Running, Walking, Standing: the interplay between the running coupling and IR physics

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Focus:

The focus will be on the **role that Lattice numerical simulations can play** in the study of possible strong interactions in **Beyond the Standard Model (BSM) physics**, and in particular within the following topic areas:

- Composite dark matter
- Composite Higgs models and EWSB
- Theoretical applications in conformal field theory, string theory, and holography
- Strongly coupled models, including many-fermion gauge theories and SUSY

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Composite Higgs

Assume a new system with N_f fermions coupled to SU(N_T) gauge fields
Couple it to Standard Model fields:

- The Higgs could be a $\bar{q}q$ (possibly qq) bound state
- 3 Goldstone pions break EW symmetry
- Tower of additional hadronic states appear in experiments

What models could be compatible with EW data?

- Chirally broken
- Most likely strongly coupled
- Walking

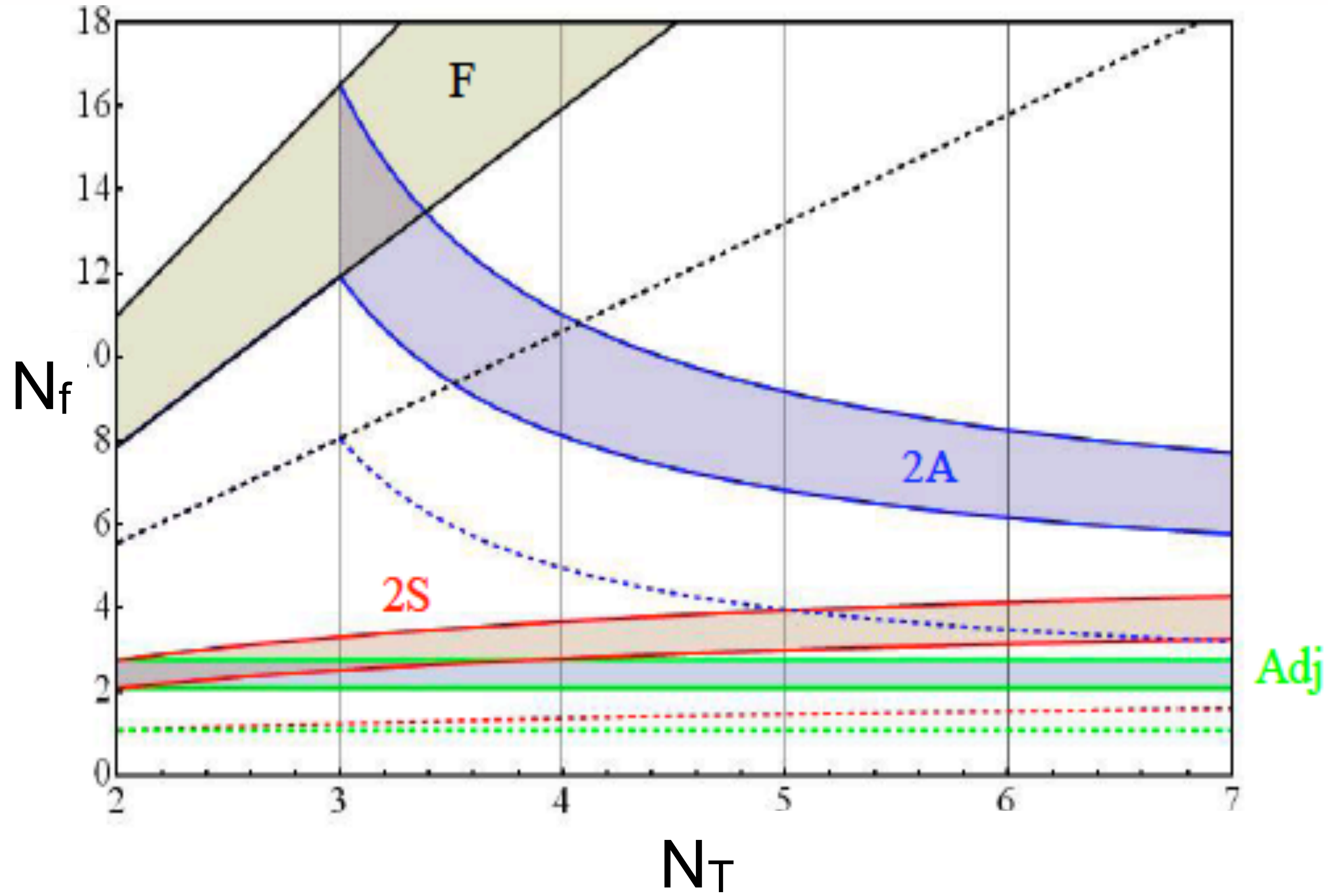
What are the generic properties of strongly coupled models?

- Is walking necessary ? Is large anomalous dimension necessary?
- Spectrum ? Where is M_{0++} compared to M_ρ ?

Lattice can investigate/answer most of the relevant questions

Roadmap: Theory Space

Dietrich, Sannino



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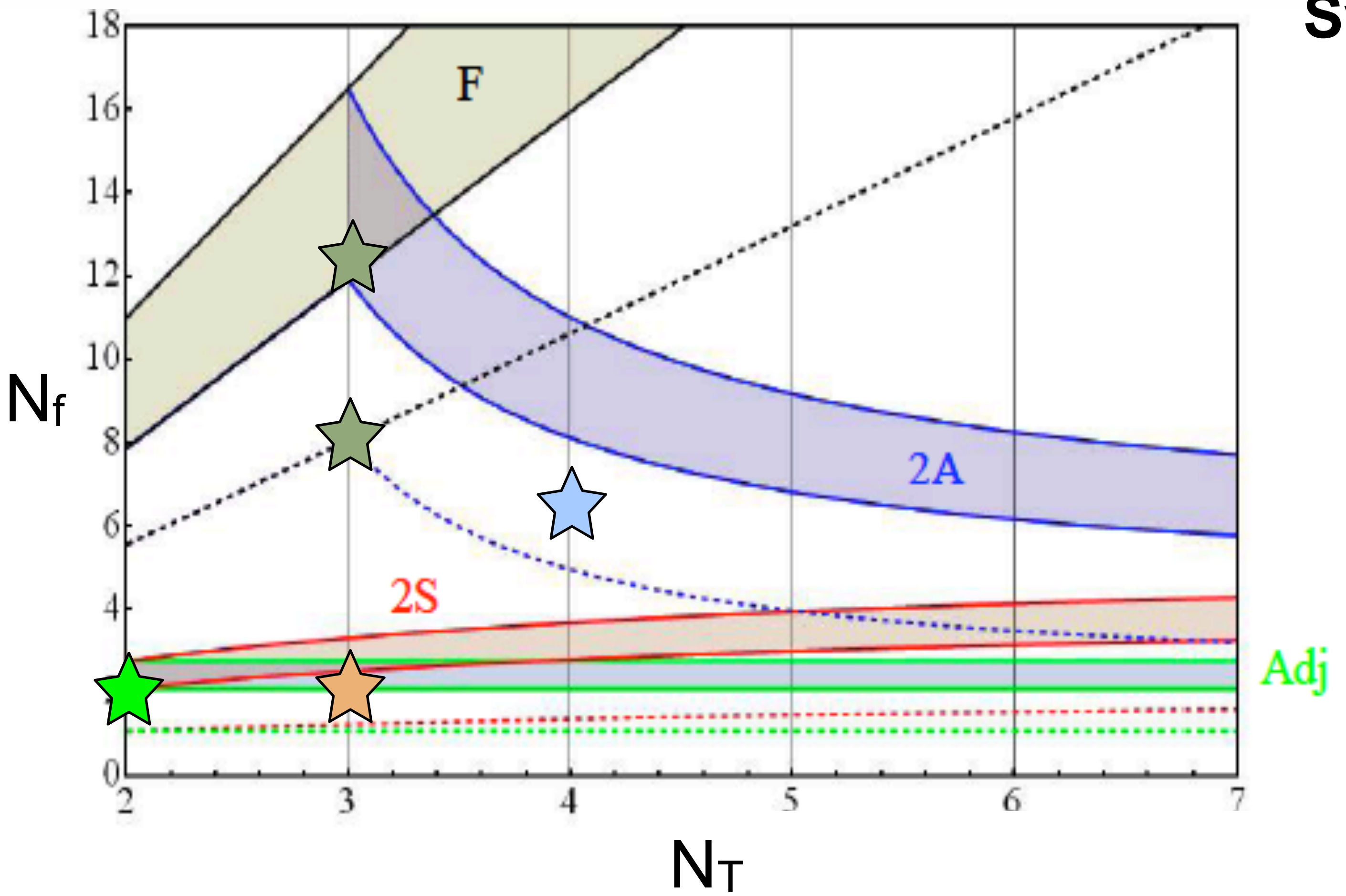
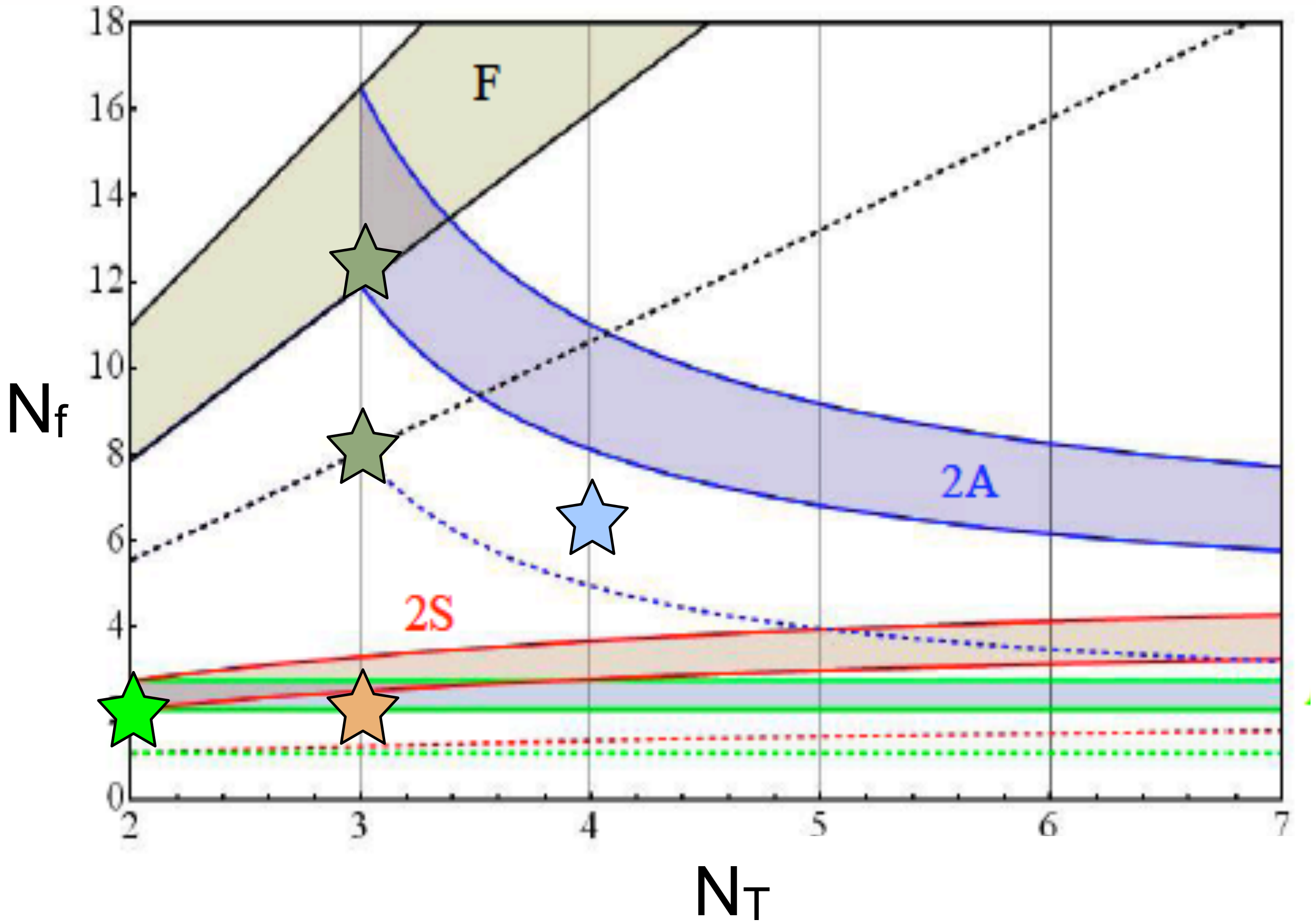


Systems near the conformal boundary:

Roadmap:

Dietrich, Sannino



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SU(2), 2-flavor adjoint - conformal

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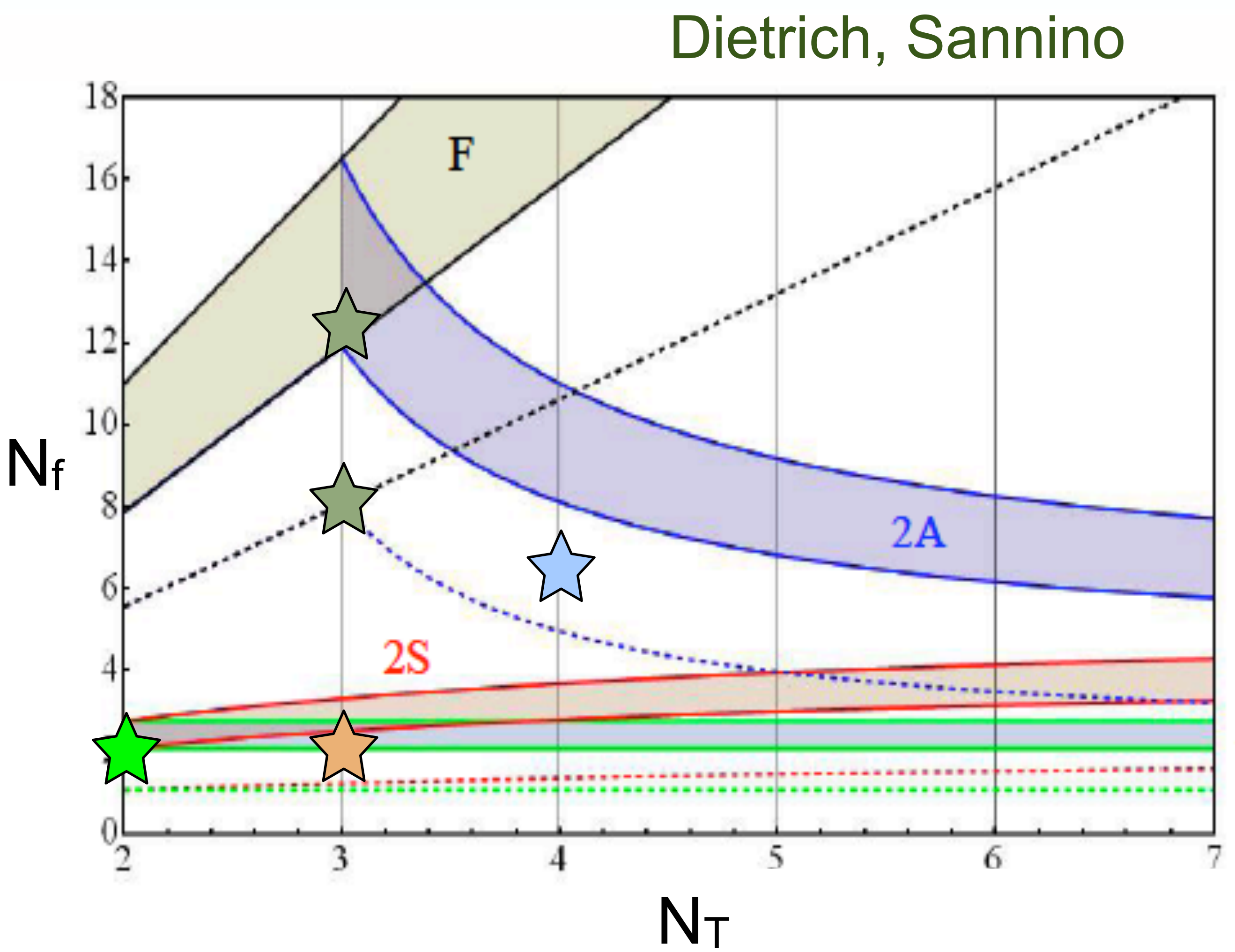

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SU(2), 2-flavor adjoint - conformal

SU(3), 12 flavor fundamental - looks conformal

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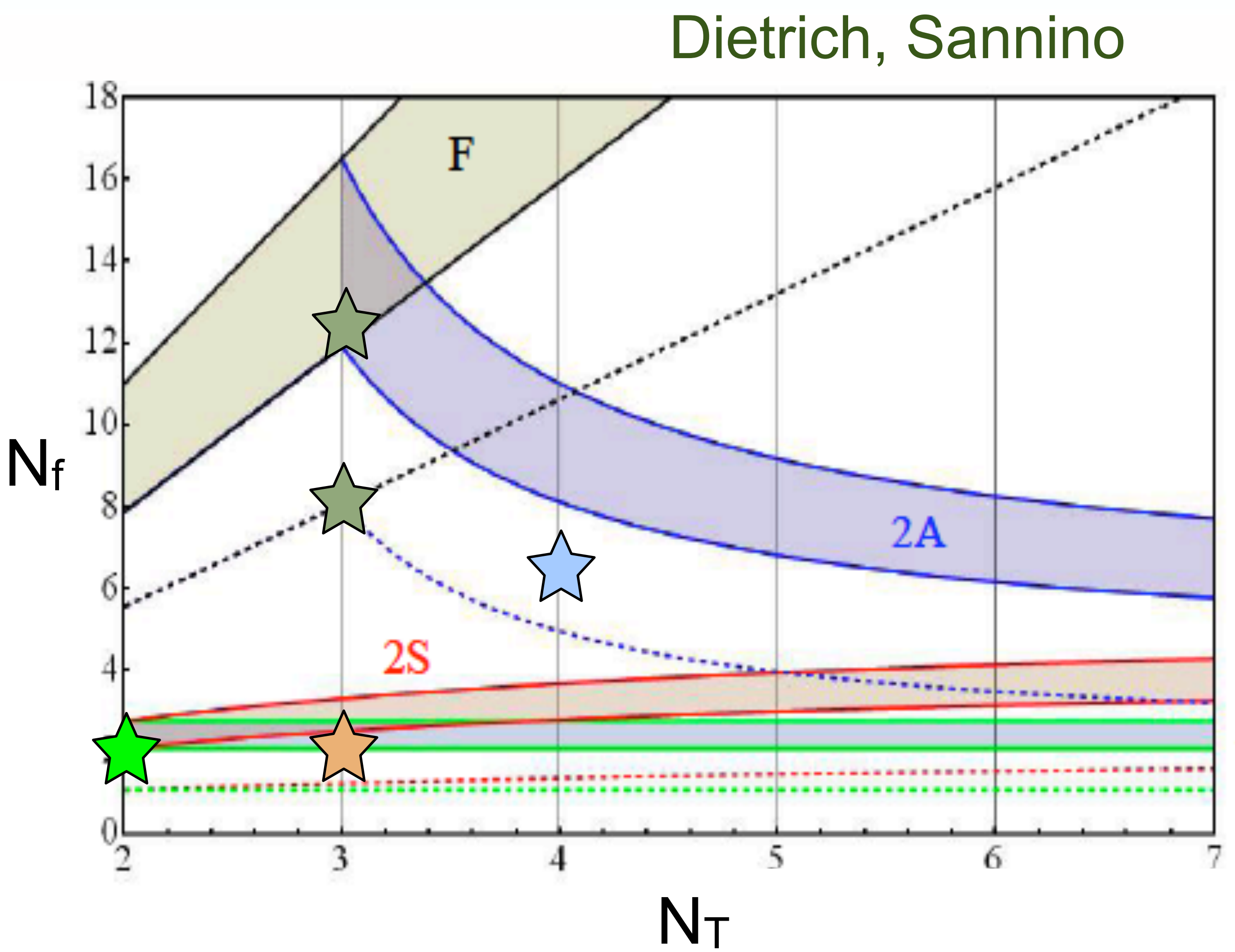

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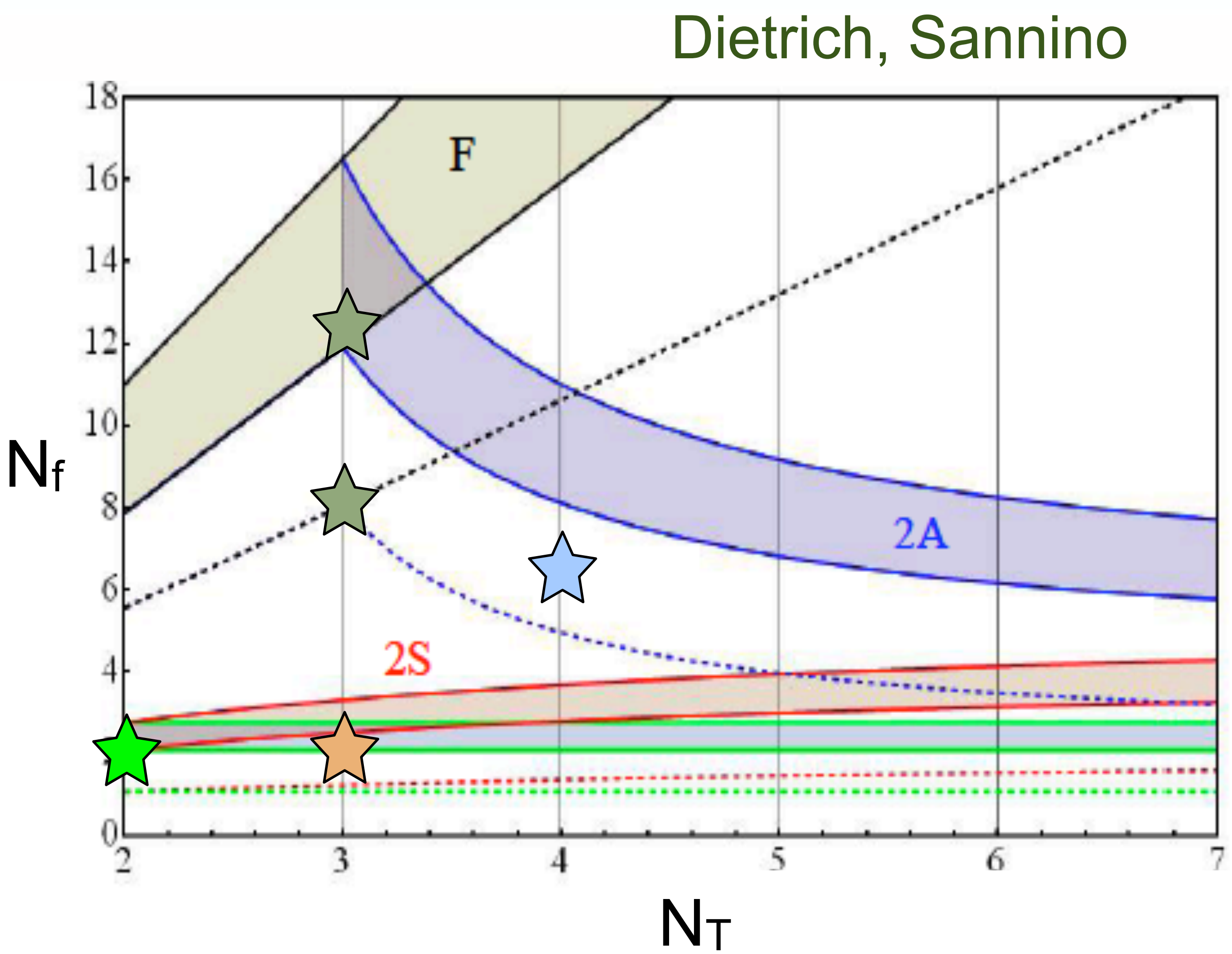

