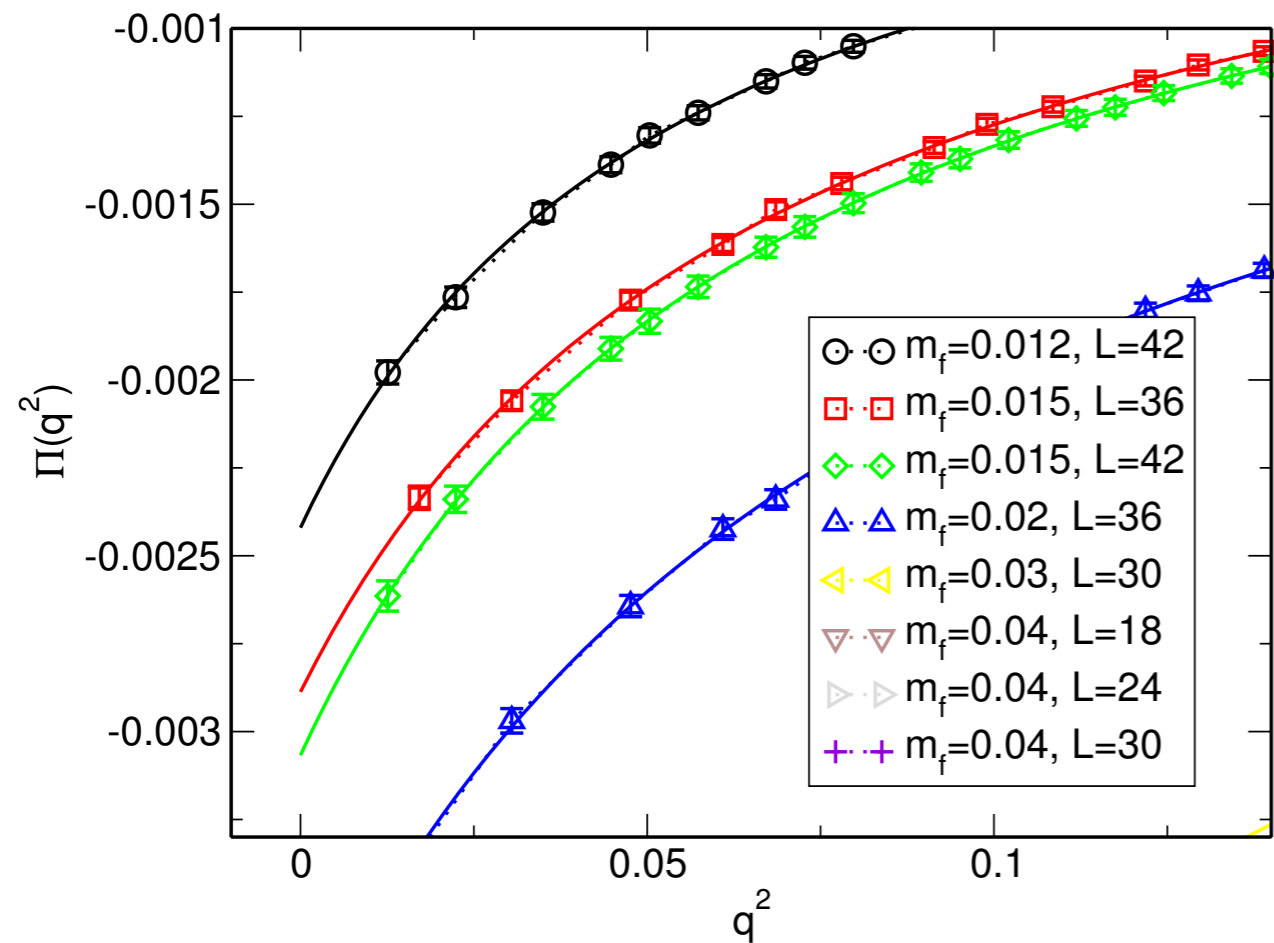


V-A vacuum polarization function: Pade(1,2) fit

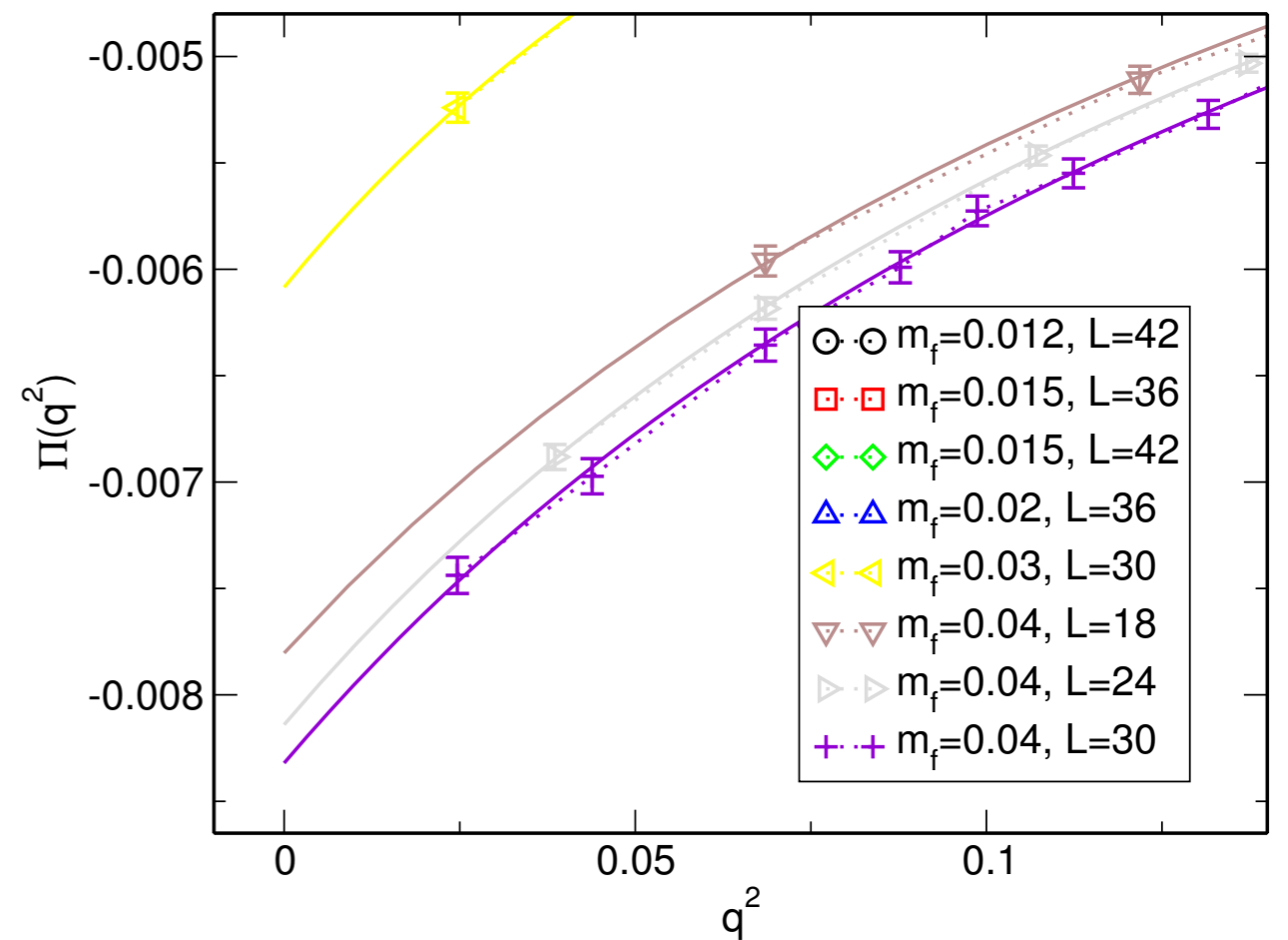


