Running, Walking, Standing: the interplay between the running coupling and IR physics

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Focus:
The focus will be on the role that Lattice numerical simulations can play in the study of possible strong interactions in Beyond the Standard Model (BSM) physics, and in particular within the following topic areas:

- Composite dark matter
- Composite Higgs models and EWSB
- Theoretical applications in conformal field theory, string theory, and holography
- Strongly coupled models, including many-fermion gauge theories and SUSY
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- **Composite Higgs models and EWSB**
- Theoretical applications in conformal field theory, string theory, and holography
- Strongly coupled models, including many-fermion gauge theories and SUSY
Composite Higgs

Assume a new system with \(N_f\) fermions coupled to SU(\(N_T\)) gauge fields
Couple it to Standard Model fields:

- The Higgs could be a \(\bar{q}q\) (possibly \(qq\)) bound state
- 3 Goldstone pions break EW symmetry
- Tower of additional hadronic states appear in experiments

**What models could be compatible with EW data?**
- Chirally broken
- Most likely strongly coupled
- Walking

**What are the generic properties of strongly coupled models?**
- Is walking necessary? Is large anomalous dimension necessary?
- Spectrum? Where is \(M_{0^{++}}\) compared to \(M_\rho\)?

Lattice can investigate/answer most of the relevant questions
Roadmap: Theory Space
Roadmap:

Systems near the conformal boundary:
Roadmap:

Dietrich, Sannino

Systems near the conformal boundary:

SU(2), 2-flavor adjoint - conformal
Roadmap:

Systems near the conformal boundary:

- SU(2), 2-flavor adjoint - conformal
- SU(3), 12 flavor fundamental - looks conformal
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Dietrich, Sannino
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\( N_f = 8 \) RG step scaling function (basically negative RG \( \beta \) function) based on gradient flow coupling

Results follow 4-loop MS prediction up to \( g^2 = 15 \)


\( N_f = 8 \)
Roadmap:

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- SU(3) 2 flavor sextet - J. Kuti tomorrow

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Sixtet RG step scaling function (\(\beta\) function) based on gradient flow coupling

Results similar to 4-loop MS prediction up to \(g^2 = 5\)

A.H, C. Huang, Y. Liu, B. Svetitsky, in prep.

- Is this system chirally broken&walking or conformal?
- Are Wilson and rooted staggered fermions equivalent here? (Universality)
So many possibilities!
Is there any guiding principle to help choose?
Is there some general behavior near conformality?
Simple model - I

SU($N_c$) gauge with $N_\ell$ light ($m_\ell \approx 0$) and $N_h$ heavy ($m_h$) fermions

In the IR the heavy flavors decouple, $N_\ell$ light remain

$N_\ell + N_h =$ small: gauge coupling runs fast, heavy flavors have limited effect on the IR (QCD)

Continuum limit:

- tune $g^2 \to 0$
- $m_h \to 0$

Perturbative UVFP

RG flow from UV to IR
Simple model - II

SU($N_c$) gauge with $N_\ell$ light ($m_\ell \approx 0$) and $N_h$ heavy ($m_h$) fermions

$N_\ell + N_h = \text{near but below the conformal window}$

IF the gauge coupling is “walking” the IR can be very different

$\beta \propto 1/g^2$

There is no guarantee that any $N_\ell + N_h$ system will walk
Simple model - III

SU($N_c$) gauge with $N_\ell$ light ($m_\ell \approx 0$) and $N_h$ heavy ($m_h$) fermions

$N_\ell + N_h = \text{above the conformal window, } N_\ell \text{ is below}$

guarantees that the gauge coupling is “walking”; the IR will be very different
Simple model - III

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What are the properties of these strongly coupled “walking” systems?
$N_\ell + N_h$: Parameter space

3 independent parameters: $(g^2, m_\ell, m_h)$

- $g^2$ does not matter once the flow reaches the RG trajectory
- sufficient to work at $g^2 = \text{const}$, vary $m_h$ only ($m_\ell = 0$)

Continuum limit: tune $m_h \to 0$

$(m_\ell \ll m_h)$
$N_\ell + N_h : \text{Parameter space}$

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Continuum limit:
tune $m_h \to 0$