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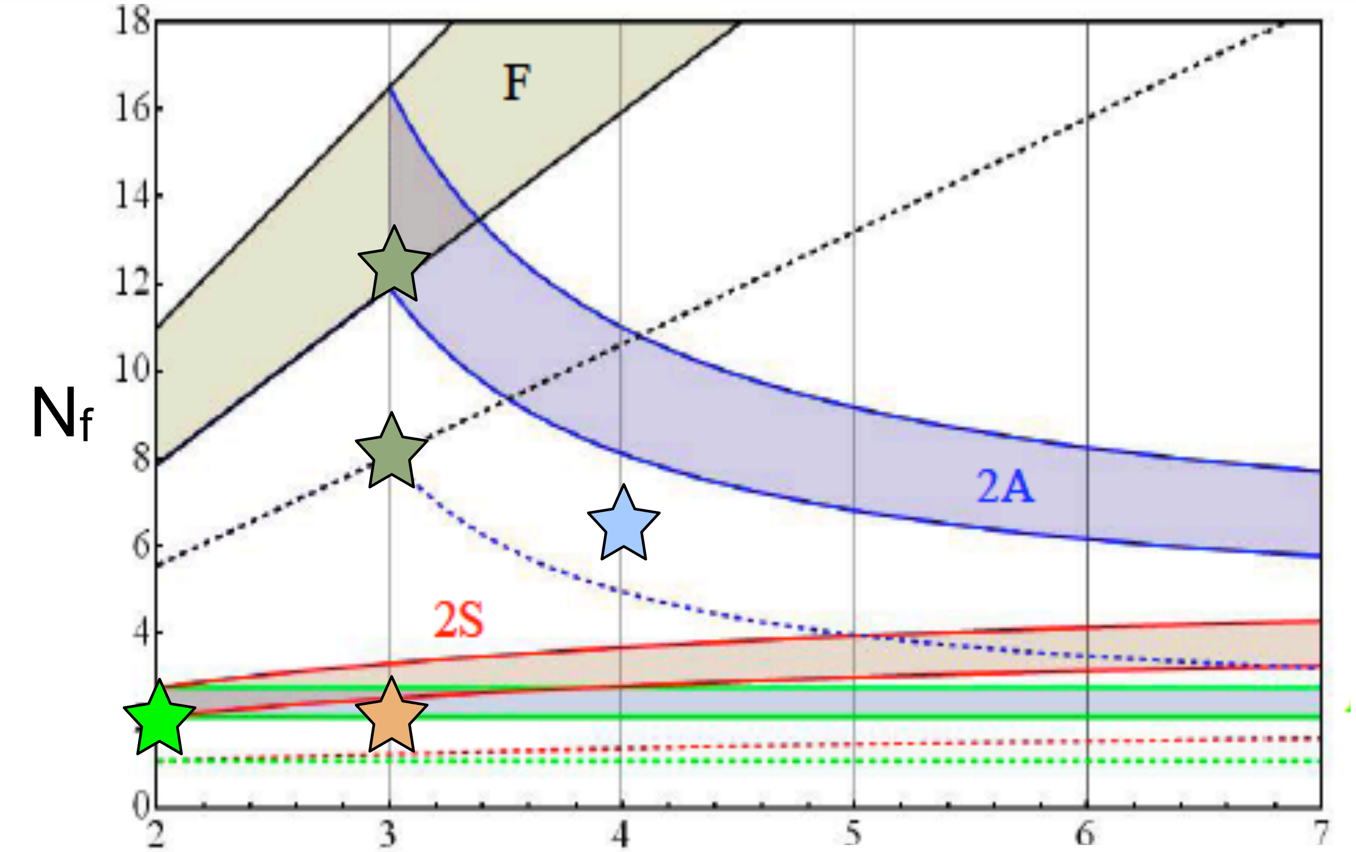

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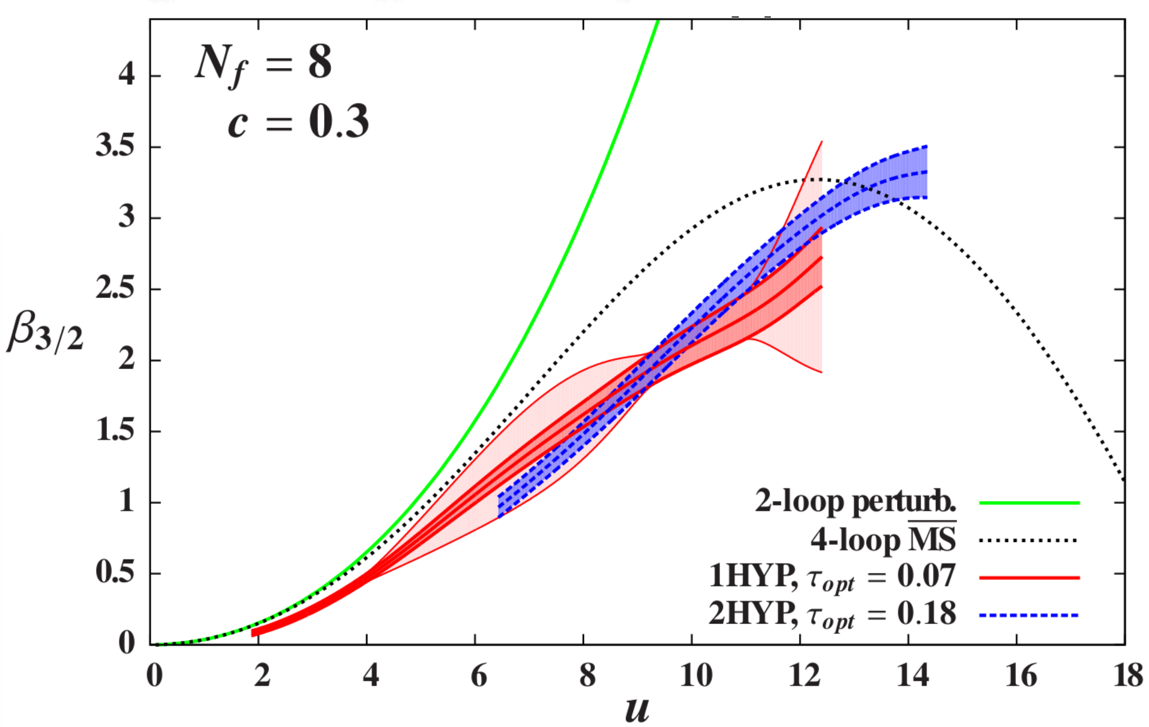


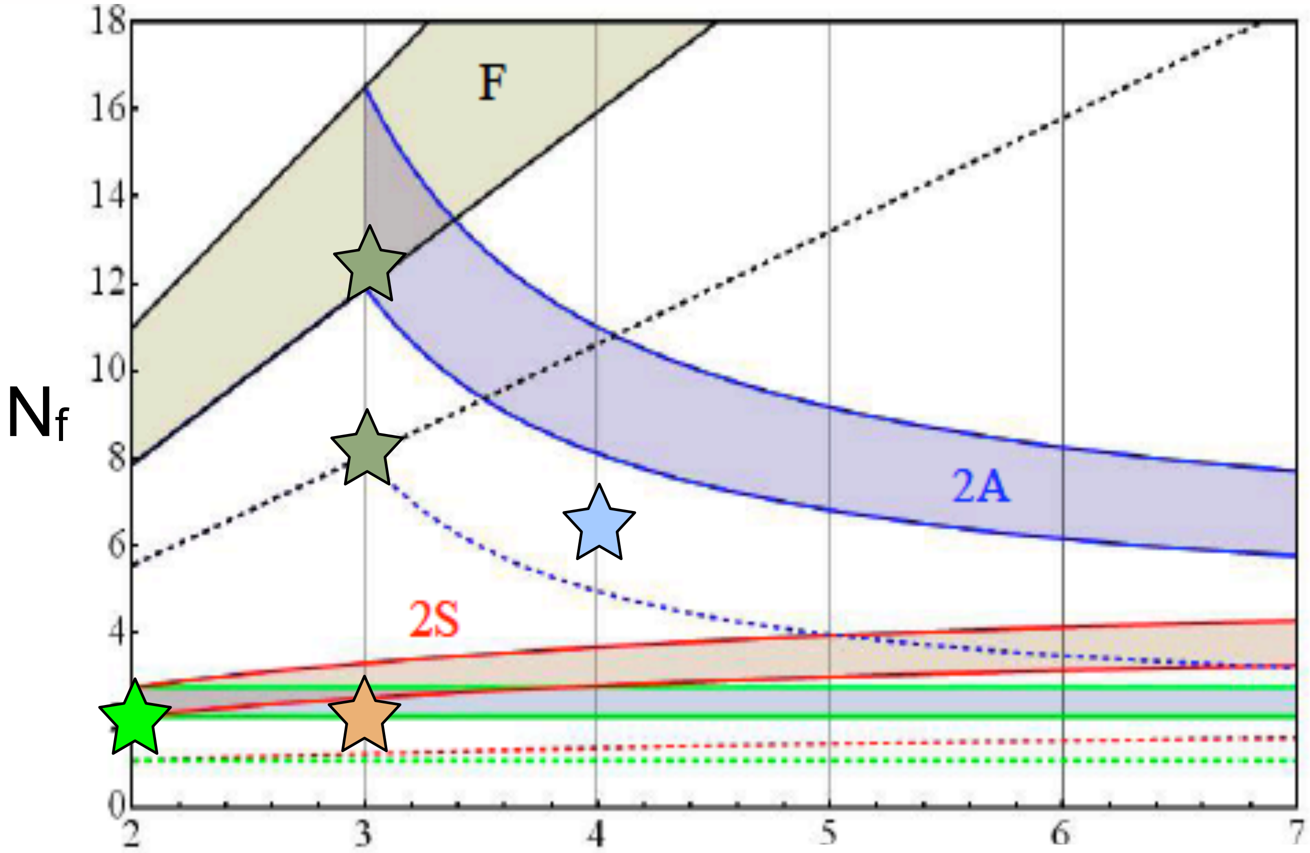
N_f=8 RG step scaling function (basically negative RG β function) based on gradient flow coupling

Results follow 4-loop MS prediction up to $g^2 = 15$

A.H., D.Schaich, A.Veernala, arXiv:1410:5886)

Roadmap:

Dietrich, Sannino



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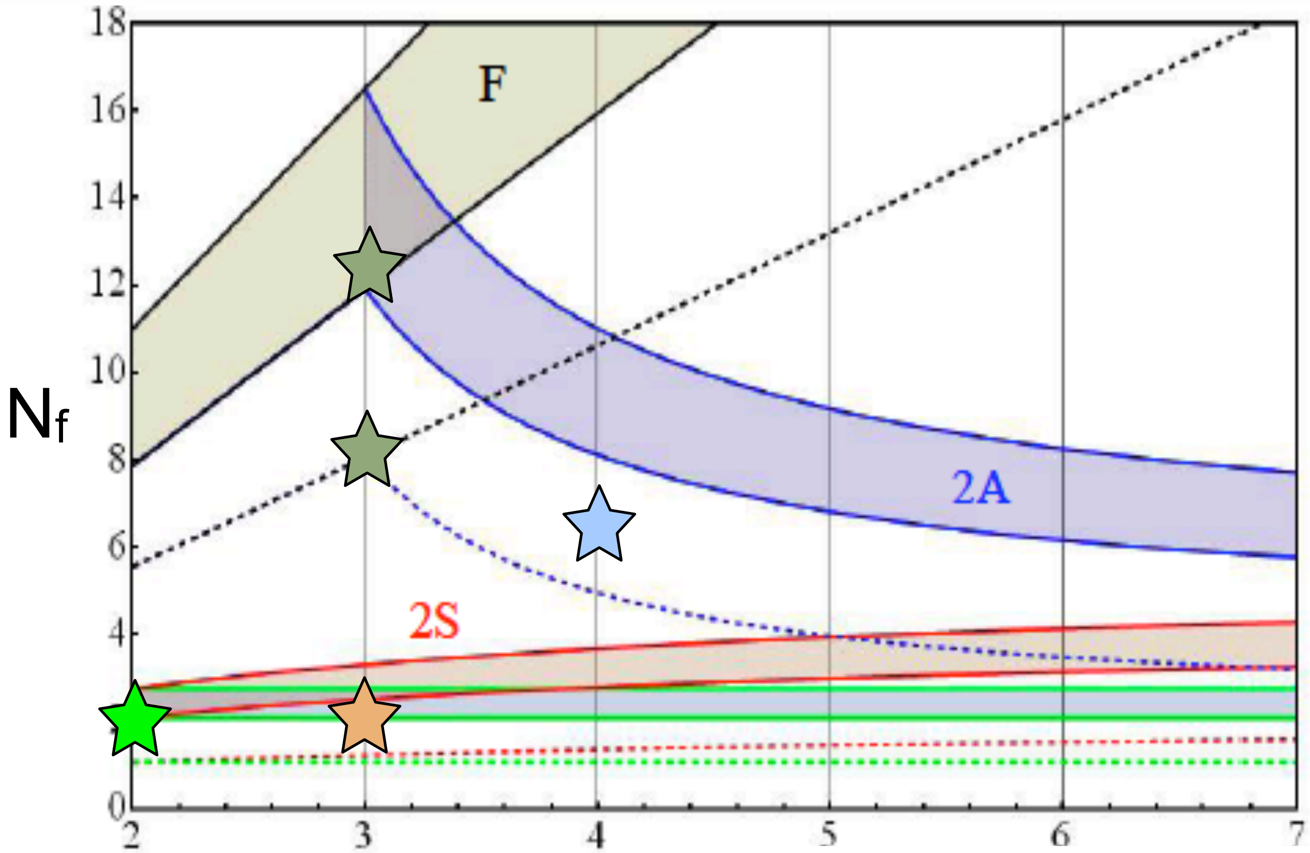

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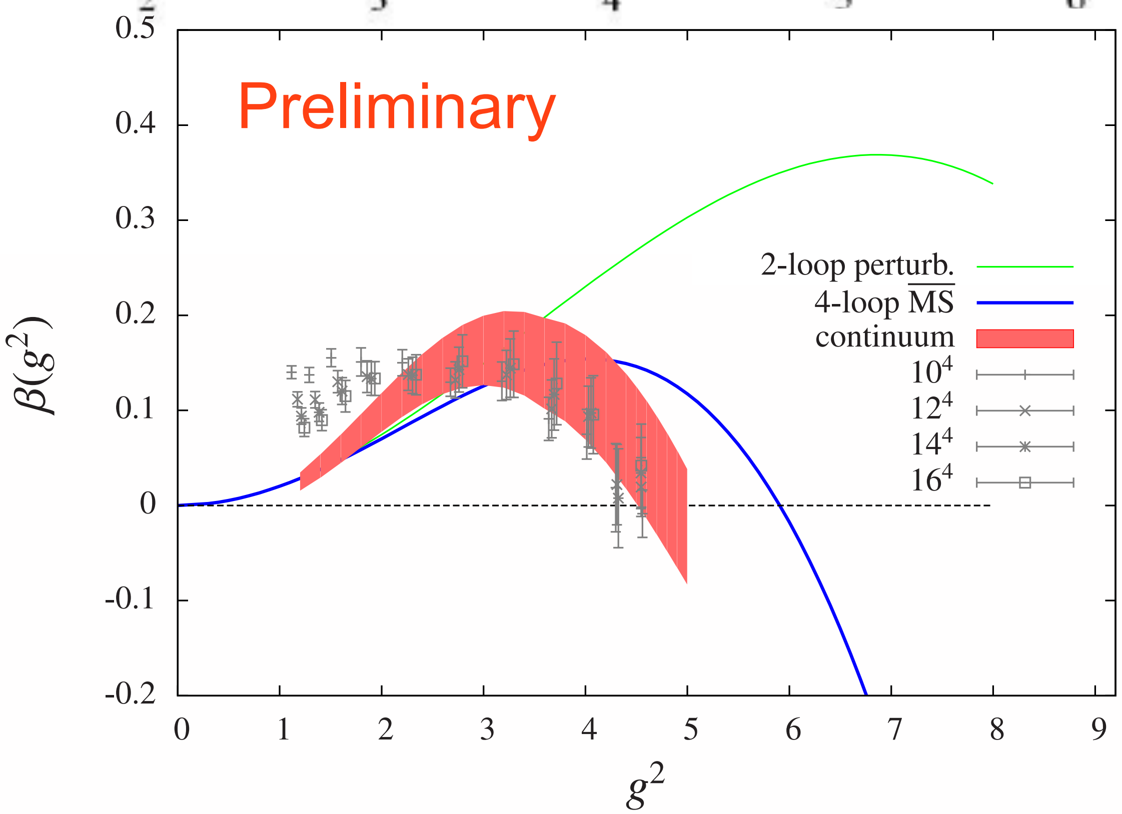


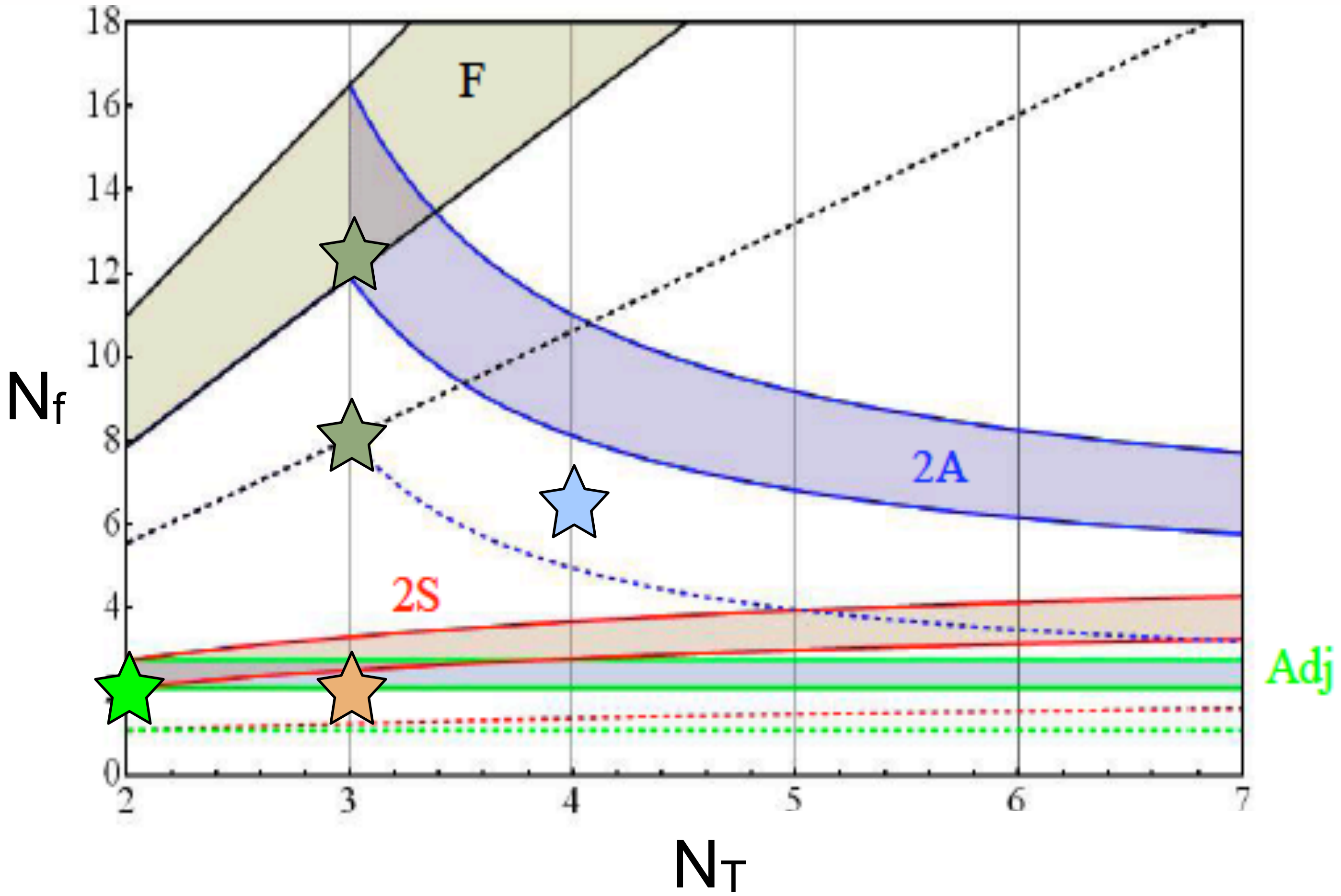
Sextet RG step scaling function (~ negative RG β function) based on gradient flow coupling

Results similar to 4-loop MS prediction up to $g^2 = 5$
A.H, C. Huang, Y.Liu, B. Svetitsky, in prep.)

- Is this system chirally broken&walking or conformal?
- Are Wilson and rooted staggered fermions equivalent here? (Universality)

Roadmap:

Dietrich, Sannino



So many possibilities!

Is there any guiding principle to help choose?