V-A vacuum polarization function: close-up

for $m_f=0.015$

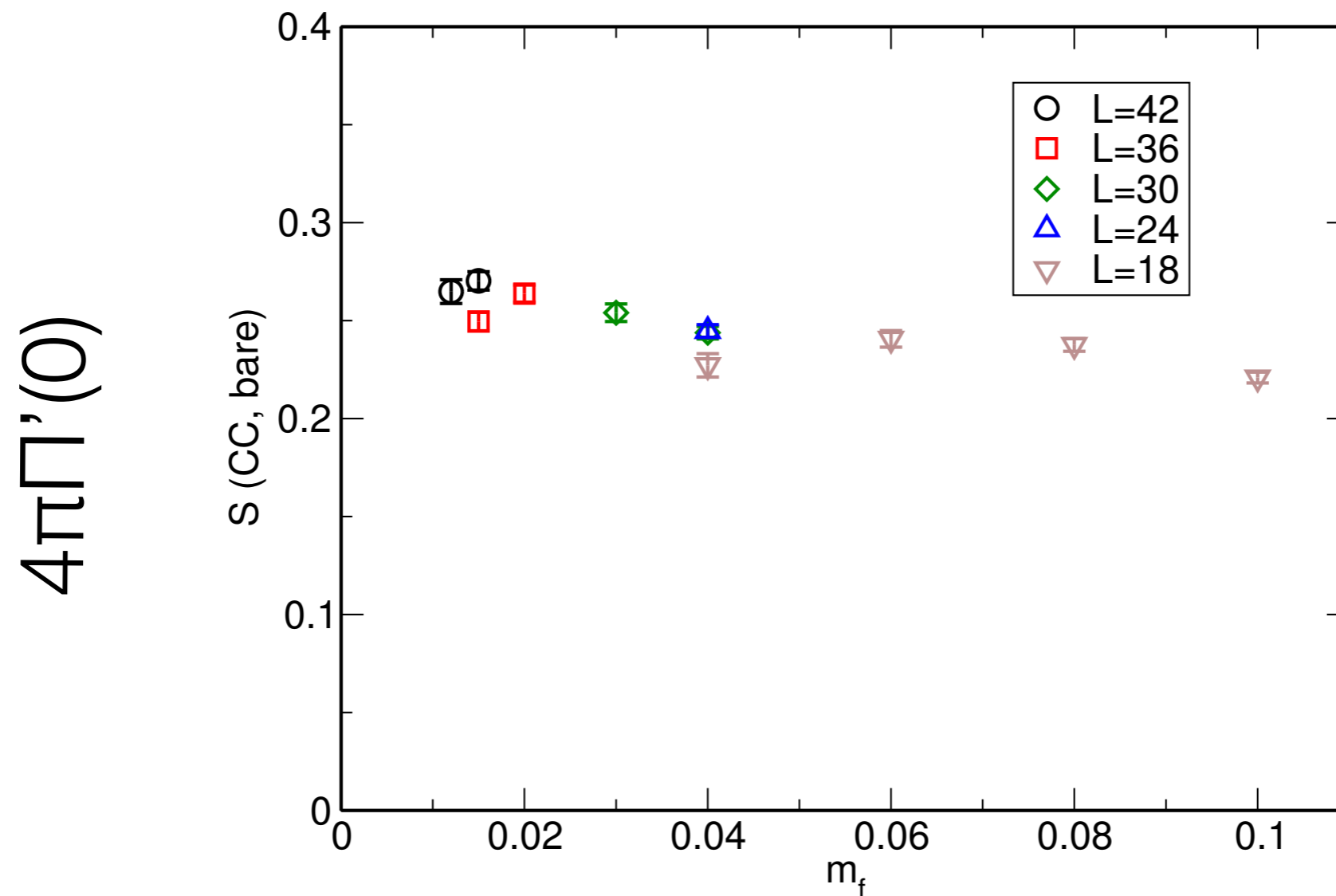