$(m_\ell << m_h)$
\( N_{\ell} + N_h : \) Parameter space

3 independent parameters: \((g^2, m_{\ell}, m_h)\)
- \(g^2\) does not matter once the flow reaches the RG trajectory
- sufficient to work at \(g^2=\text{const}\), vary \(m_h\) only \((m_{\ell}=0)\)

Continuum limit:
- tune \(m_h \rightarrow 0\)
- \((m_{\ell} \ll m_h)\)
\(N_\ell + N_h\) systems

\(N_\ell + N_h = 2 + 6\) if \(N_f = 8\) is the UV model

or

\(N_\ell + N_h = 2 + 10\) for \(N_f = 12\) conformal behavior in the UV

Pilot study:

\(N_\ell + N_h = 4 + 8\) : conformal in the UV, \(N_l=4\) flavor in the IR

in collaboration with R. Brower, C. Rebbi, E. Weinberg, O. Witzel

arXiv:1411.3243

Why \(4+8\)? We use staggered fermions:

4 and 8 flavors do not require rooting
\( N_\ell + N_h = 4+8 \) : Parameter space

- \( \beta = 4.0 \) (close to the 12-flavor IRFP)
- \( m_h = 0.10, 0.08, 0.06, 0.05 \)
- \( m_\ell = 0.003, 0.005, 0.010, 0.015, 0.025, 0.035 \)

Volumes:
24\(^3\)x48, (dots)
32\(^3\)64 (circle), 36\(^3\)64
48\(^3\)x96 (square)

Color: volume \text{OK} / marginal/ squeezed

20,000 MDTU, most still in progress
Lattice scale

Use gradient flow to estimate the lattice scale $\sqrt{8t_0}$

Significant variation with $m_{\ell}, m_h$, but finite volume effects are controlled

$\sqrt{8t_0} \lesssim L/5$

is usually sufficient

$\to$ color coding

$N_f=12$

$m_h=0.040$

$m_h=0.050$

$m_h=0.060$

$m_h=0.080$

$m_h=0.100$

$N_f=4$
Running coupling

Gradient flow is a gauge field transformation that defines a renormalized coupling

\[ g^2_{GF}(\mu = \frac{1}{\sqrt{8t}}) = \frac{1}{N} t^2 \langle E(t) \rangle \]

\[ g^2_{GF} \] is used for scale setting as

\[ g^2_{GF}(t = t_0) = \frac{0.3}{N} \]

Is it appropriate to determine the renormalized running coupling?

Yes:

– on large enough volumes
– at large enough flow time
– in the continuum limit

arXiv:1006.4518
Running coupling

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Is it appropriate to determine the renormalized running coupling?

Yes:

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- at large enough flow time

\} use t-shift improved coupling

- in the continuum limit
Improved running coupling : 4+8 flavors

There are error bars on this plot!

$g^2_{GF}(\mu)$ develops a “shoulder” as $m_h \to 0$ : this is walking!

Walking range can be tuned arbitrarily with $m_h$
Improved running coupling : 4+8 flavors

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Improved running coupling : 4+8 flavors

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Walking range can be tuned arbitrarily with $m_h$

There are error bars on this plot!
Anomalous dimension

A scale dependent anomalous dimension $\gamma_{\text{eff}}(\mu)$ can be predicted from the Dirac operator mode number:

$$\mu(\lambda) \propto \lambda^{\frac{4}{(\gamma_{\text{eff}}(\lambda)+1)}}, \quad \lambda \propto \mu$$

$\gamma_{\text{eff}}(\mu)$
- matches perturbative value at large $\mu \sim \lambda$
- matches universal IRFP value at $\lambda=0$ for conformal system (meaningless once chiral symmetry breaks)
Anomalous dimension

Scale dependent anomalous dimension $\gamma_{\text{eff}}(\mu)$

In this system the anomalous dimension is not large but still $O(1)$ and can persist.
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$N_f=12: \gamma_{\text{IRFP}} = 0.235(15)$

Preliminary
Hadron spectrum (sketch)

Chirally broken

Conformal (hyperscaling)

$M_\rho$

$M_{\pi}$

$M_{0++}$

$m_f$

$m_f$
Hadron spectrum (sketch)

Chirally broken

Conformal (hyperscaling)

Chirally broken, near conformal

$M_\rho$

$M_{\pi \pi}$

$M_{0^{++}}$

$m_f$

$M_{0^{++}}$ light relative to $M_\rho$

$m_f$

$m_f$

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Chirally broken, near conformal

Is this correct?