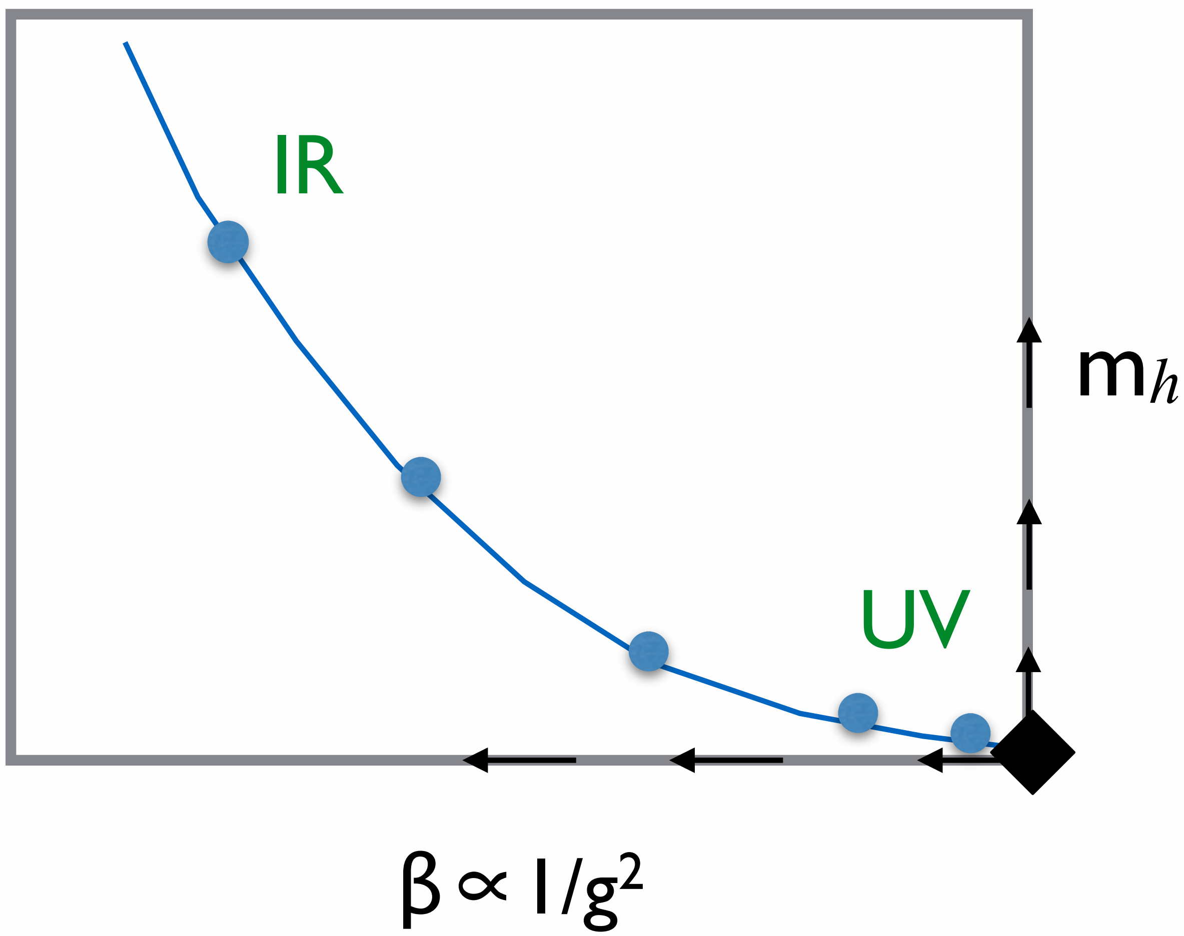
Is there some general behavior near conformality?

Simple model - I

SU(N_c) gauge with N_ℓ light ($m_\ell \approx 0$) and N_h heavy (m_h) fermions

In the IR the heavy flavors decouple, N_ℓ light remain

$N_\ell + N_h$ =small: gauge coupling runs fast, heavy flavors have limited effect on the IR (QCD)



◆ Perturbative UVFP

● RG flow from UV to IR

Continuum limit:

tune $g^2 \to 0$

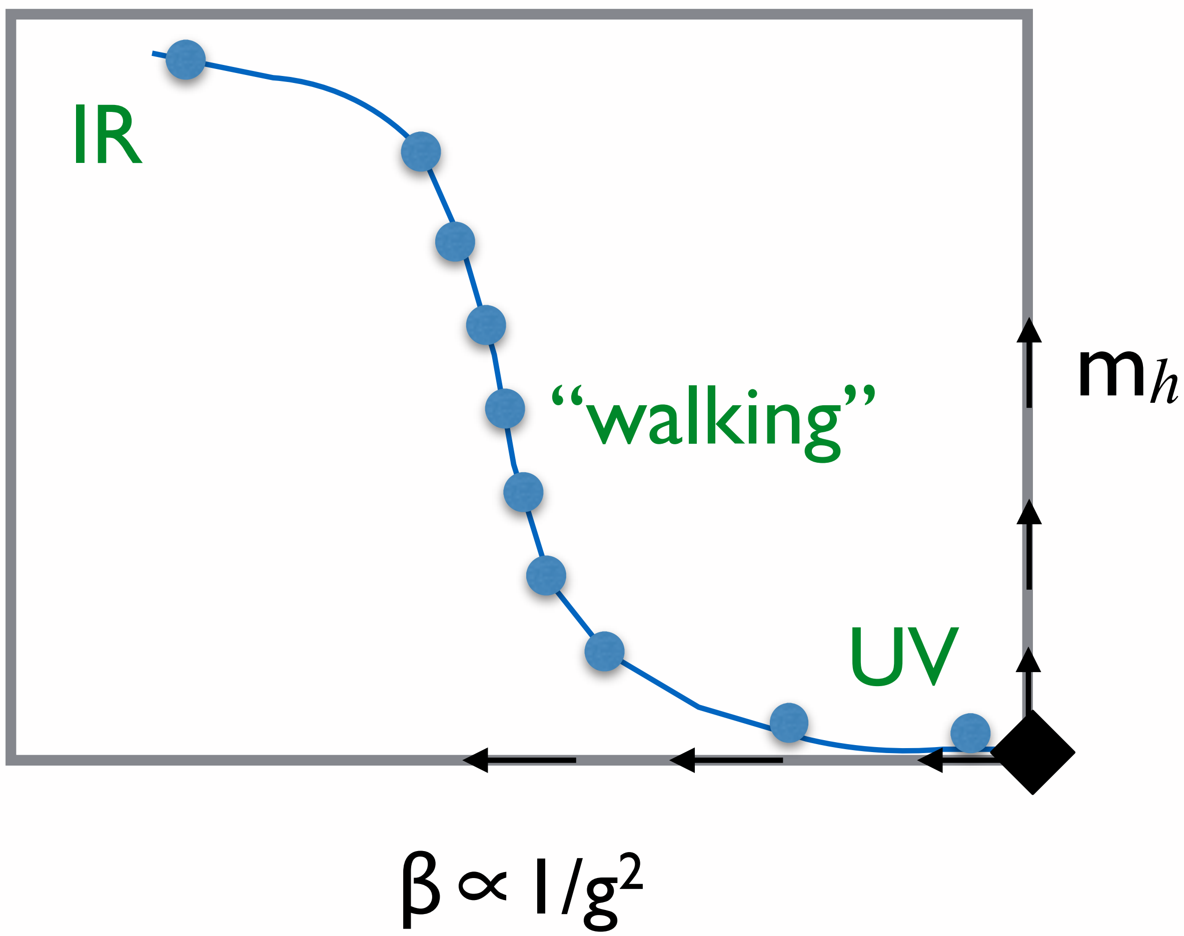
$m_h \to 0$

Simple model - II

SU(N_c) gauge with N_ℓ light ($m_\ell \approx 0$) and N_h heavy (m_h) fermions

$N_\ell + N_h$ = near but below the conformal window

IF the gauge coupling is "walking" the IR can be very different



◆ Perturbative UVFP

● RG flow from UV to IR

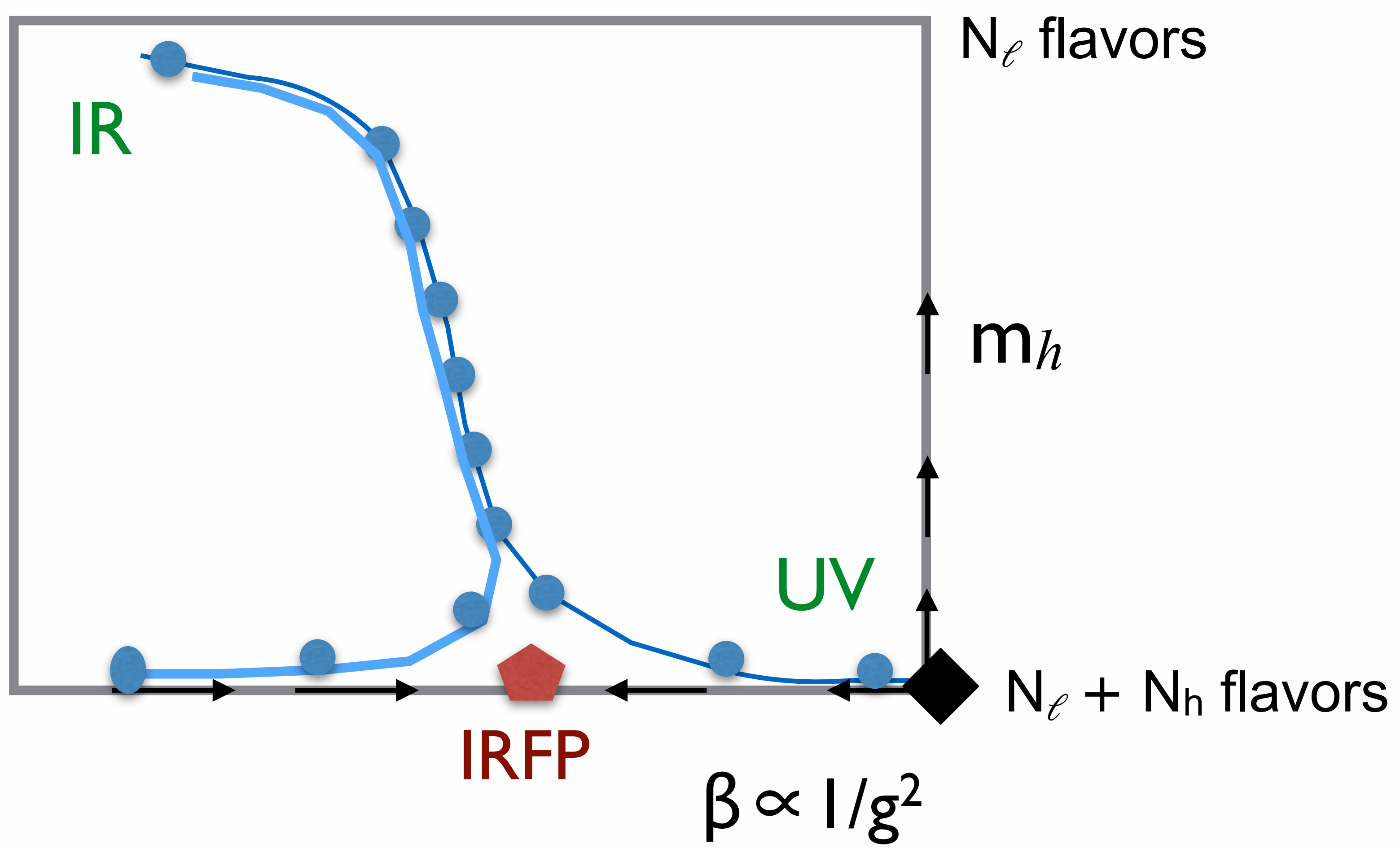
There is no guarantee that any $N_\ell + N_h$ system will walk

Simple model - III

SU(N_c) gauge with N_ℓ light ($m_\ell \approx 0$) and N_h heavy (m_h) fermions

$N_\ell + N_h$ = above the conformal window, N_ℓ is below

guarantees that the gauge coupling is “walking”;
the IR will be very different



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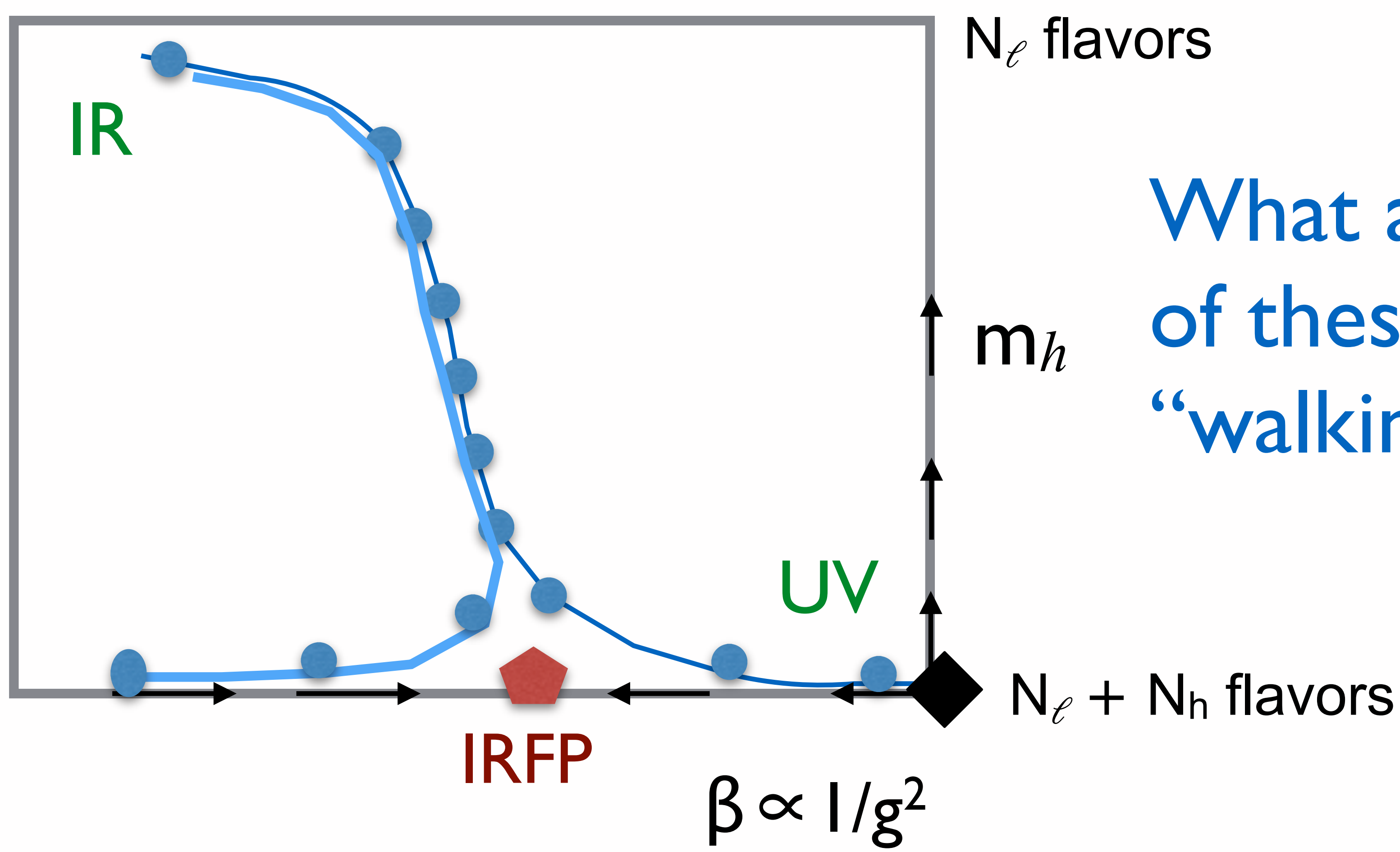
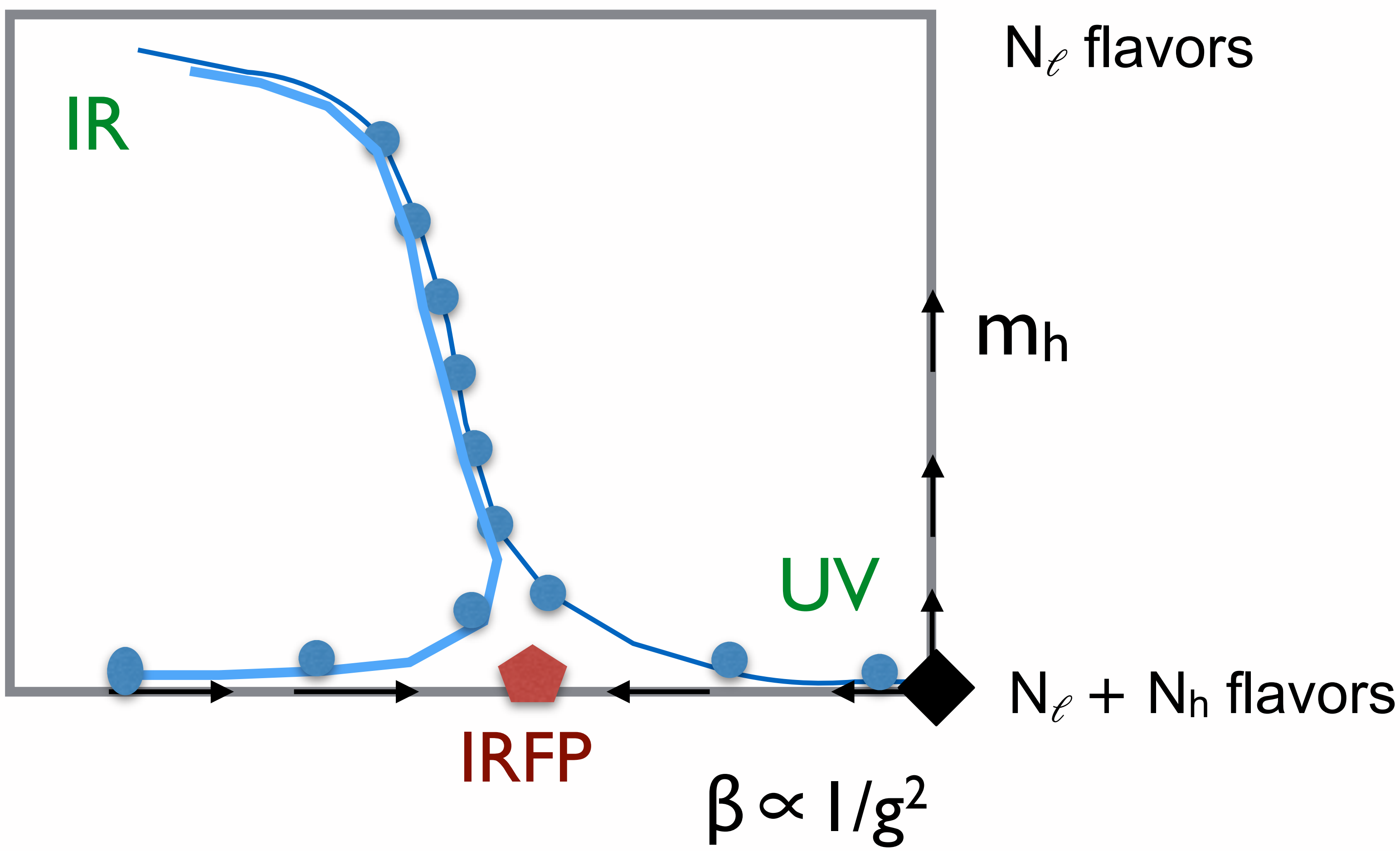


What are the properties of these strongly coupled "walking" systems?

$N_\ell + N_h$: Parameter space

3 independent parameters: (g^2, m_ℓ, m_h)

- g^2 does not matter once the flow reaches the RG trajectory
- sufficient to work at g^2 =const, vary m_h only ($m_\ell=0$)



Continuum limit:
tune $m_h \to 0$
($m_\ell << m_h$)

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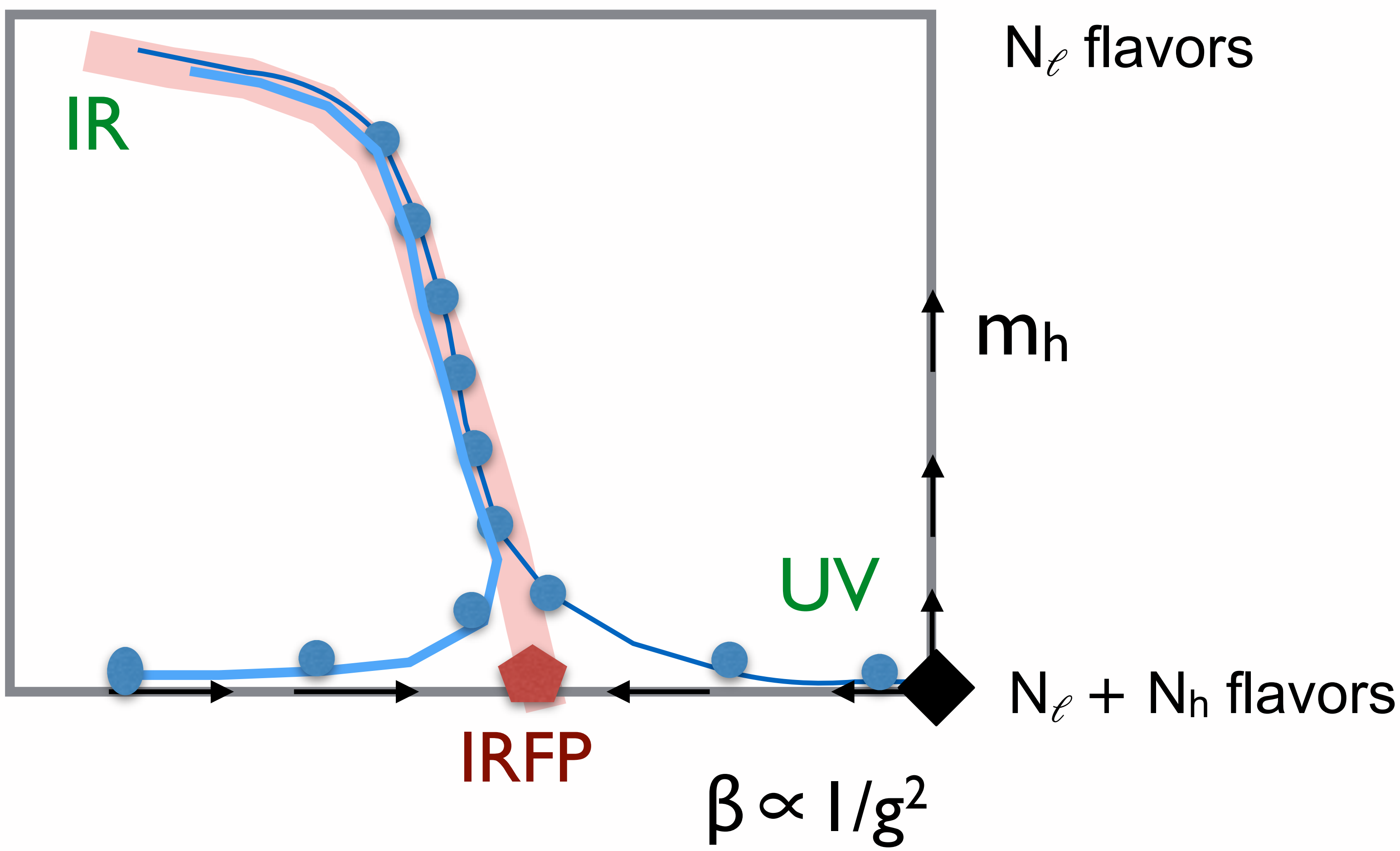