for $m_f=0.04$



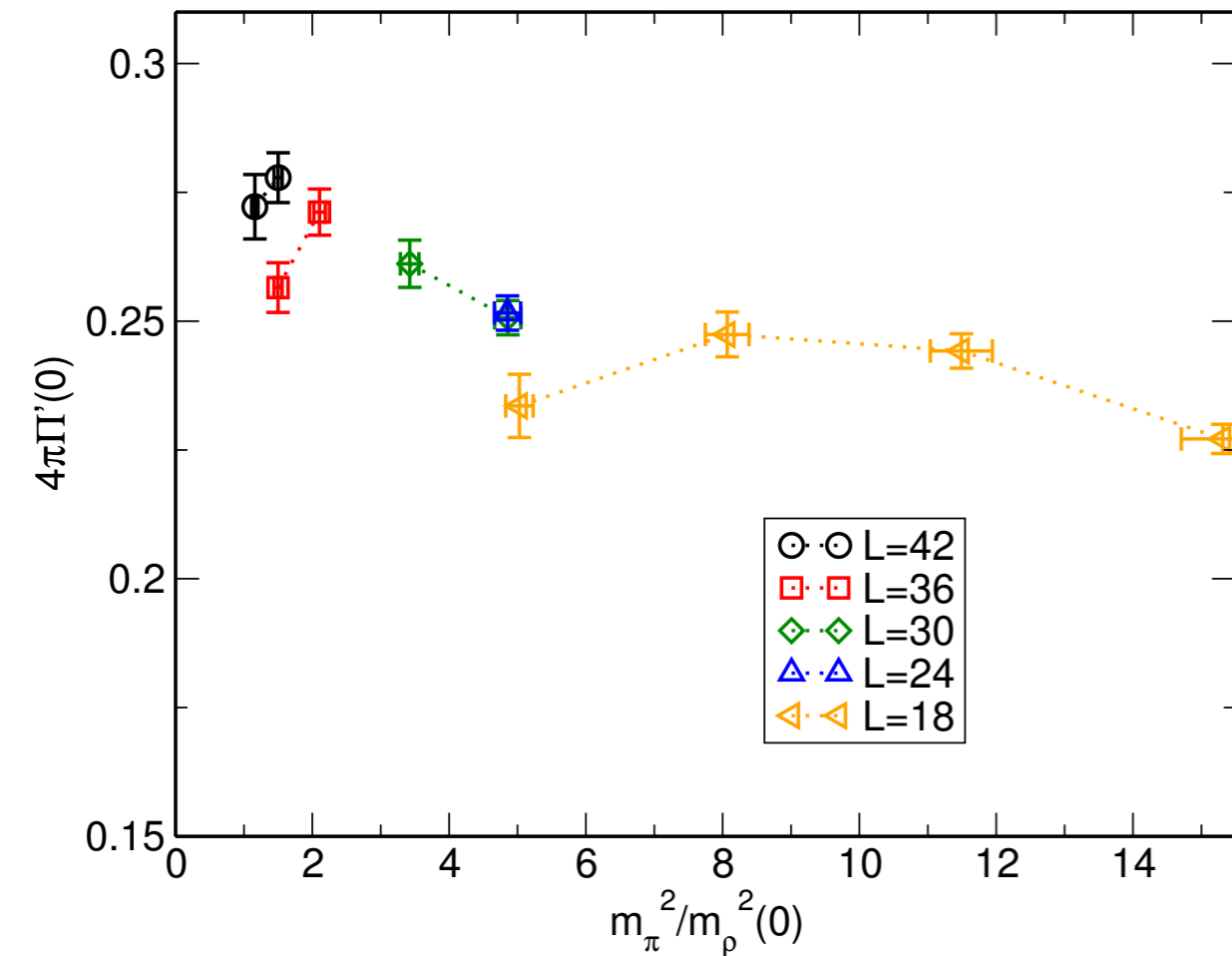
- finite volume effect on $\Pi(q^2)$ and $\Pi'(0)$?

$S(m_f)$: TC contribution per EW doublet (preliminary)



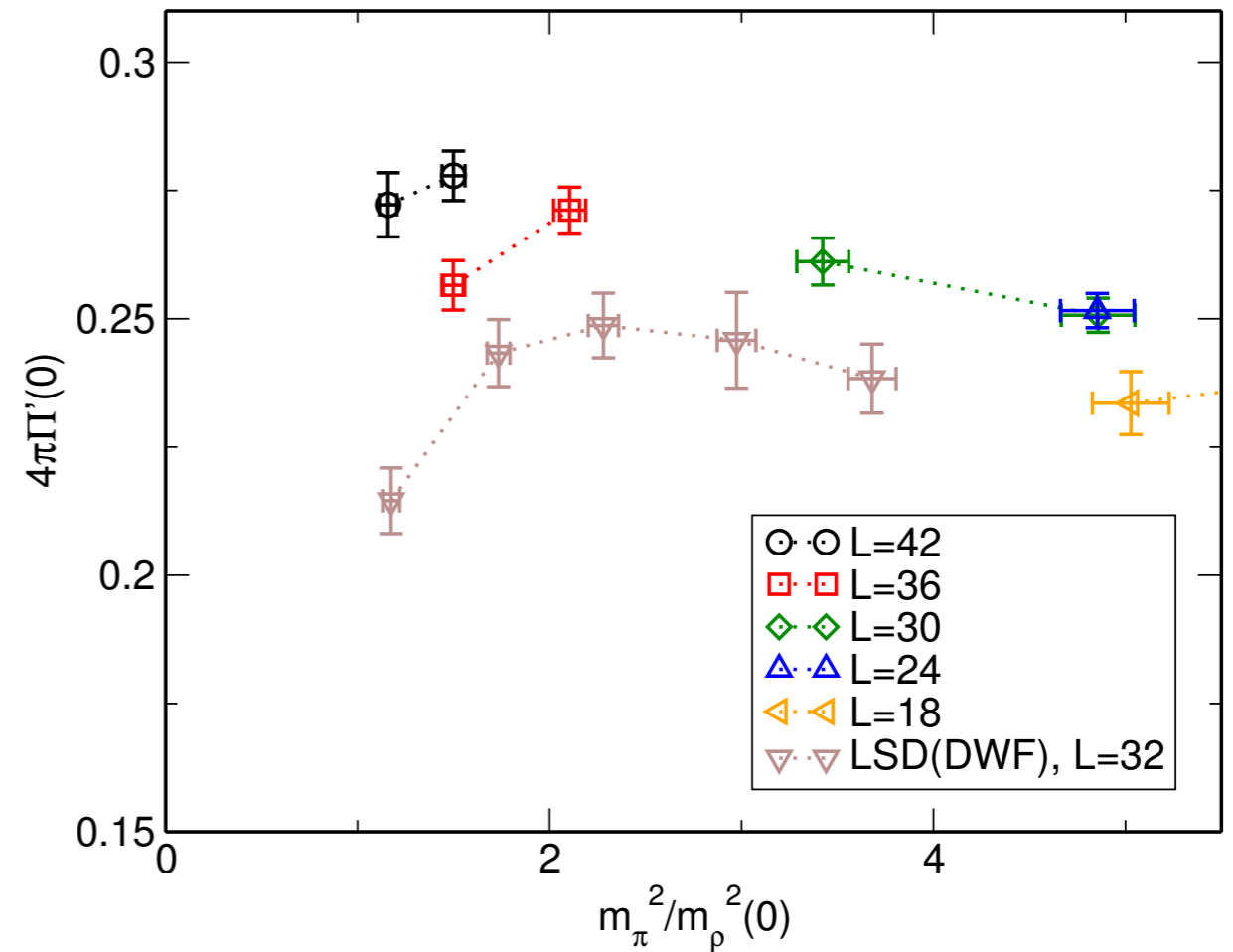
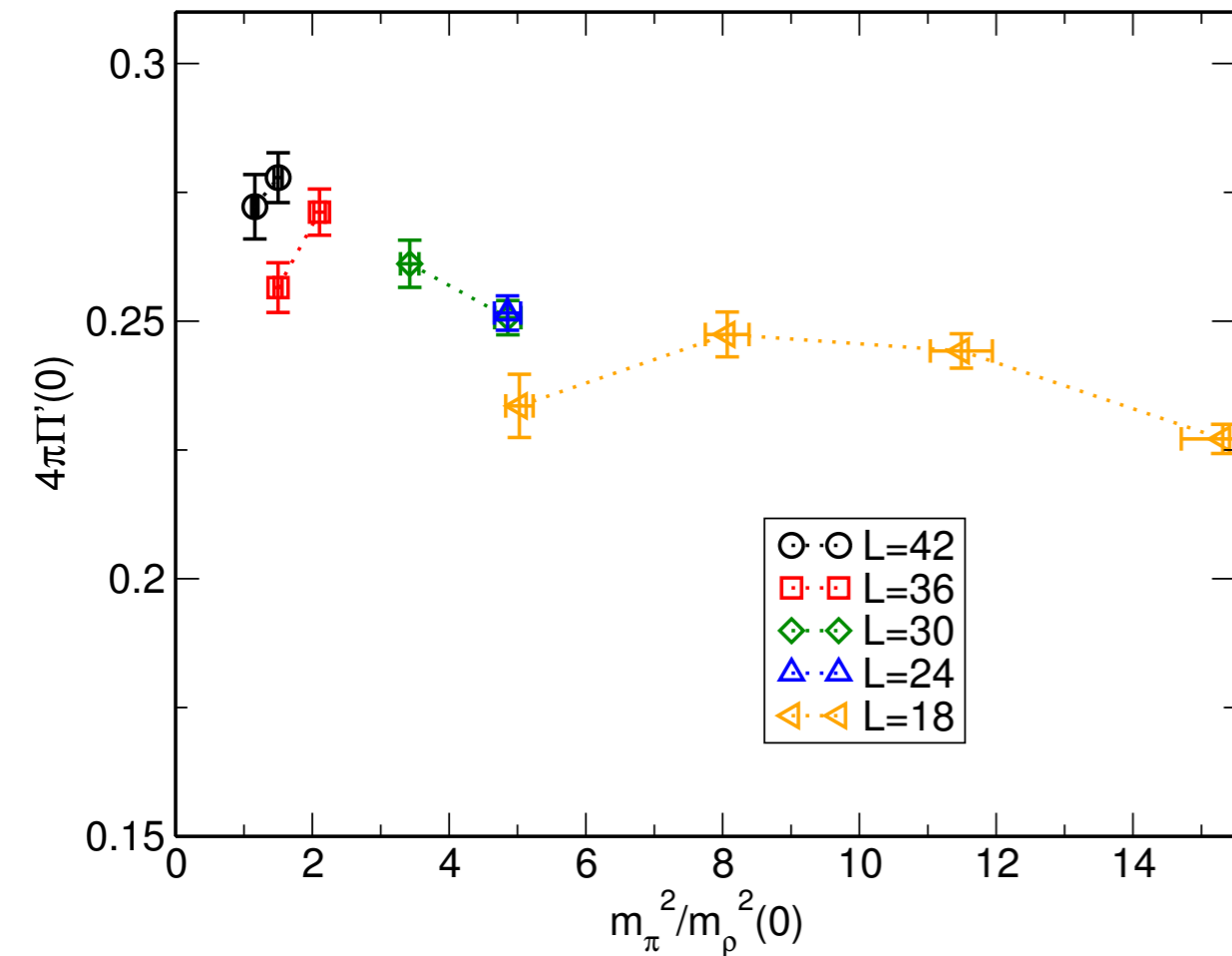
- finite size effect, somehow large, observed
- 8% \downarrow @ $m_f=0.02$; $L=42 \rightarrow 36$ c.f. pion mass: 0.04% \downarrow (zero consistent)

$S(m_\pi^2)$: TC contribution per EW doublet (preliminary)