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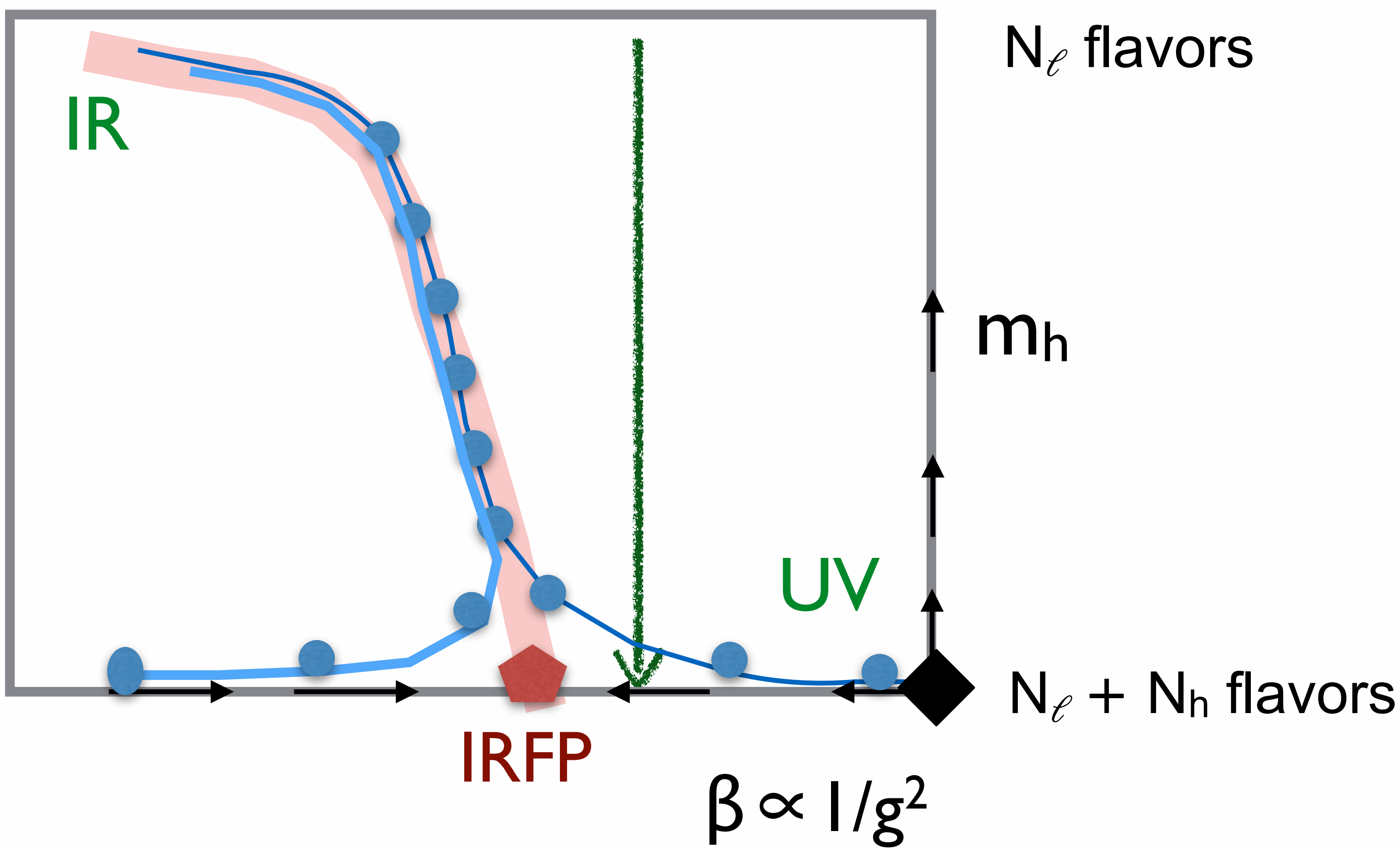

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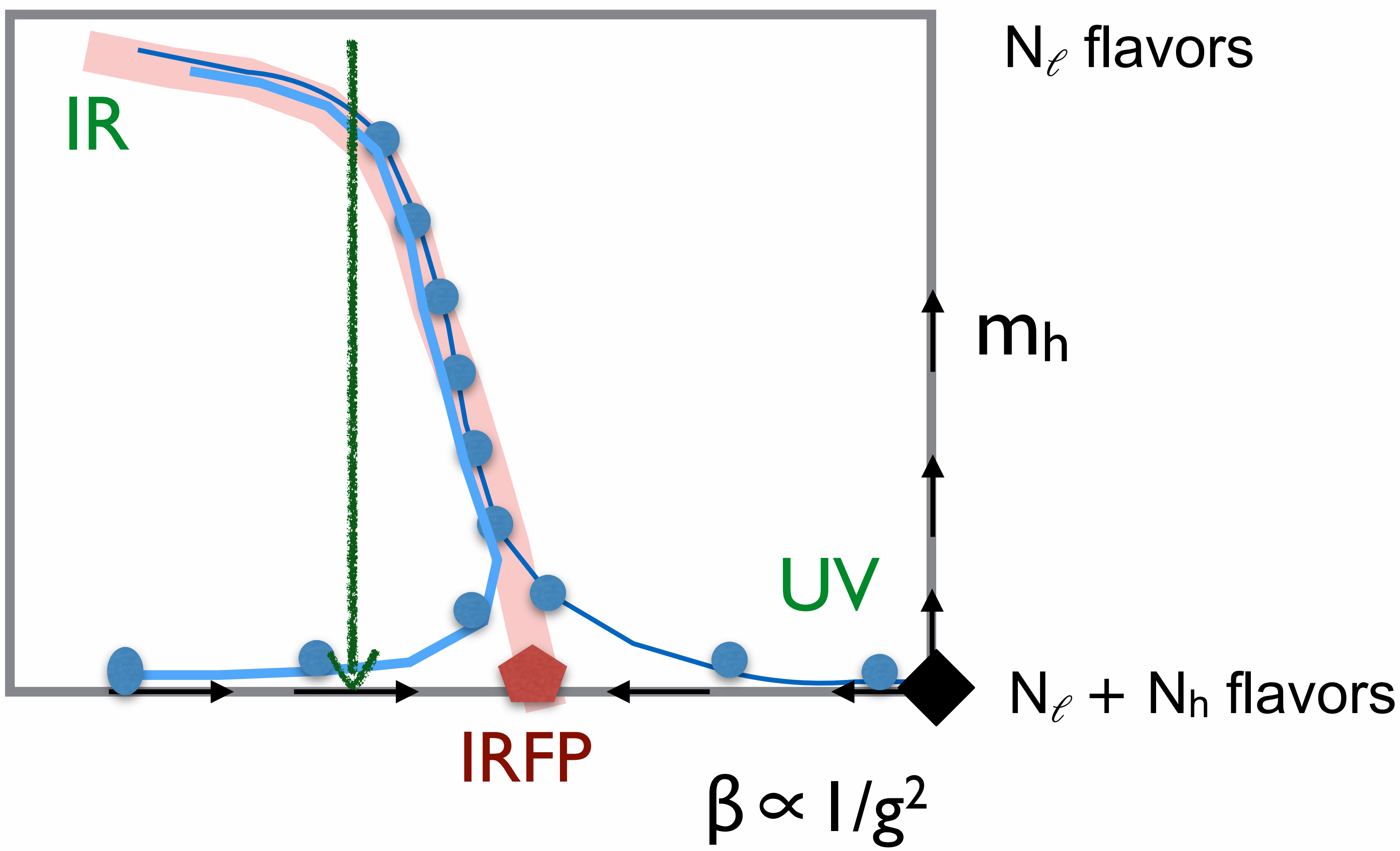

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N_ℓ + N_h systems

$N_\ell + N_h = 2 + 6$ if $N_f = 8$ is the UV model

or

$N_\ell + N_h = 2 + 10$ for $N_f = 12$ conformal behavior in the UV

Pilot study:

$N_\ell + N_h = 4 + 8$: conformal in the UV, N_l=4 flavor in the IR

in collaboration with R. Brower, C. Rebbi, E. Weinberg, O. Witzel

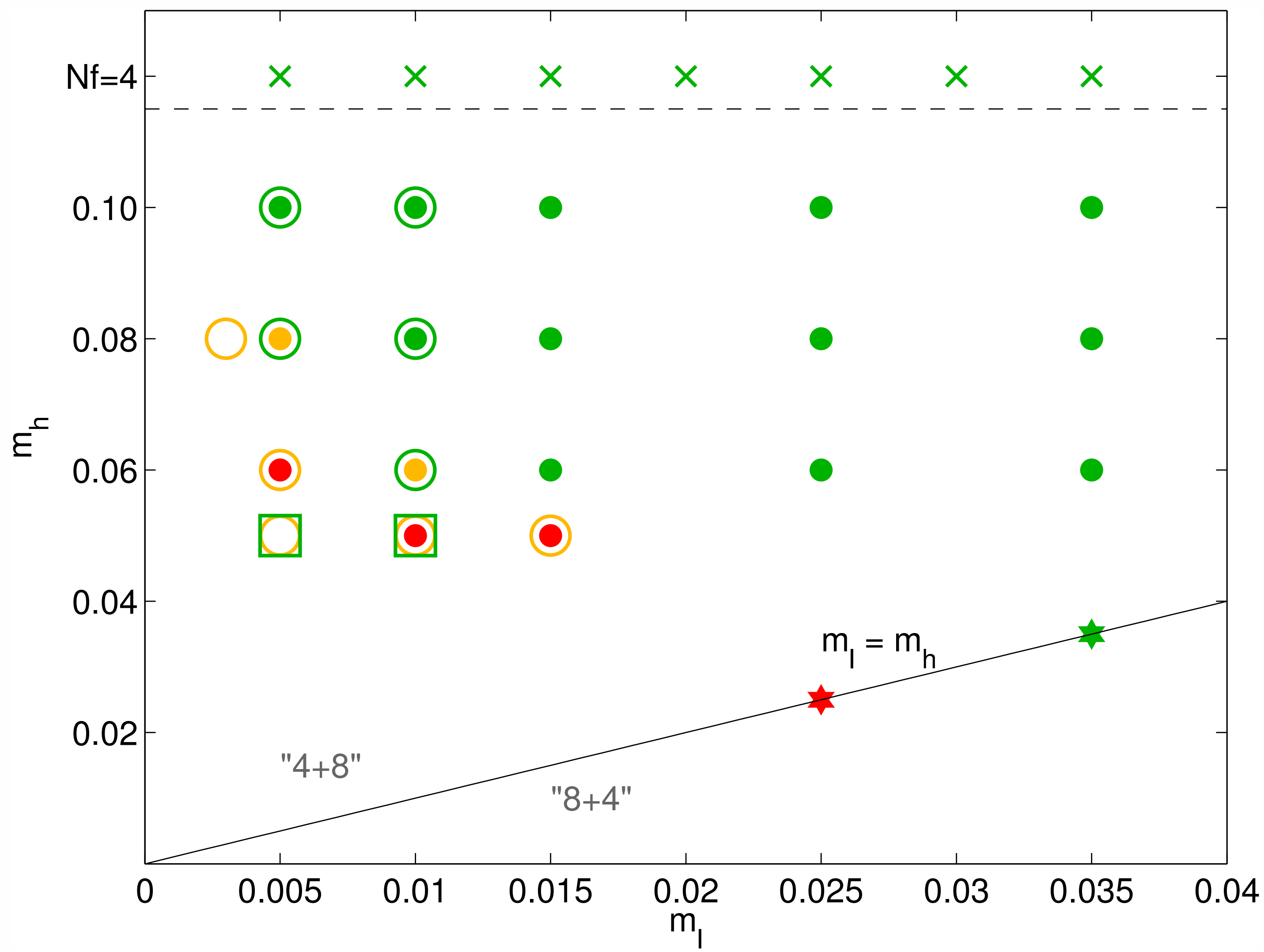
arXiv:1411.3243

Why 4+8 ? We use staggered fermions:

4 and 8 flavors do not require rooting

$N_\ell + N_h = 4 + 8$: Parameter space

- β=4.0 (close to the 12-flavor IRFP)
- m_h=0.10, 0.08, 0.06, 0.05
- m_ℓ=0.003, 0.005, 0.010, 0.015, 0.025, 0.035



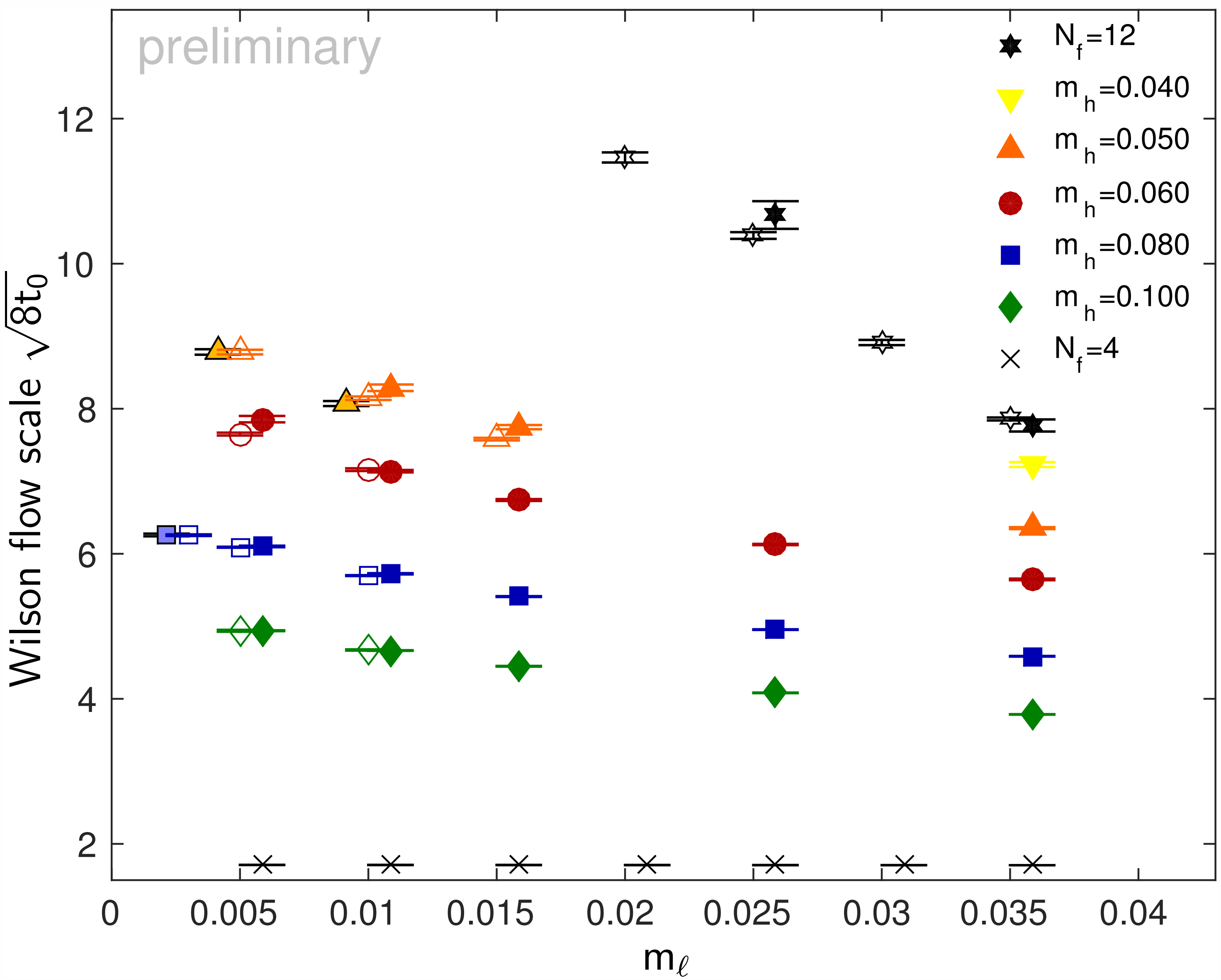
Volumes :
24^3x48, (dots)
$32^3$64 (circle), $36^3$64
48^3x96 (square)
Color: volume OK / marginal/ squeezed

20,000 MDTU, most still in progress

Lattice scale

Use gradient flow to estimate the lattice scale $\sqrt{8t_0}$

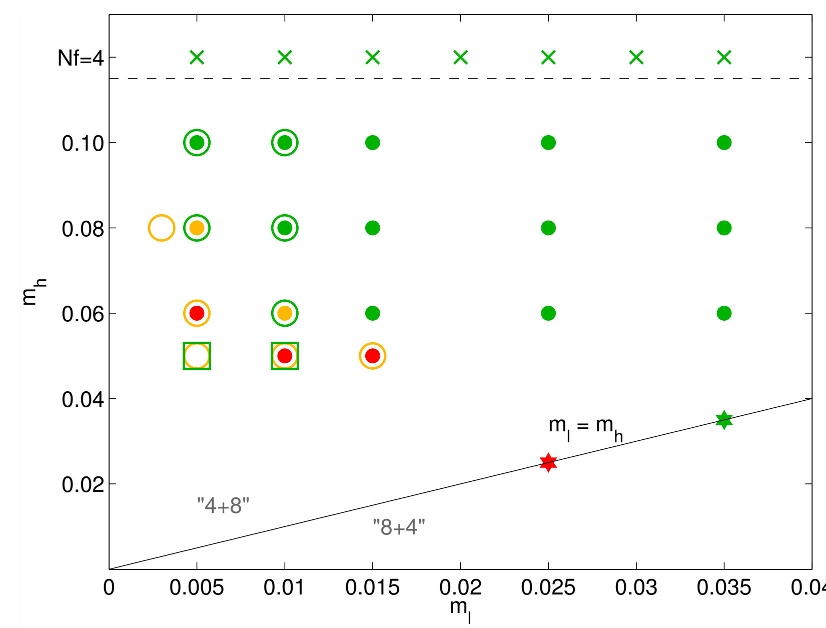
Significant variation with m_ℓ, m_h, but finite volume effects are controlled



$\sqrt{8t_0} \lesssim L/5$

is usually sufficient

→ color coding



Running coupling

Gradient flow is a gauge field transformation that defines a renormalized coupling

arXiv:1006.4518

$$g^2_{GF}(\mu=\frac{1}{\sqrt{8t}})=\frac{1}{\mathcal{N}}t^2\langle E(t)\rangle$$

t: flow time;
E(t):energy density

g^2_{GF} is used for scale setting as

$$g^2_{GF}(t=t_0)=\frac{0.3}{\mathcal{N}}$$

Is it appropriate to determine the renormalized running coupling?

Yes :

- on large enough volumes
- at large enough flow time
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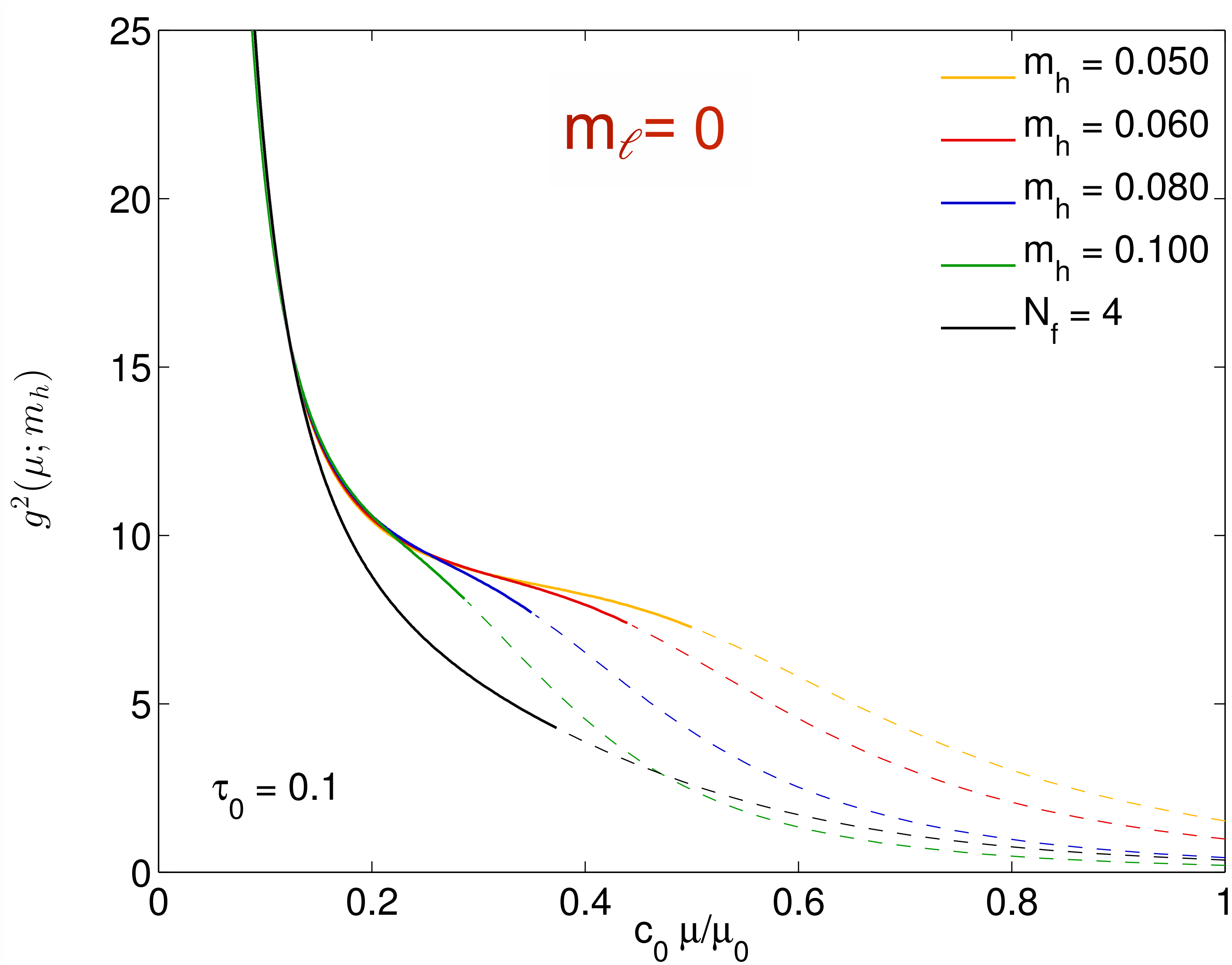
} use t-shift improved coupling

Improved running coupling : 4+8 flavors



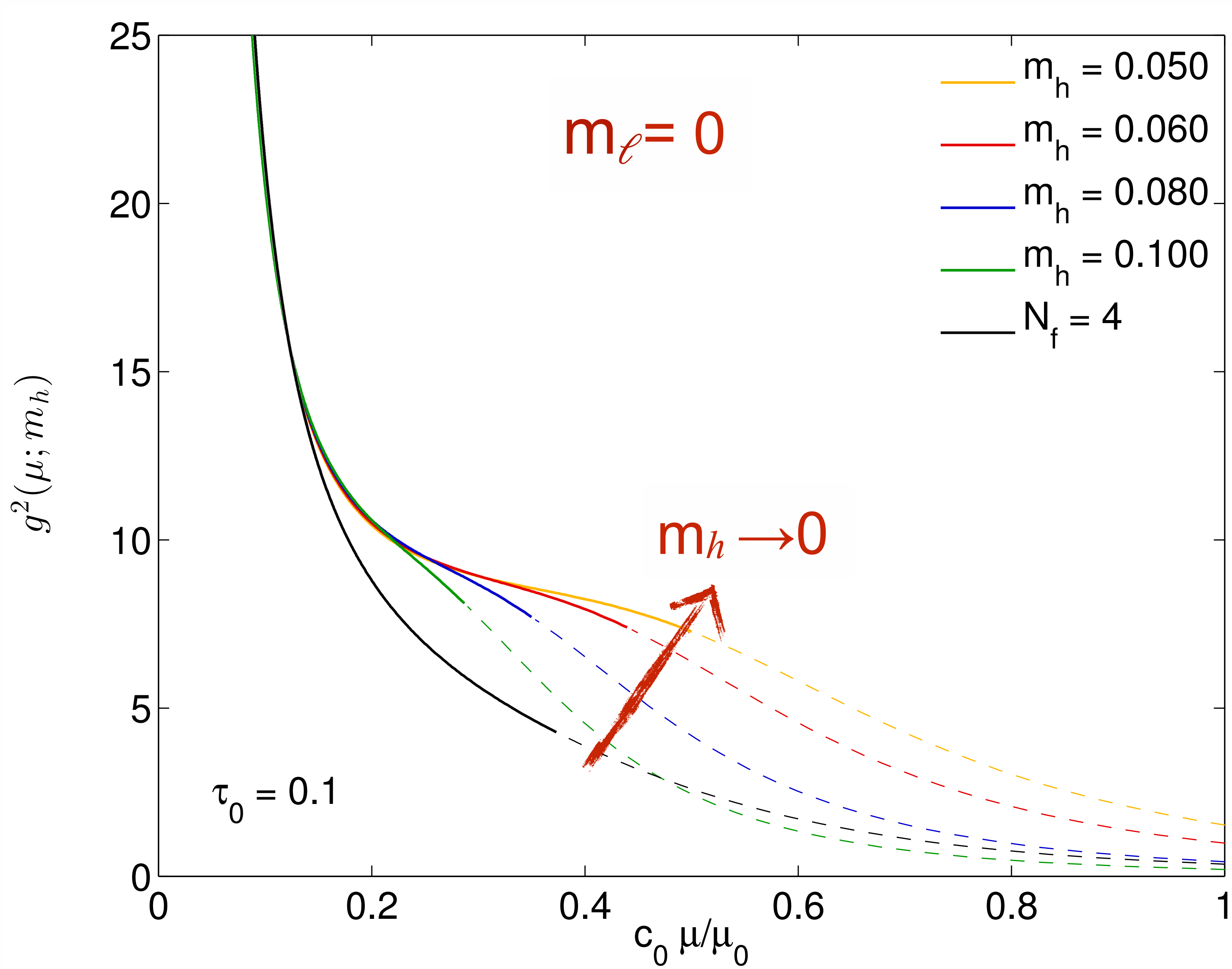
There are error bars on this plot!