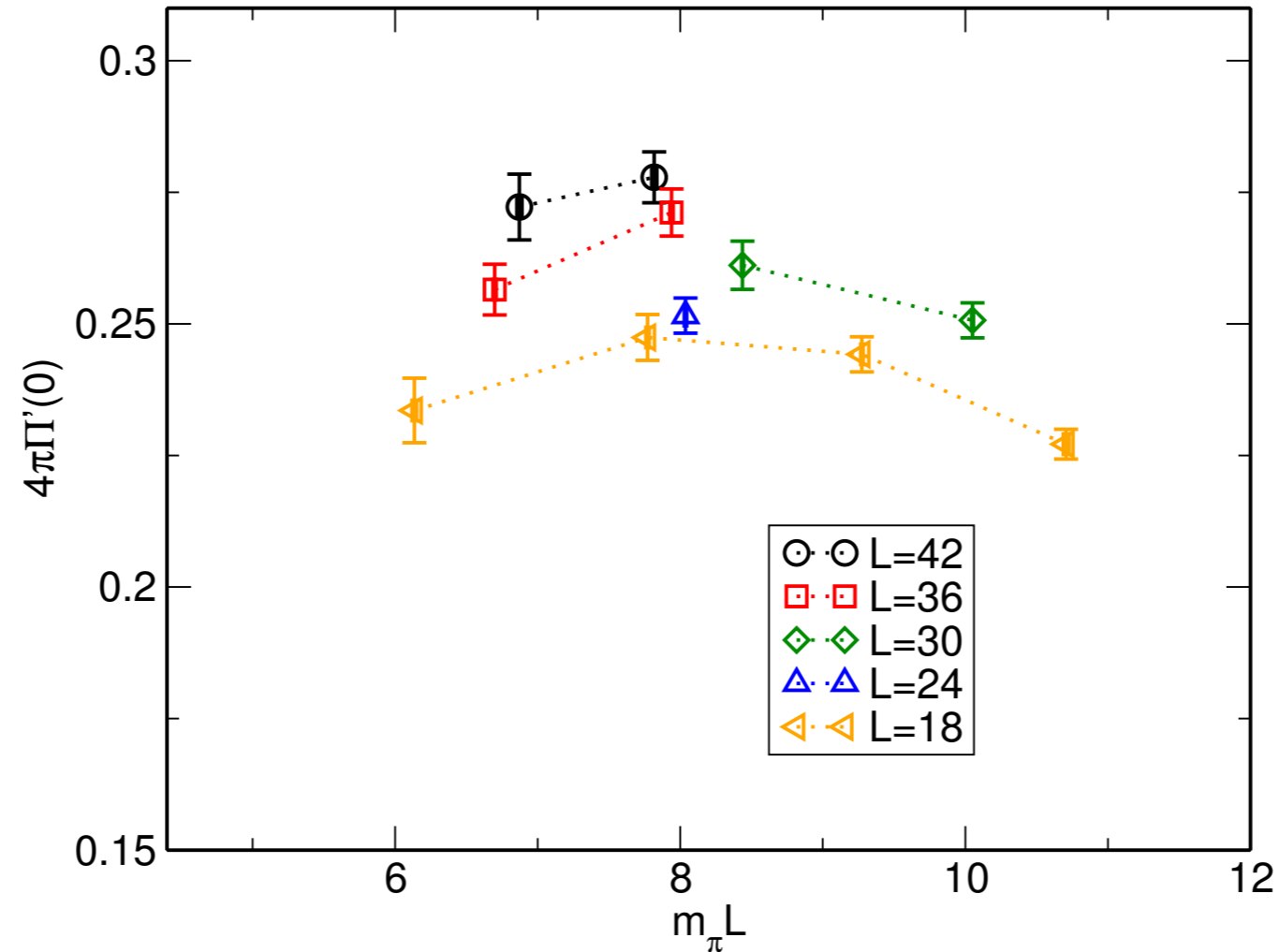
- x axes: normalized with ρ mass in $m_f \rightarrow 0$ (linearly extrapolated)
- finite volume effect tends to reduce S

$S(m_\pi^2)$: TC contribution per EW doublet (preliminary)