N_f=4 : running fast

$g^2_{GF}(\mu)$ develops a “shoulder” as $m_h \to 0$: this is walking !

Walking range can be tuned arbitrarily with m_h

Improved running coupling : 4+8 flavors



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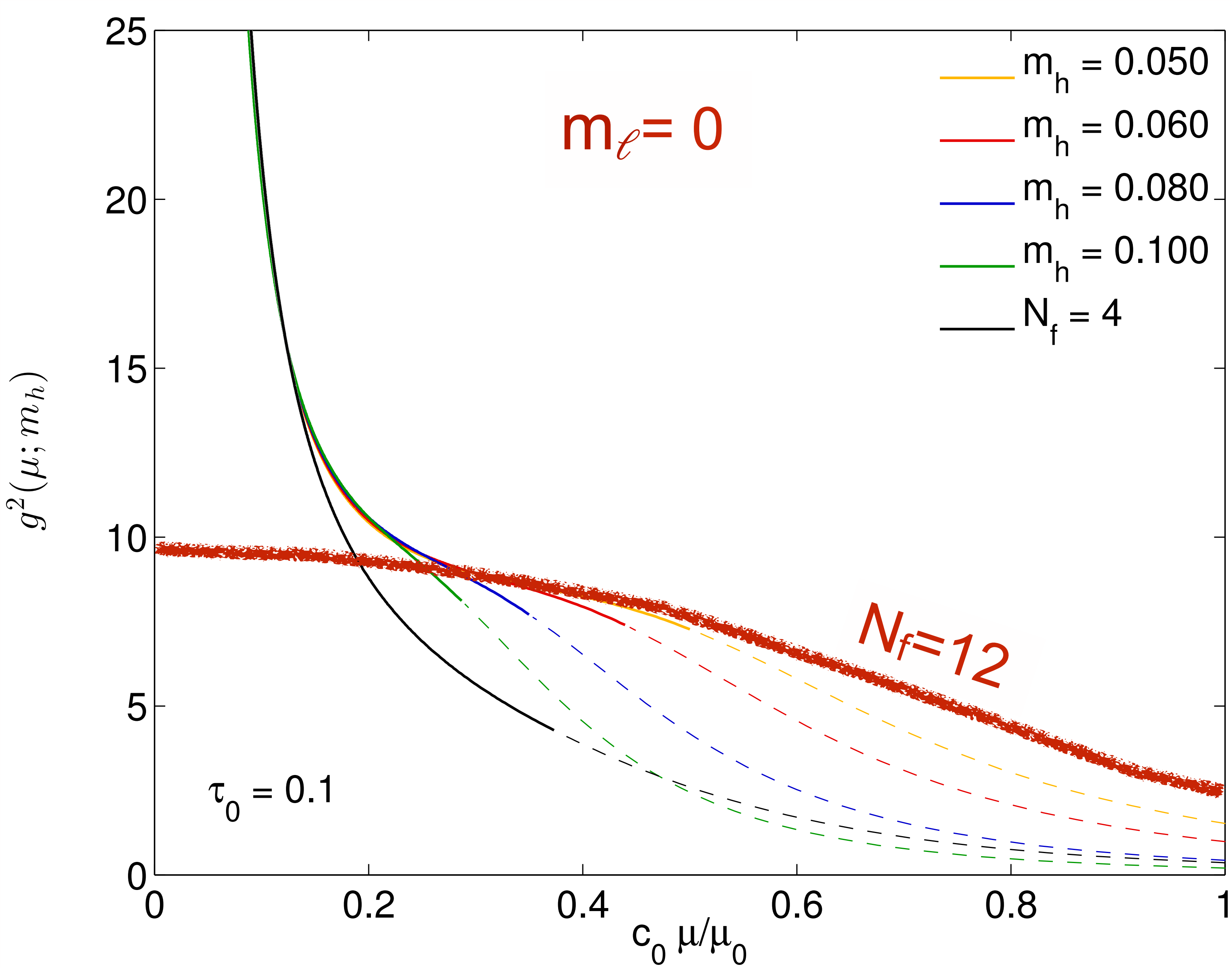

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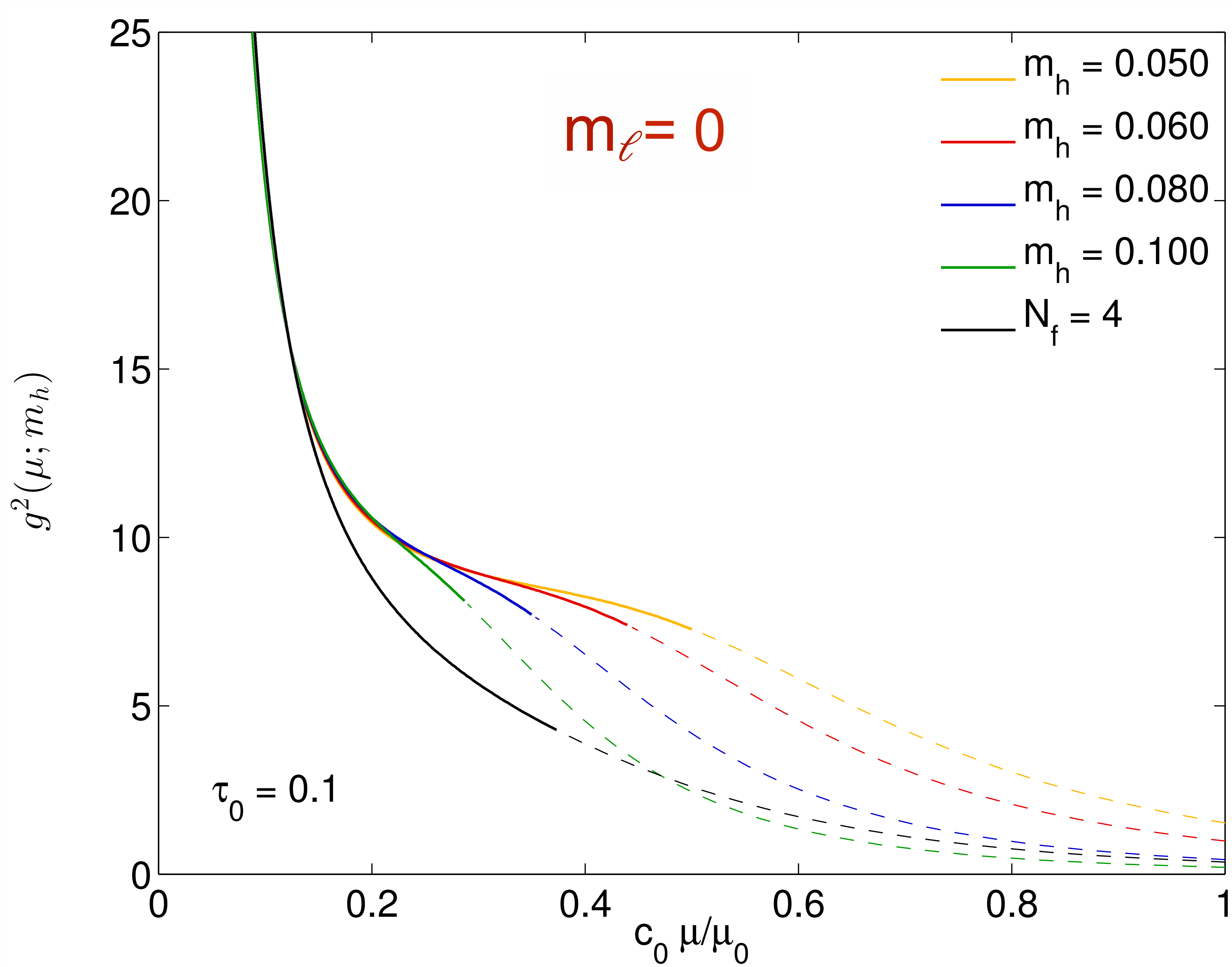

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Anomalous dimension

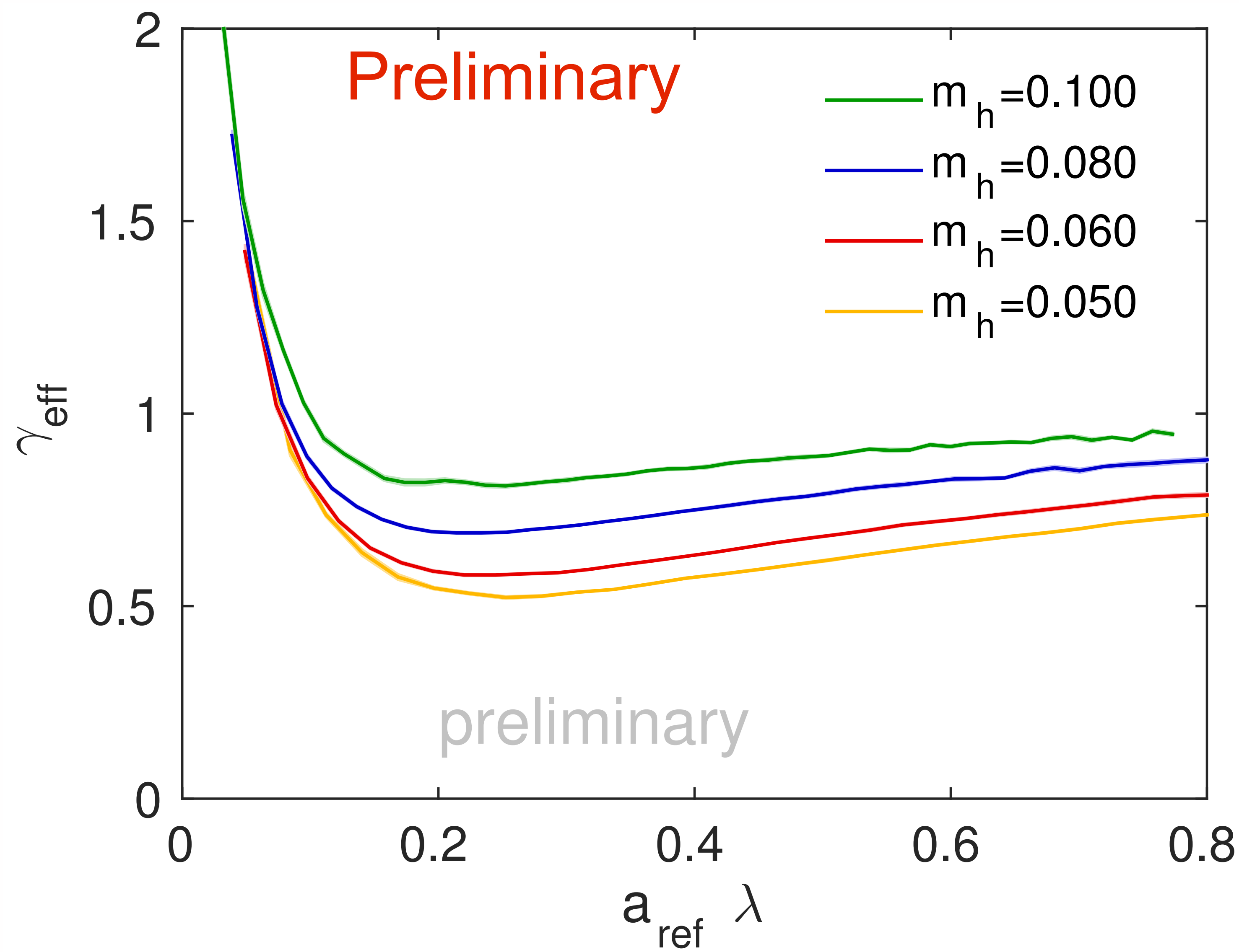
A scale dependent anomalous dimension $\gamma_{eff}(\mu)$ can be predicted from the Dirac operator mode number:

$$\mu(\lambda) \propto \lambda^{4/(\gamma_{eff}(\lambda)+1)} , \qquad \lambda \propto \mu$$

$\gamma_{eff}(\mu)$
- matches perturbative value at large μ ~ λ
- matches universal IRFP value at λ=0 for conformal system (meaningless once chiral symmetry breaks)

Anomalous dimension

Scale dependent anomalous dimension $\gamma_{eff}(\mu)$



In this system the anomalous dimension is not large but still O(1) and can persist

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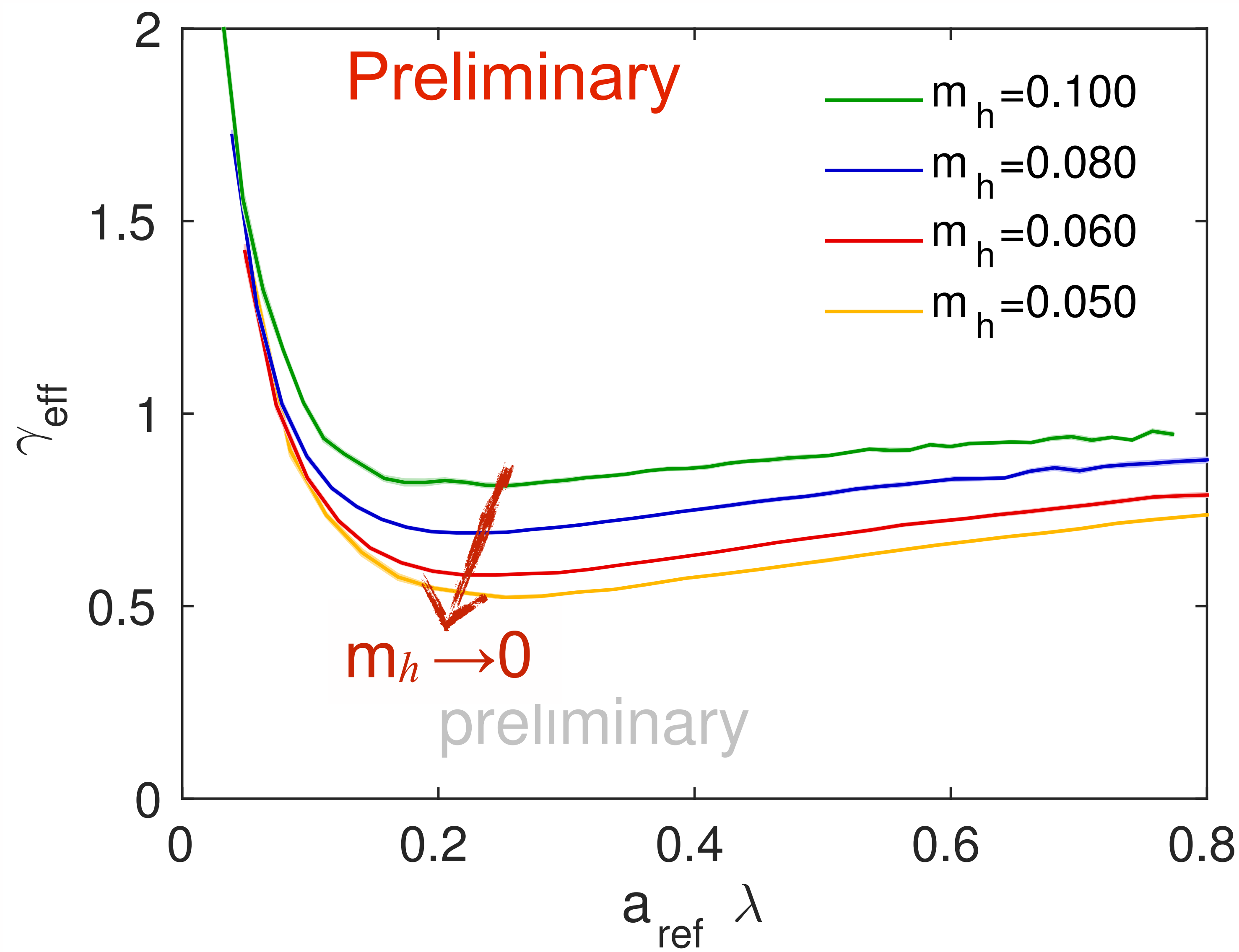

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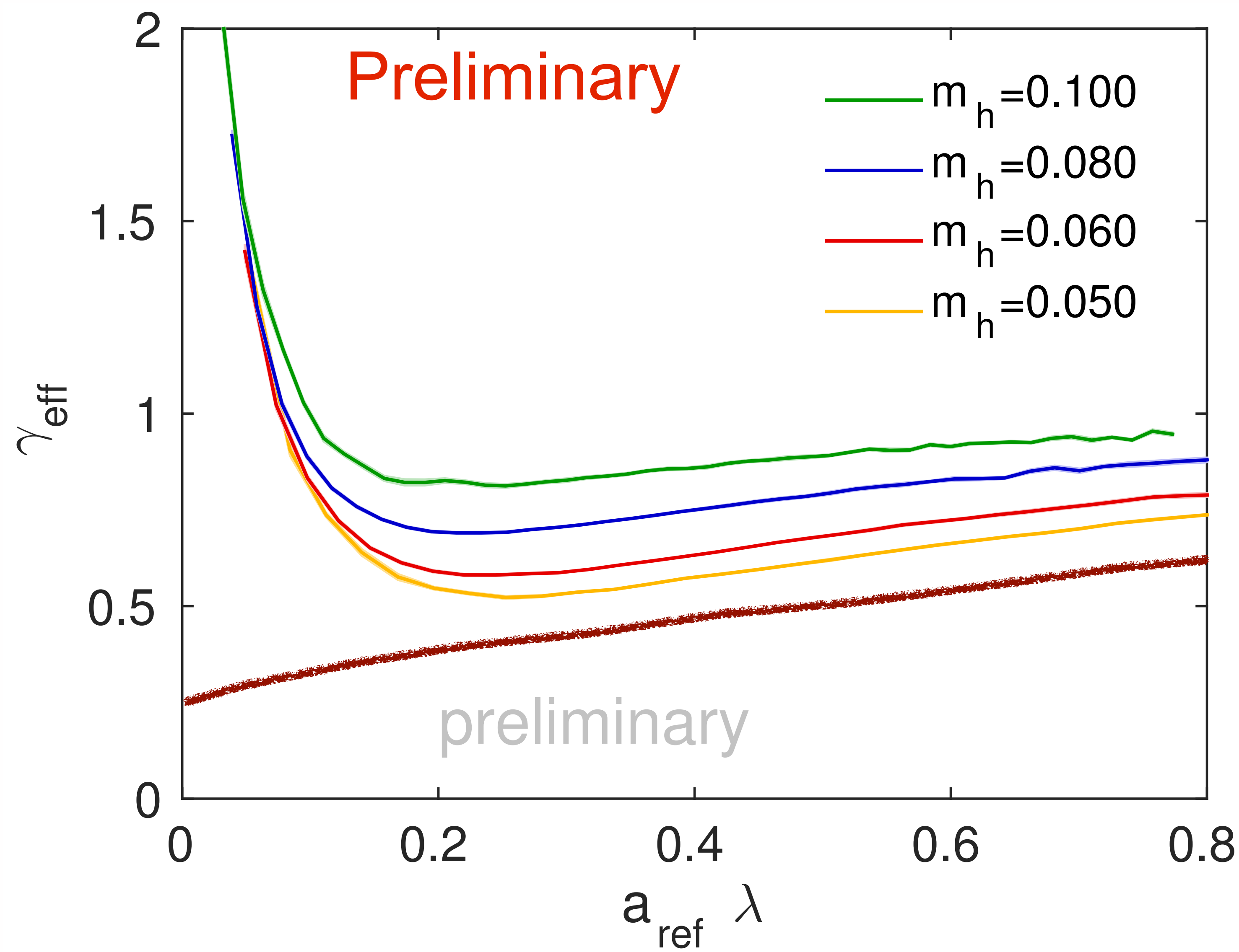
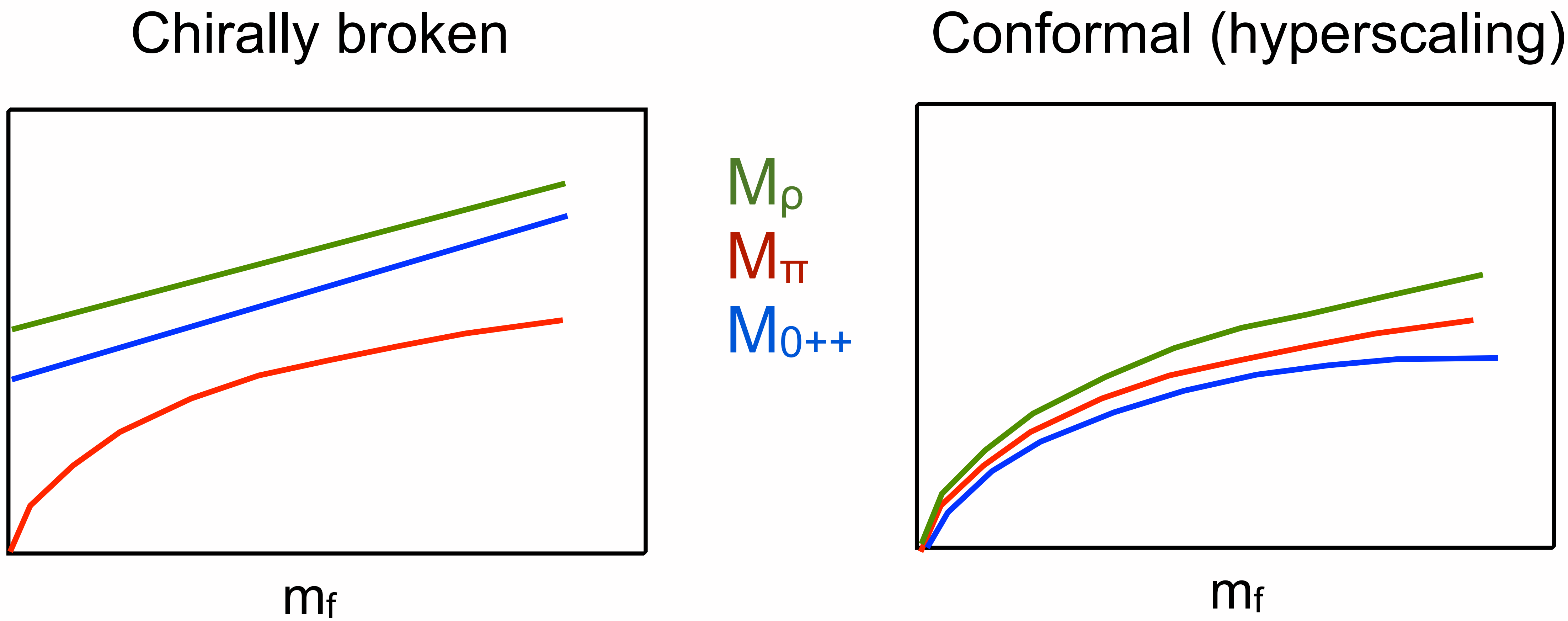