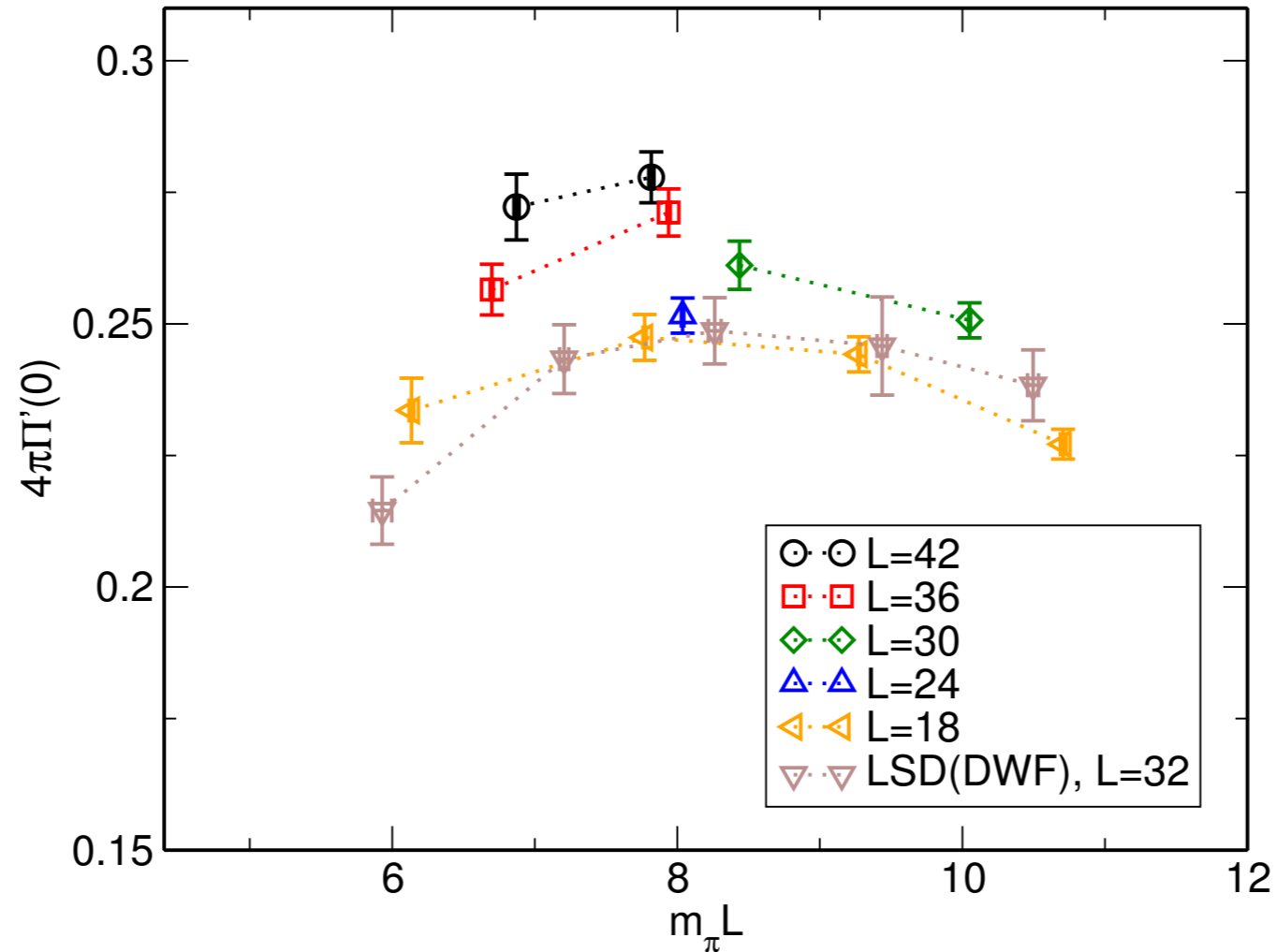
- x axes: normalized with ρ mass in $m_f \rightarrow 0$ (linearly extrapolated)
- finite volume effect tends to reduce S

$S(m_\pi L)$: TC contribution per EW doublet (preliminary)



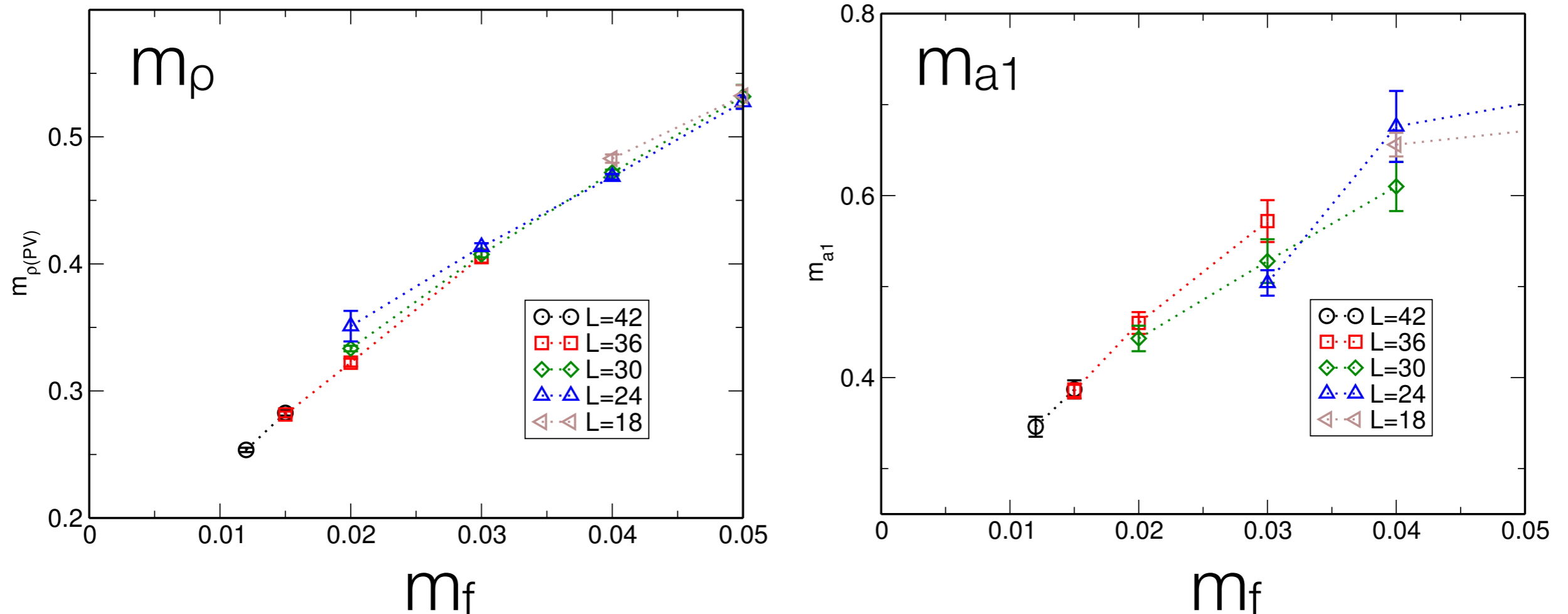
- finite volume effect tends to reduce S
- $m_\pi L \lesssim 7$ likely affected by finite volume effect