$N_f=12$: $\gamma_{IRFP}=0.235(15)$

In this system the anomalous dimension is not large but still O(1) and can persist

Hadron spectrum (sketch)



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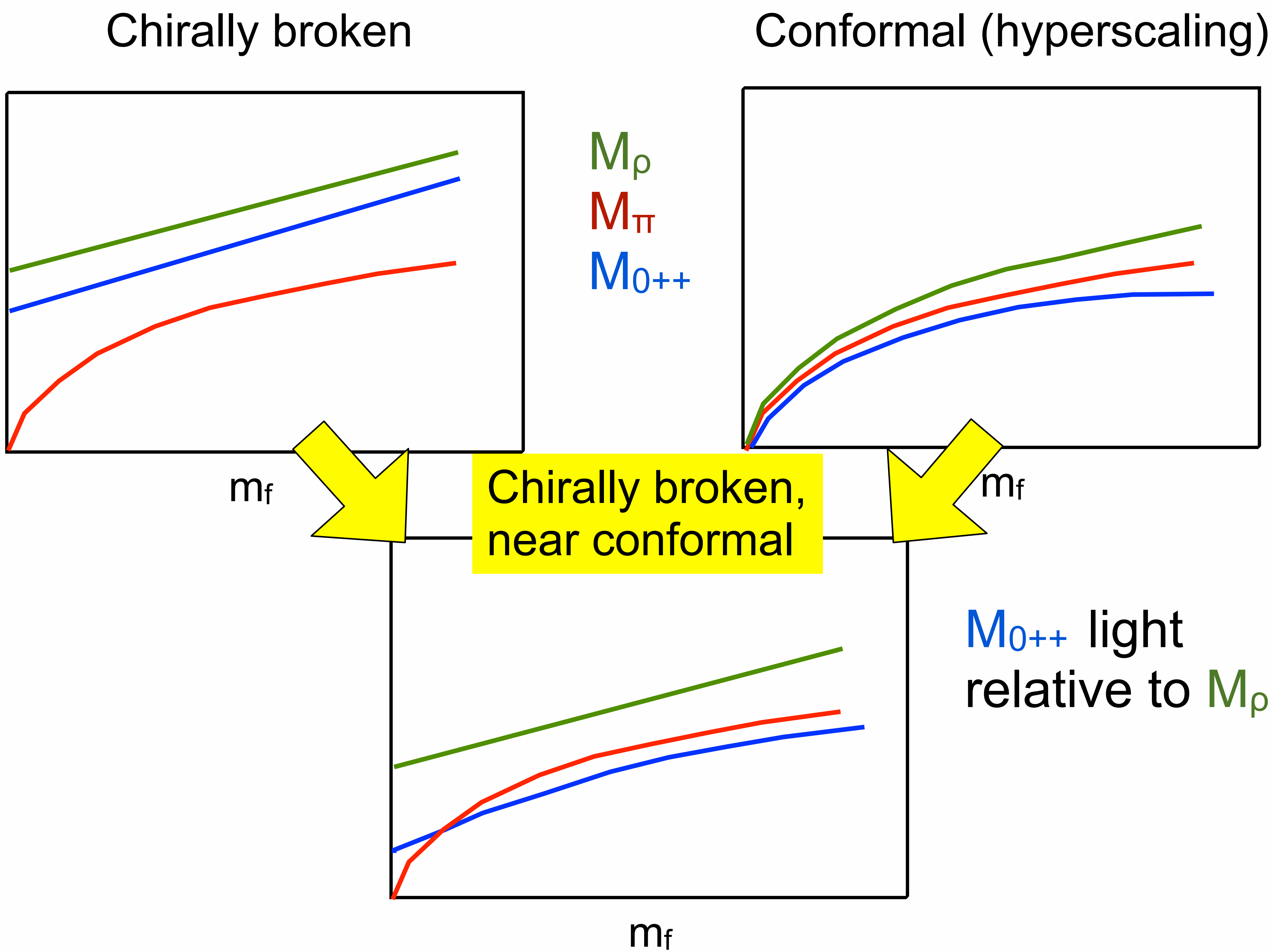

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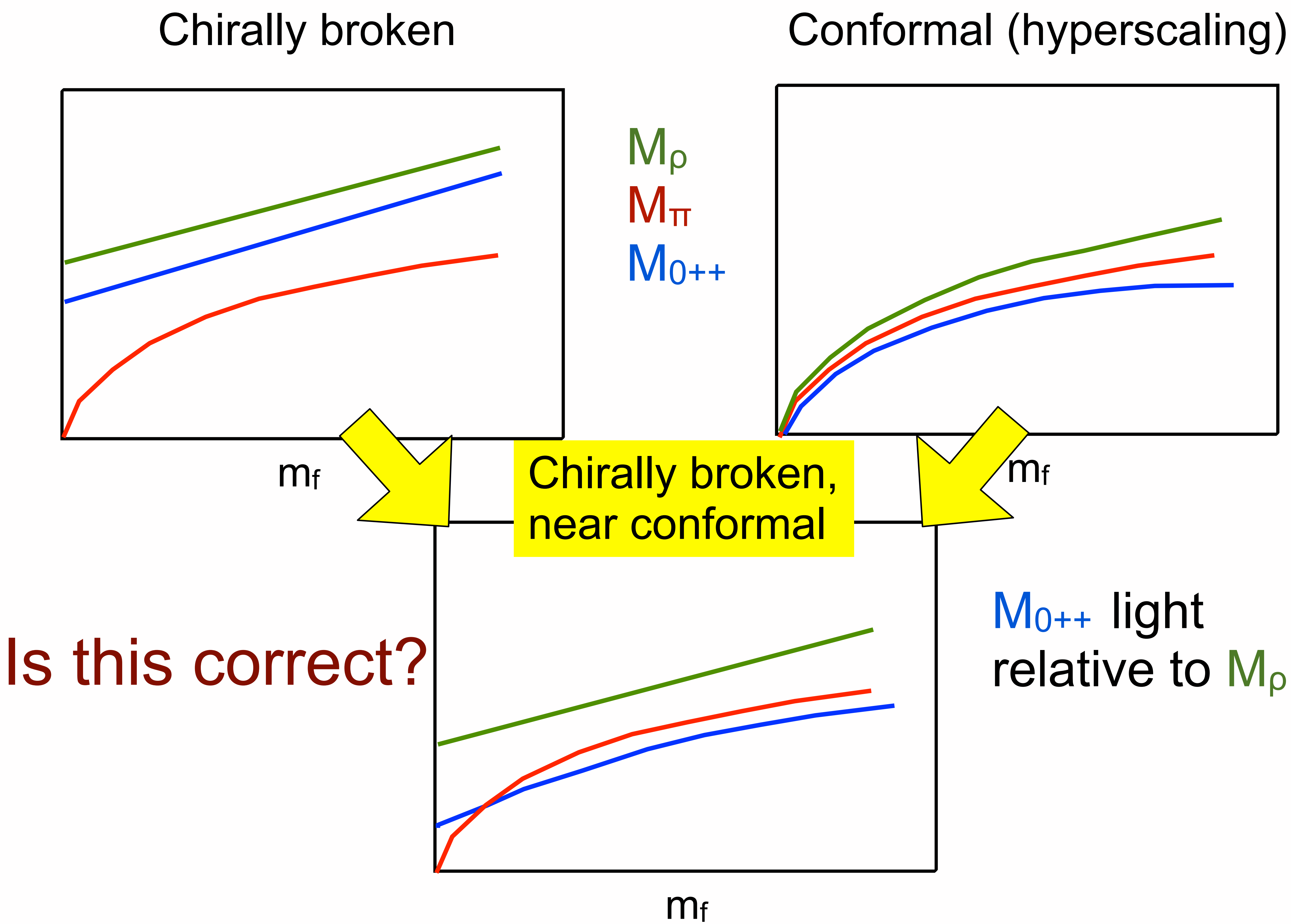
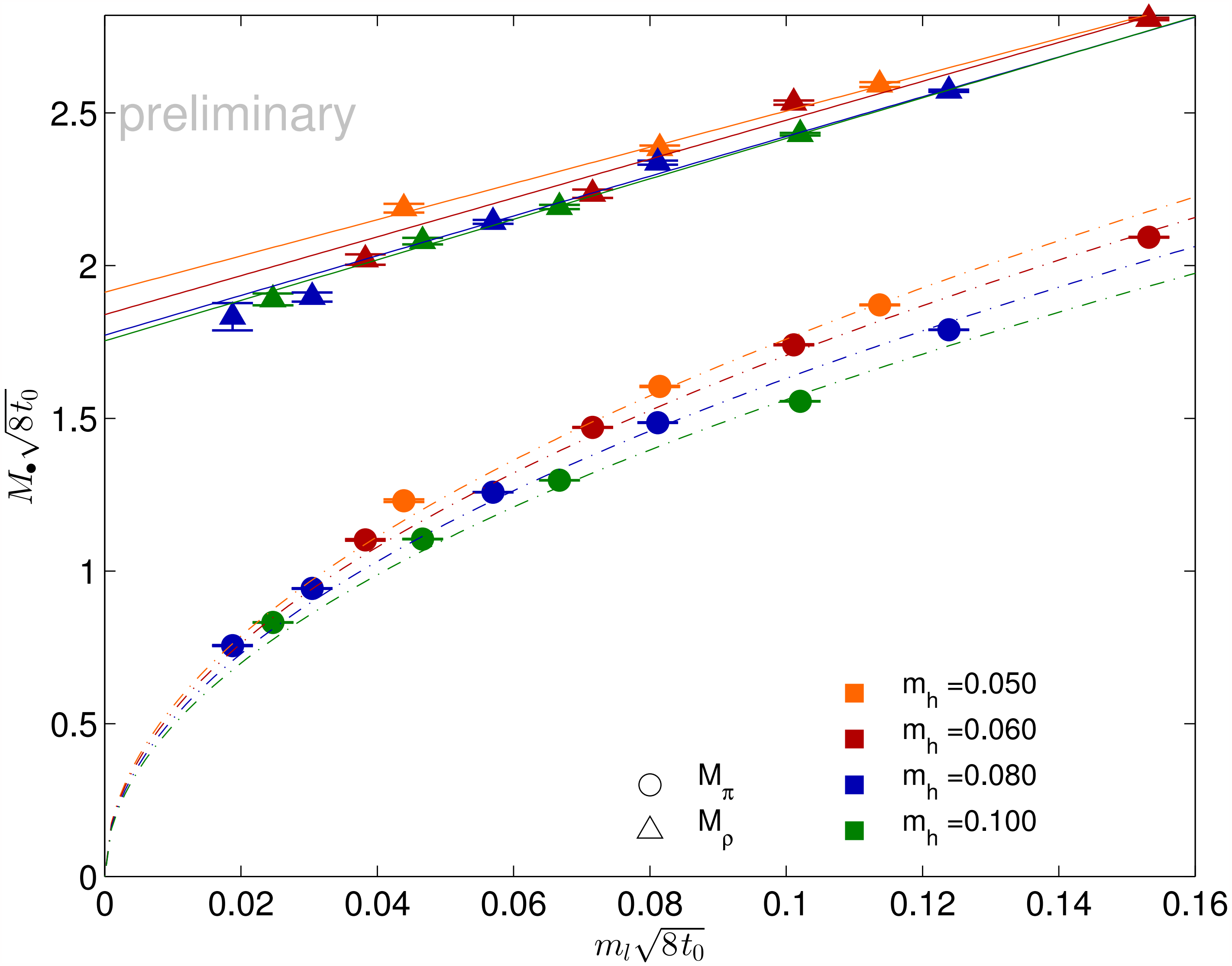


Connected spectrum, **4+8** flavors



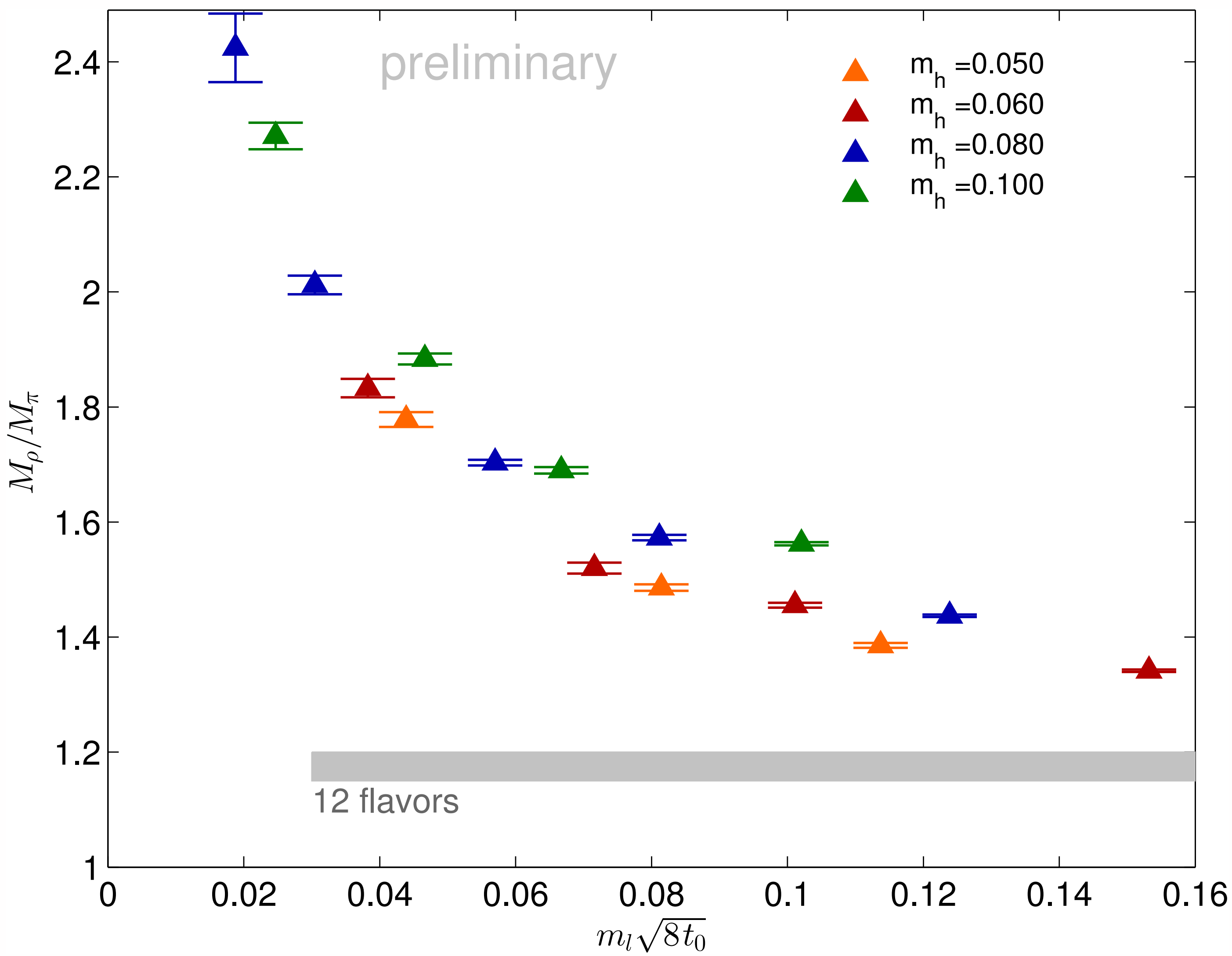
> M_π, M_ρ vs m_ℓ

(rescaled by the gradient flow scale $\sqrt{8t_0}$)

– little variation with m_h

Is the system chirally broken ?

M_ρ/M_π shows that we approach the chiral regime



< N_f=12 predicts an almost constant ratio (as should be in a conformal system)

(arXiv:1401.0195)

Finally : the 0^{++} scalar state

We use the same method to construct and fit the correlators as with N_f = 8 joint LSD project (E. Weinberg's talk)

- Disconnected correlators:
  - 6 U(1) sources
  - diluted on each timeslice, color, even/odd spatial
  - variance reduced $\langle \bar{\psi}\psi \rangle$
- Fit:
  - correlated fits to both parity (staggered) states
  - the vacuum subtraction introduces very large uncertainties
    - it is advantageous to add a (free) constant to the fit

$$C(t) = c_{0^{++}} \cosh\left(M_{0^{++}}\left(N_T/2 - t\right)\right) + c_{\pi_{\overline{sc}}} (-1)^t \cosh\left(M_{\pi_{\overline{sc}}}\left(N_T/2 - t\right)\right) + v$$

–this is equivalent to fitting the finite difference of the correlator

$$C(t+1) - C(t)$$

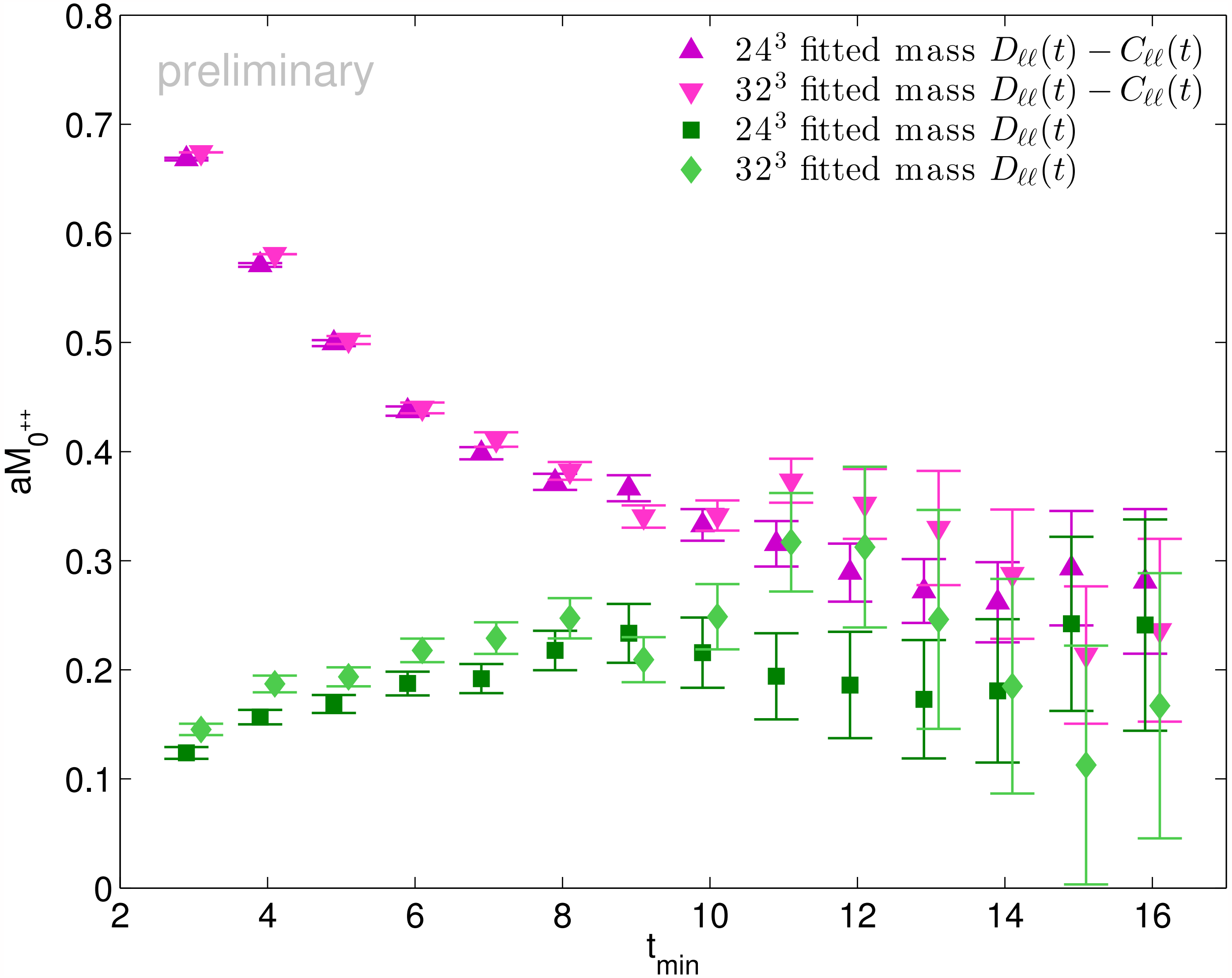
The 0^{++} mass

We compare predictions from $D_{\ell\ell}$ and $D_{\ell\ell}$ - $C_{\ell\ell}$ correlators