$S(m_\pi L)$: TC contribution per EW doublet (preliminary)



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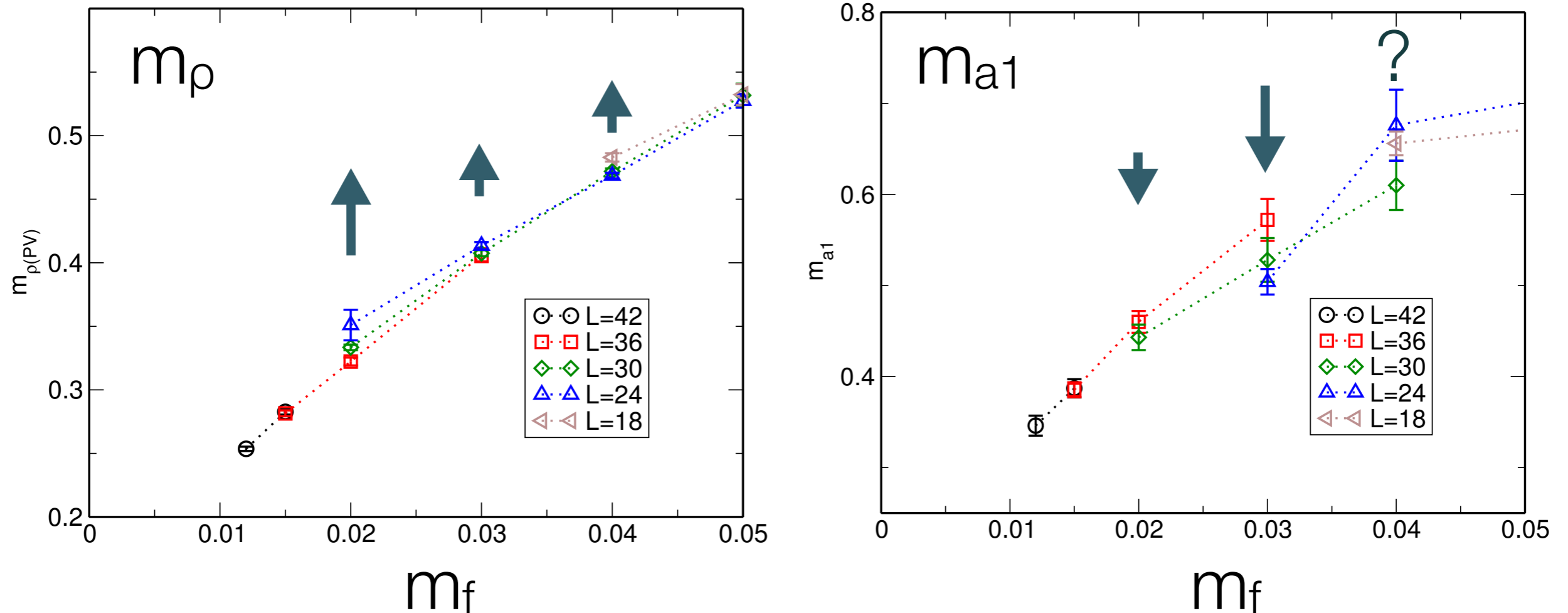
spectrum in vector and axialvector channel



- measured with local operators (spin-taste: PV)
- indicating finite volume effect tends to push toward parity doubling

spectrum in vector and axialvector channel

finite volume effect



- measured with local operators (spin-taste: PV)
- indicating finite volume effect tends to push toward parity doubling

summary and outlook

- $N_f=8$ QCD is still a candidate of Walking Technicolor
- Spectrum (rough for the moment) is predicted
- Important to go light towards chiral limit: we are still doing such effort
- S parameter is investigated for $N_f=8$ QCD
- staggered $SU(N_f/4)$ vector & axial exact symmetry allows a clean calculation
- large “finite volume effect” on S, to make it reduced, is observed
- axial vector meson mass has interesting volume effect
- further checks required for establish the observation (on-going)
- for sure, one needs careful volume analysis for the S parameter