- in the $t \to \infty$ limit they should agree

Also compare different volumes

$m_h = 0.06$, $m_\ell = 0.010$:



M_{0++} predicted from non-linear range fits (t_{min} - $N_T/2$)

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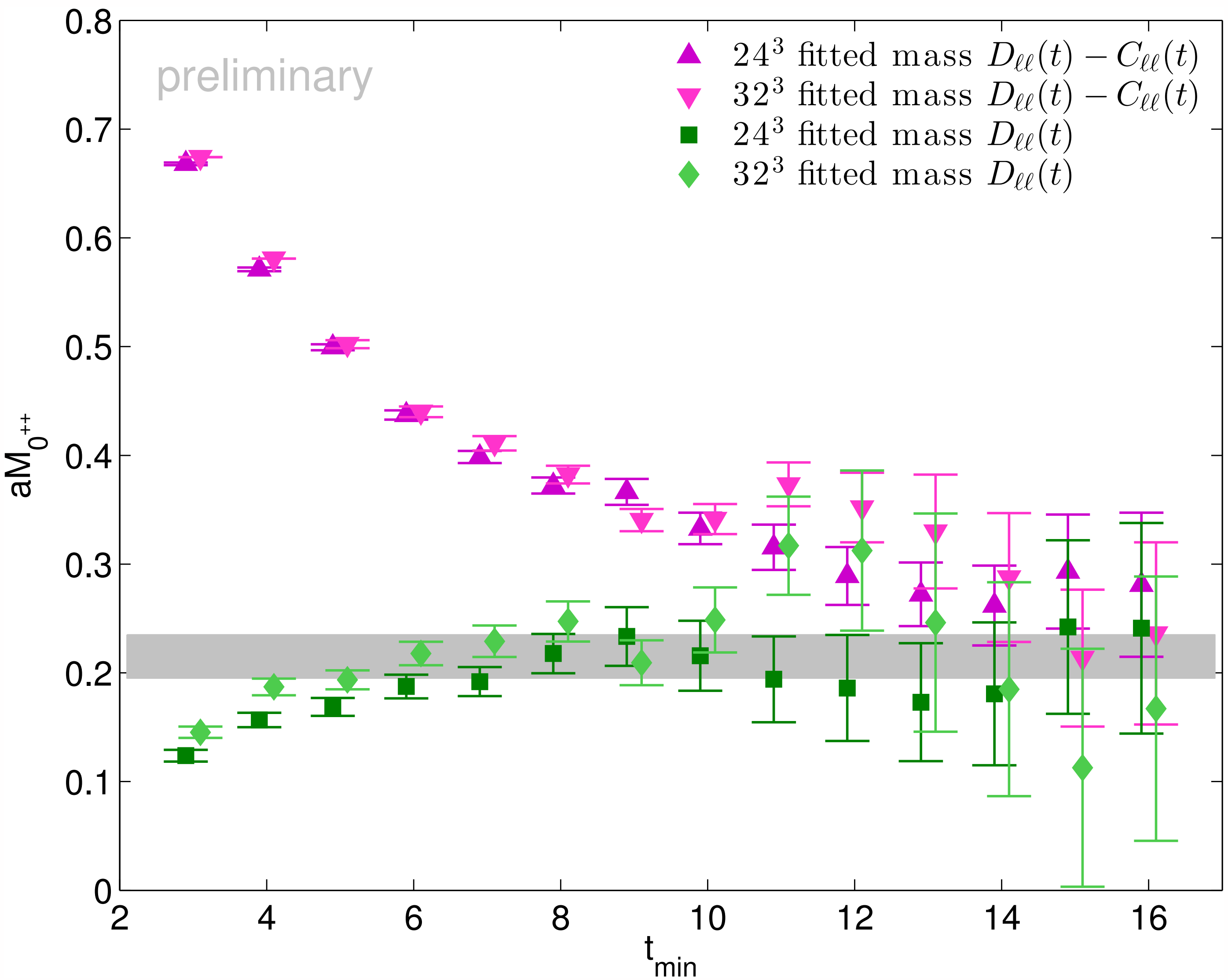

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both volumes, both correlators predict a consistent value

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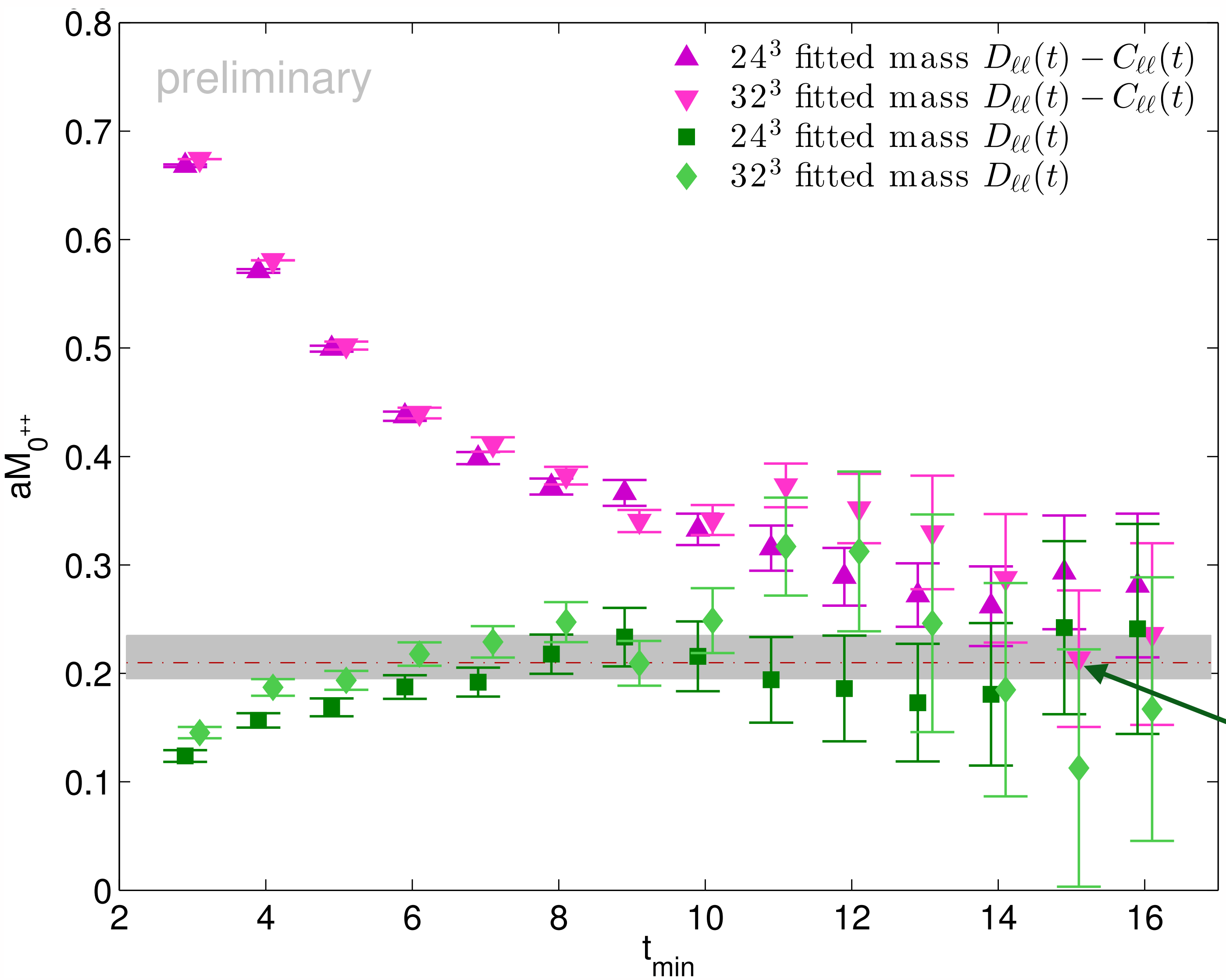

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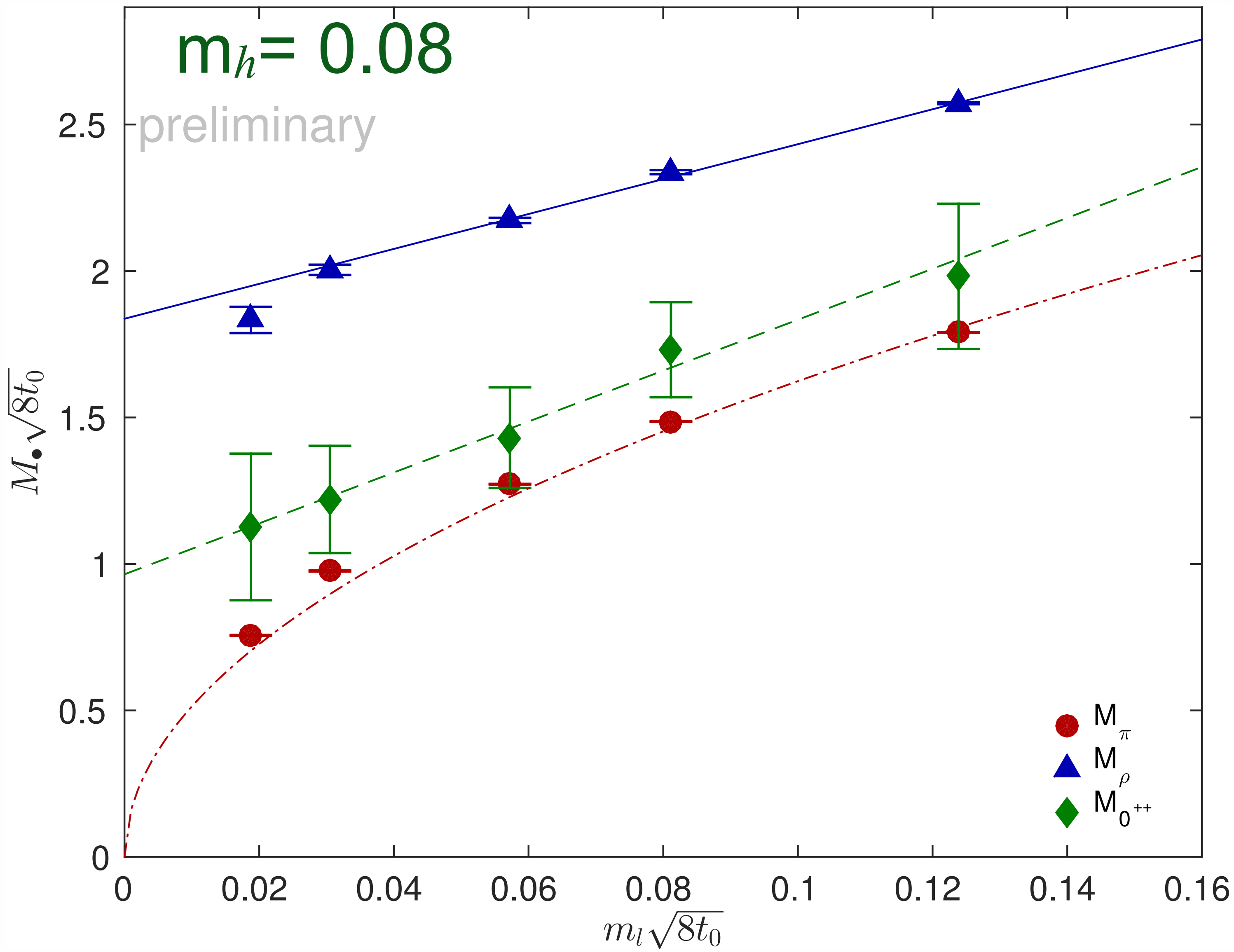
Spectrum

Compare the pion, rho and 0^{++} masses:

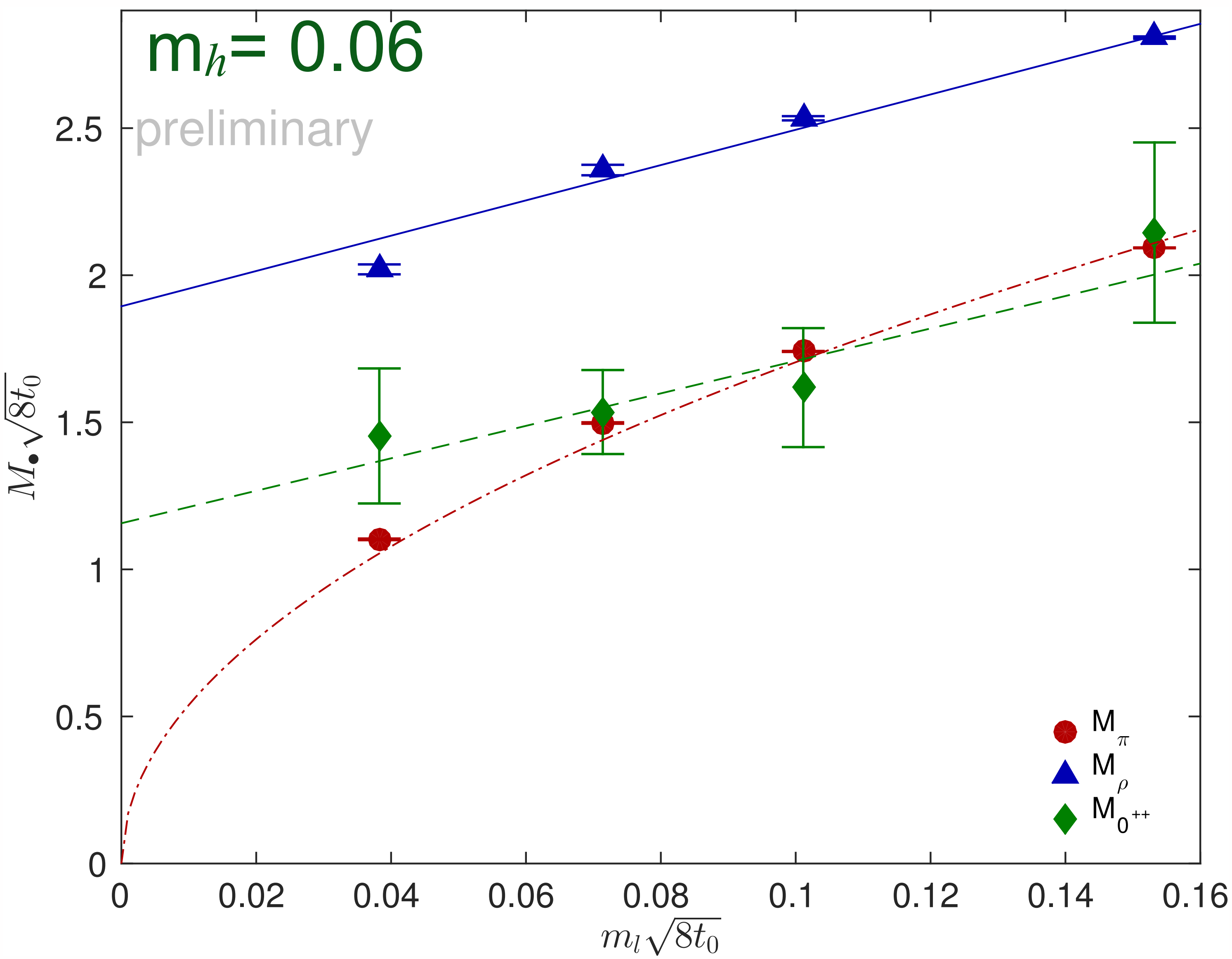


m_h = 0.08: the 0++

- is just above the pion,
- not Goldstone
- well below the rho

Spectrum

Compare the pion, rho and 0^{++} masses:



m_h = 0.06: the 0++

- is degenerate with pion at heavier m_ℓ
- need larger volumes, more statistics to resolve the small m_ℓ region

Conclusion & Summary

Lots of interesting possibilities ….

Lattice studies are needed to investigate strongly coupled systems

- individual and generic properties

Even models without apparent phenomenological importance can teach us to:

- understand universality
  - Wilson vs staggered vs rooted staggered vs domain wall fermions
- understand general properties of strongly coupled systems
  - walking near the conformal window
  - 0^{++} near the conformal window

Models with split fermion masses, like the 4+8 flavor model, can help us navigate the landscape

EXTRA SLIDES

$N_\ell + N_h = 4 + 8$: Parameter space

Action: nHYP smeared staggered fermions,
fundamental + adjoint gauge plaquette

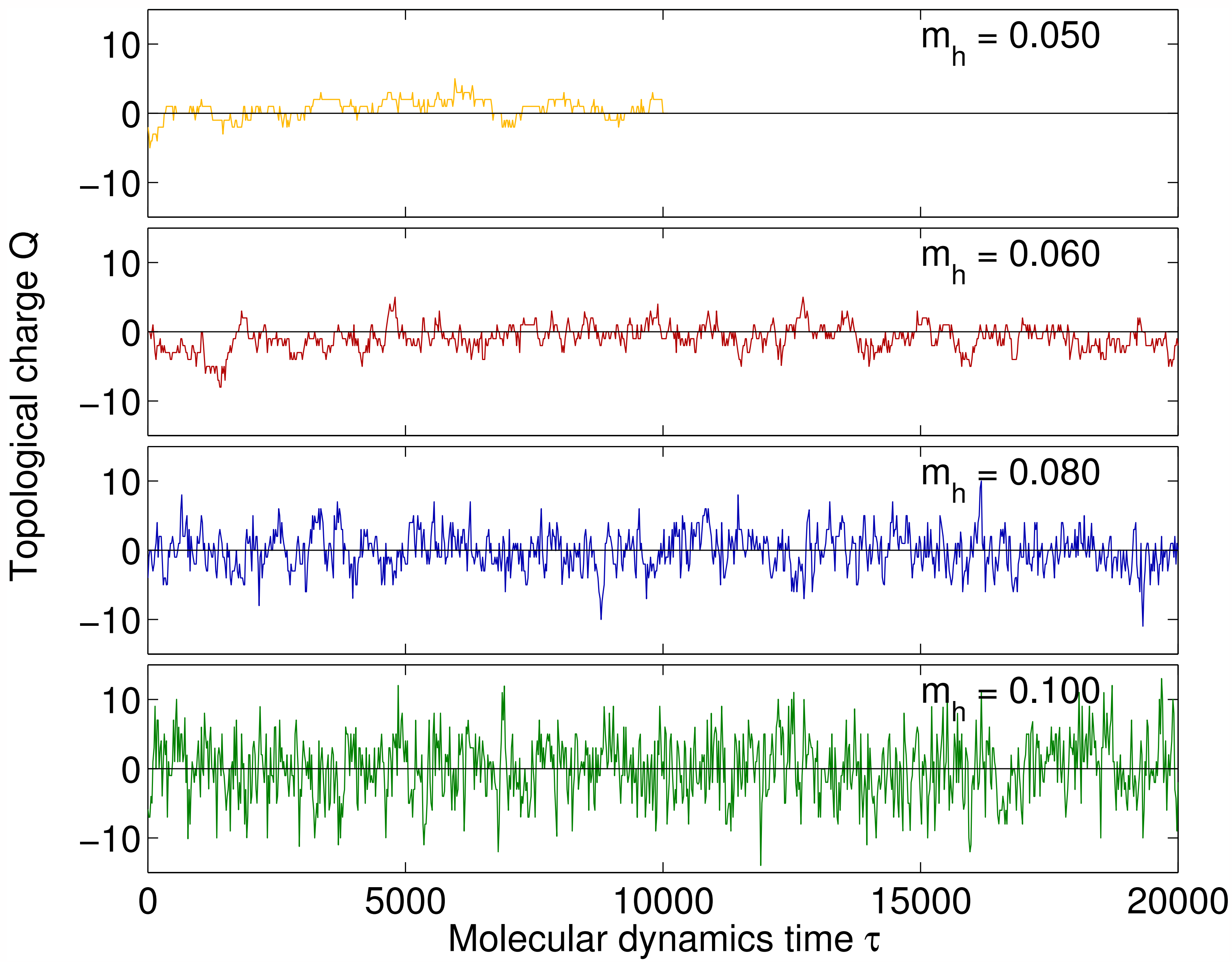
This action was used in the Boulder 4, 8, and 12 flavor studies
(1106.5293, 111.2317, 1404.0984)

It is the action used in the 8 flavor joint project with LSD
(E. Weinberg's talk)

We understand this action well

Topology evolution

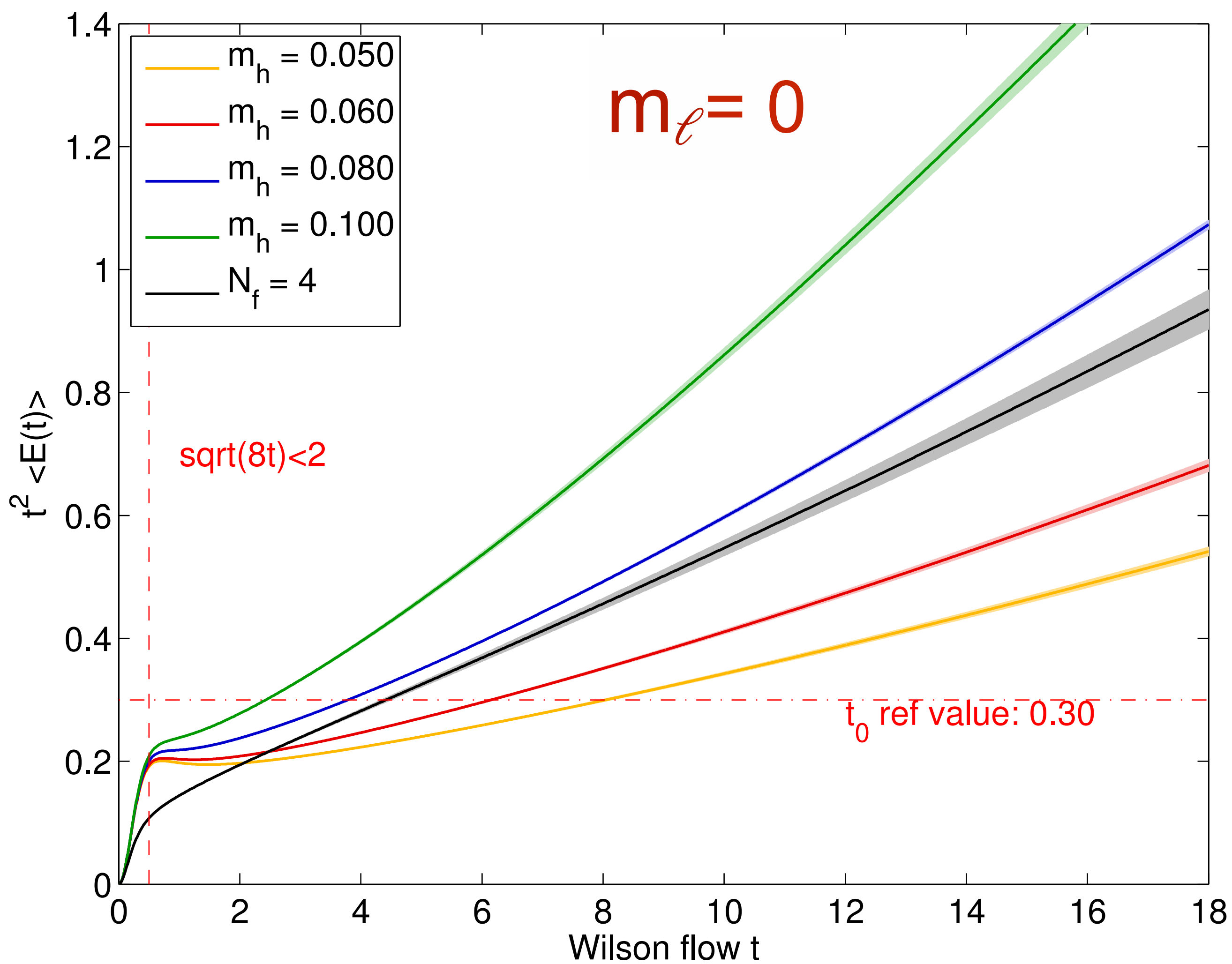
Topology is moving well even with the lightest mass



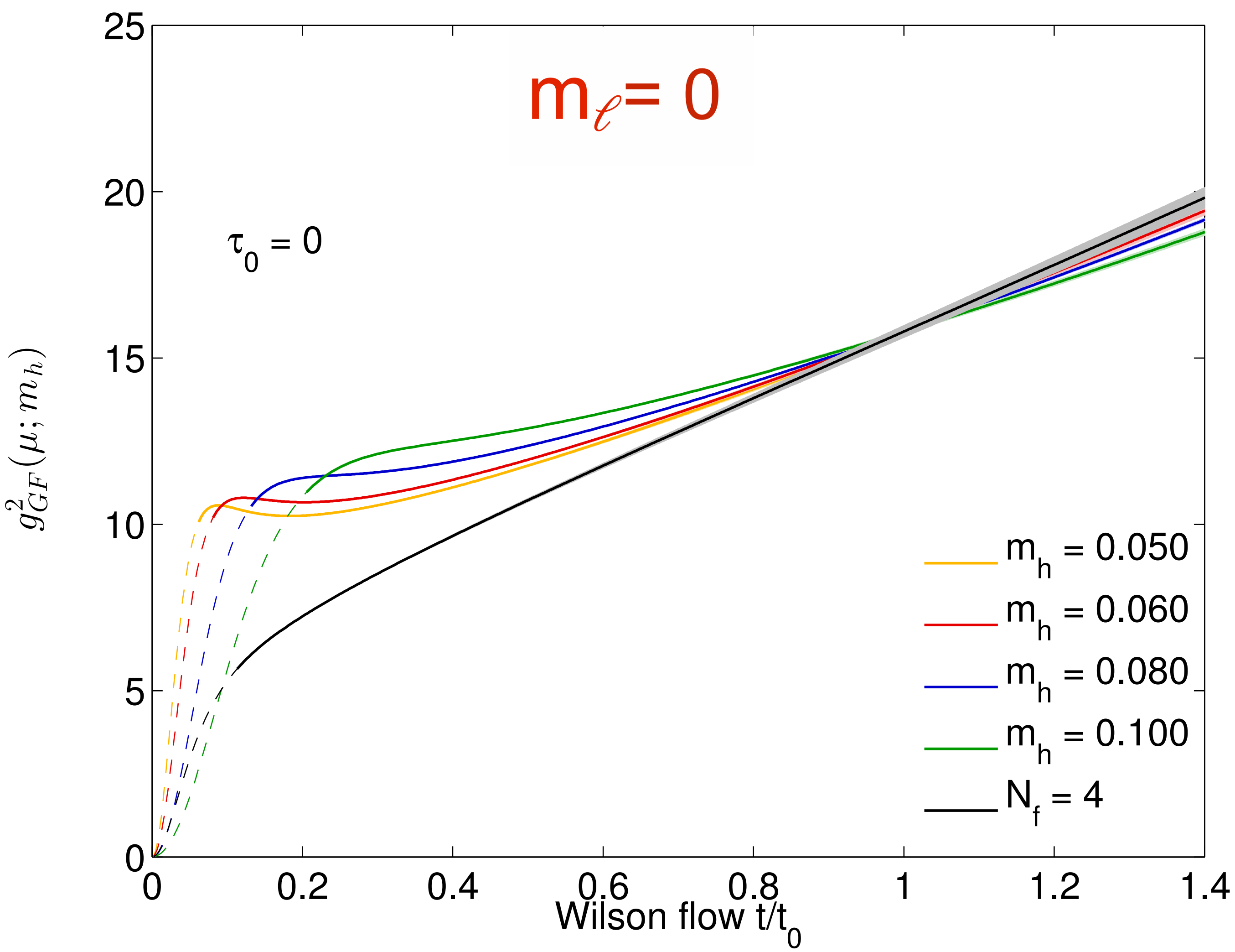
mص=0.010,
24^3x48 volume

Running coupling

$t^2\langle E(t)\rangle$ in the chiral limit at various m_h values



$g^2_{GF}(t/t_0)$ rescaled by t_0 at various m_h values



Rescaling forces the renormalized couplings to agree at t_0

Fan-out before and after are due to cut-off lattice artifacts

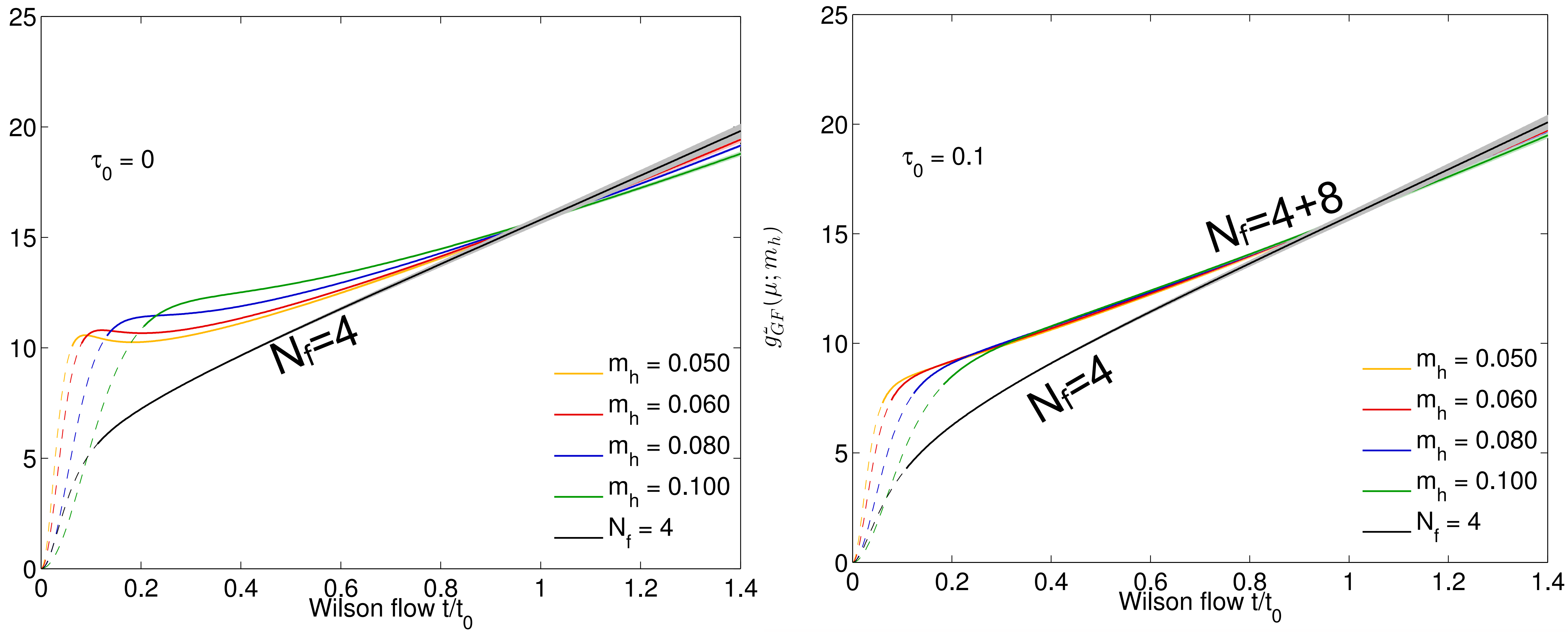
Improved running coupling

t-shift improved running coupling □□□□

$$\tilde{g}^2_{GF}(\mu = \frac{1}{\sqrt{8t}}) = \frac{1}{\mathcal{N}} t^2 \langle E(t+\tau_0) \rangle$$

by adjusting τ_0 most cut-off effects can be removed

(1404.0984, 1501.07848)



Mixing in the 0^{++} channel

There is one major difference between $N_f = 4 + 8$ and 8 :

- with non-degenerate masses the 0^{++} splits to light and heavy states
- there is mixing the heavy and light species

This is similar to η - η' mixing in QCD

→ need to diagonalize the correlator matrix

$$C(t) = \begin{pmatrix} D_{ll}(t) - C_{ll}(t) & \sqrt{2} D_{lh}(t) \\ \sqrt{2} D_{hl}(t) & 2 D_{hh}(t) - C_{hh}(t) \end{pmatrix}$$

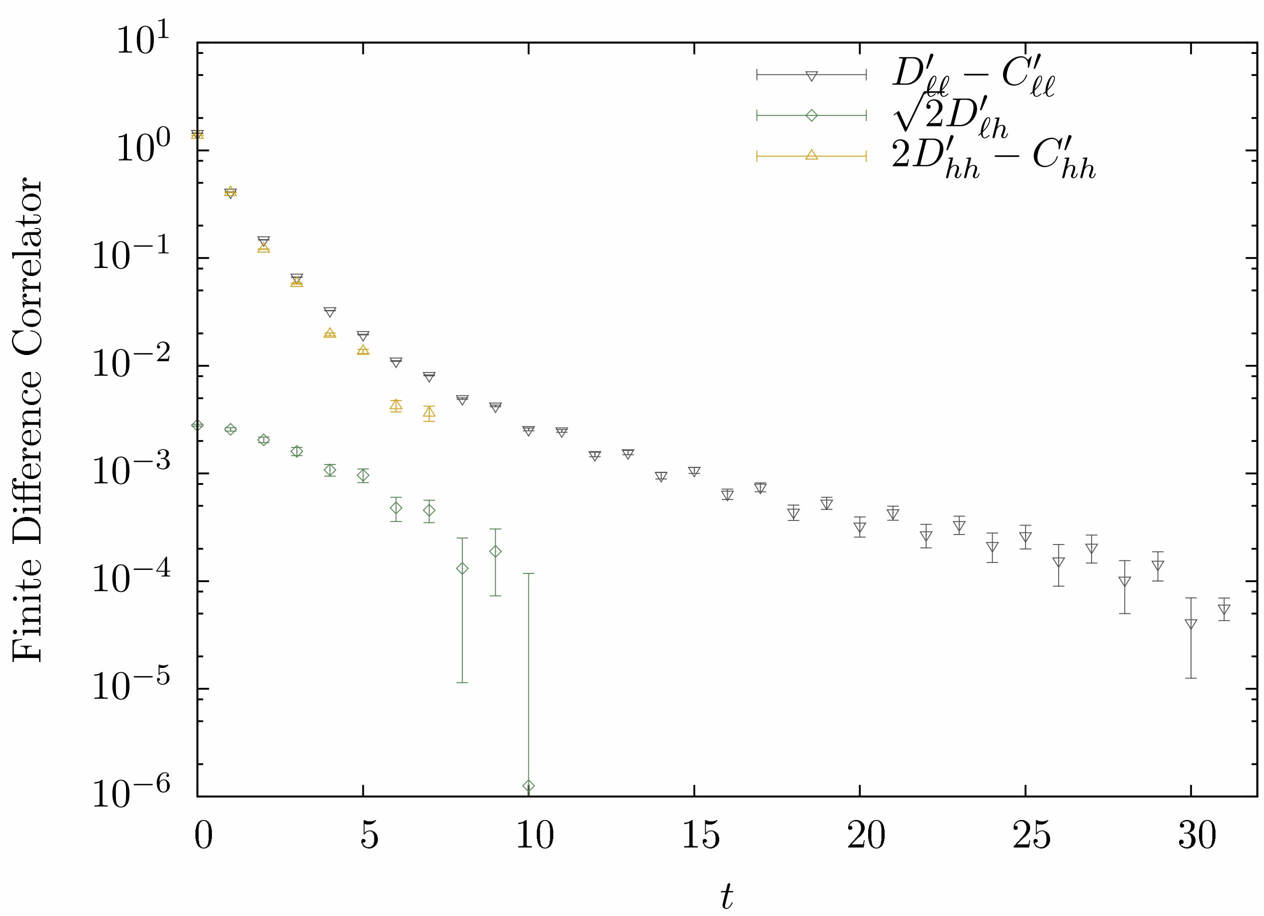
Normalization: even though we we describe 4 and 8 flavors, on the lattice they correspond to 1 and 2 staggered species

Mixing in the 0^{++} channel

$$C(t)=\begin{pmatrix} D_{ll}(t)-C_{ll}(t) & \sqrt{2}D_{lh}(t) \\ \sqrt{2}D_{hl}(t) & 2D_{hh}(t)-C_{hh}(t) \end{pmatrix}$$

Diagonalizing C(t) could lead to very large statistical errors.

Fortunately: $D_{\ell h}$ << diagonal terms for almost all parameter values



Finite difference correlators at $m_h = 0.05$, $m_\ell = 0.005$

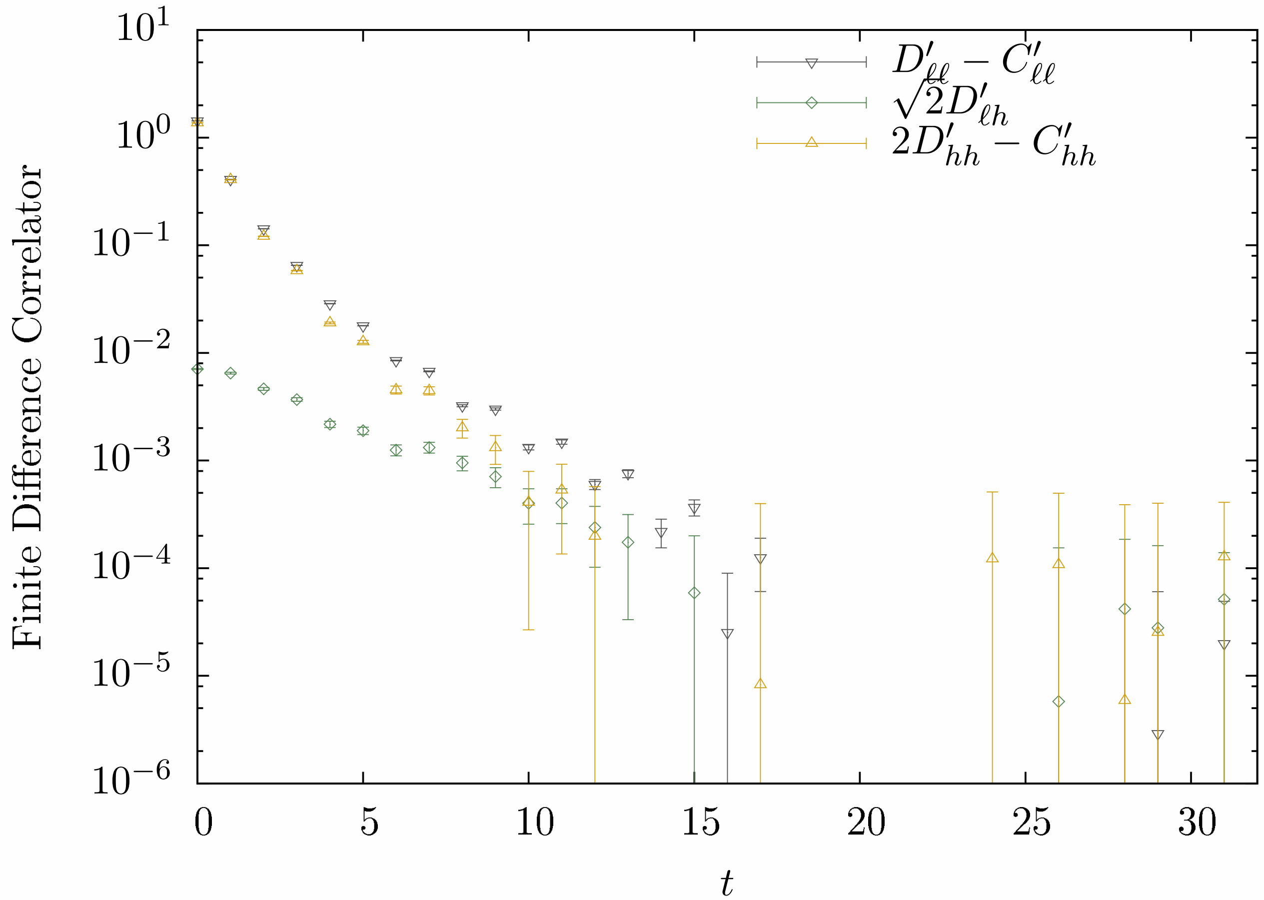
Mixing in the 0^{++} channel

$$C(t)=\begin{pmatrix} D_{ll}(t)-C_{ll}(t) & \sqrt{2}D_{lh}(t) \\ \sqrt{2}D_{hl}(t) & 2D_{hh}(t)-C_{hh}(t) \end{pmatrix}$$

Diagonalizing C(t) could lead to very large statistical errors.

Fortunately: $D_{\ell h}$ << diagonal terms for almost all parameter values

but not always!



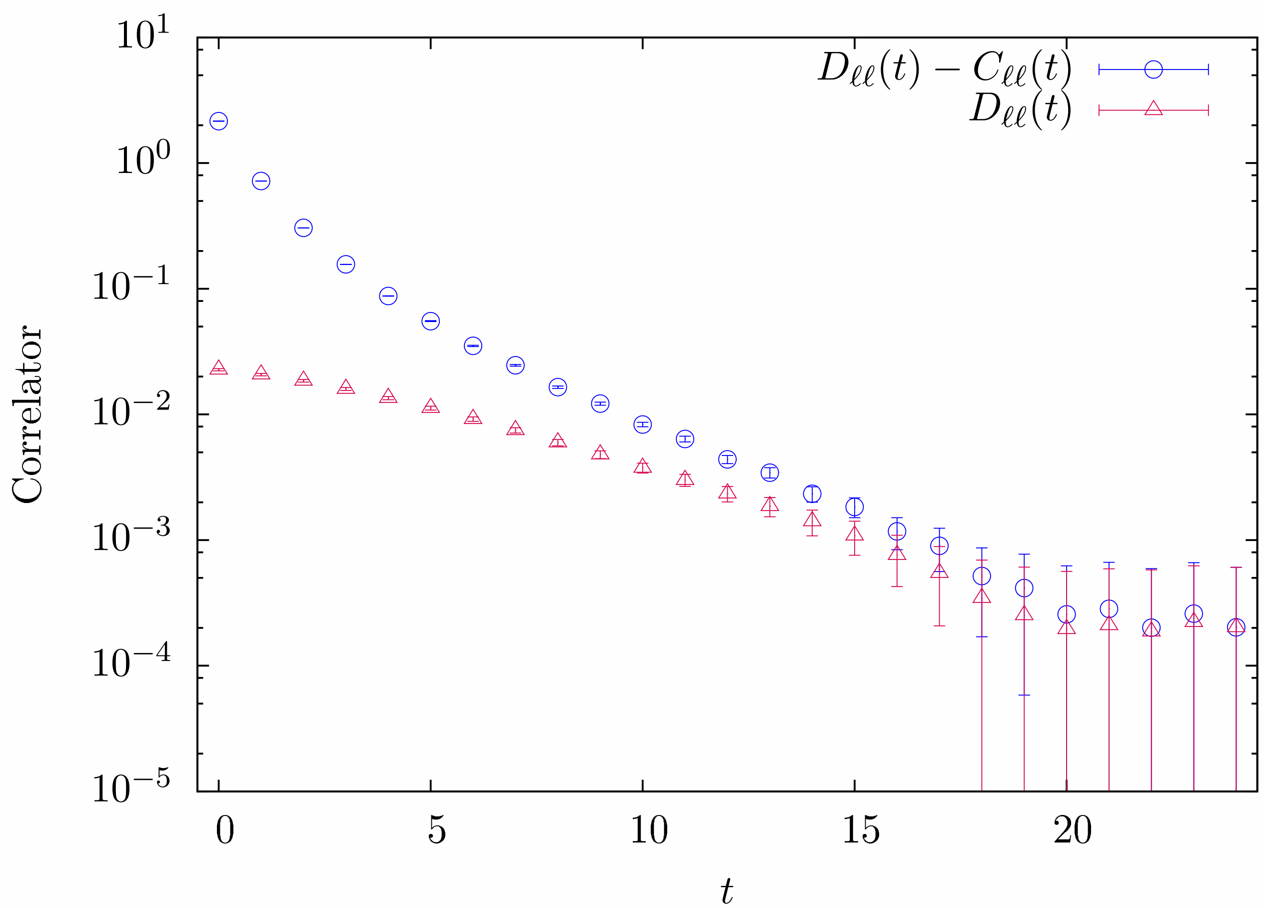
Derivative correlators at
m_h = 0.05, m_{ℓ} = 0.015

Mixing in the 0^{++} channel

$$C(t)=\begin{pmatrix} D_{ll}(t)-C_{ll}(t) & \sqrt{2}D_{lh}(t) \\ \sqrt{2}D_{hl}(t) & 2D_{hh}(t)-C_{hh}(t) \end{pmatrix}$$

Diagonalizing C(t) could lead to very large statistical errors.

Fortunately: the lightest excitation in $D_{\ell\ell}$ (and $D_{\ell h}$, D_{hh}) is the 0^{++}



Derivative correlators at
m_h = 0.06, m_ℓ = 0.010:
$D_{\ell\ell}$ and $D_{\ell\ell} - C_{\ell\ell}$