Chirally broken

Conformal (hyperscaling)

$M_\rho$

$M_{\pi\pi}$

$M_{0^{++}}$

$M_{0^{++}}$ light relative to $M_\rho$
Connected spectrum, 4+8 flavors

$\pi$, $\rho$ vs $m_\ell$ (rescaled by the gradient flow scale $\sqrt{8t_0}$)

- little variation with $m_h$
Is the system chirally broken?

$M_\rho/M_\pi$ shows that we approach the chiral regime

$\langle N_f = 12 \rangle$ predicts an almost constant ratio (as should be in a conformal system)

(arXiv:1401.0195)
Finally: the $0^{++}$ scalar state

We use the same method to construct and fit the correlators as with $N_f = 8$ joint LSD project (E. Weinberg’s talk)

- Disconnected correlators:
  - 6 U(1) sources
  - diluted on each timeslice, color, even/odd spatial
  - variance reduced $\langle \bar{\psi}\psi \rangle$

- Fit:
  - correlated fits to both parity (staggered) states
  - the vacuum subtraction introduces very large uncertainties
    - it is advantageous to add a (free) constant to the fit

$$C(t) = c_{0^{++}} \cosh\left( M_{0^{++}} \left( N_T / 2 - t \right) \right) + c_{\pi_{sc}} (-1)^t \cosh\left( M_{\pi_{sc}} \left( N_T / 2 - t \right) \right) + v$$

- this is equivalent to fitting the finite difference of the correlator

$$C(t + 1) - C(t)$$
**The 0^{++} mass**

We compare predictions from $D_{\ell\ell}$ and $D_{\ell\ell} - C_{\ell\ell}$ correlators — in the $t \to \infty$ limit they should agree.

Also compare different volumes:

$m_h = 0.06$, $m_{\ell} = 0.010$:

\[ aM_{0^{++}} \]

**M$_{0^{++}}$ predicted from non-linear range fits ($t_{\text{min}} - N_T/2$)**
The $0^{++}$ mass

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$m_h = 0.06$, $m_\ell = 0.010$:

M$_{0^{++}}$ predicted from non-linear range fits ($t_{\text{min}} - N_T/2$)

both volumes, both correlators predict a consistent value
The $0^{++}$ mass

We compare predictions from $D_{\ell\ell}$ and $D_{\ell\ell} - C_{\ell\ell}$ correlators
– in the $t \to \infty$ limit they should agree
Also compare different volumes

$m_\ell = 0.06$, $m_\ell = 0.010$: 

![Graph showing $aM_{0^{++}}$ against $t_{\text{min}}$ with data points and error bars]

M$_{0^{++}}$ predicted from non-linear range fits ($t_{\text{min}} - NT/2$)

both volumes, both correlators predict a consistent value

pion
Compare the pion, rho and 0\(^{++}\) masses:

\[ m_h = 0.08 \]

- is just above the pion,
- not Goldstone
- well below the rho

\[ m_h = 0.08: \text{ the } 0^{++} \]
Spectrum

Compare the pion, rho and $0^{++}$ masses:

$m_h = 0.06$:
- is degenerate with pion at heavier $m_\ell$
- need larger volumes, more statistics to resolve the small $m_\ell$ region
Conclusion & Summary

Lots of interesting possibilities ....
Lattice studies are needed to investigate strongly coupled systems
  - individual and generic properties

Even models without apparent phenomenological importance can teach us to:
  – understand universality
    • Wilson vs staggered vs rooted staggered vs domain wall fermions
  – understand general properties of strongly coupled systems
    • walking near the conformal window
    • $0^{++}$ near the conformal window

Models with split fermion masses, like the 4+8 flavor model, can help us navigate the landscape
EXTRA SLIDES
\( N_\ell + N_h = 4+8 : \) Parameter space

Action: nHYP smeared staggered fermions, fundamental + adjoint gauge plaquette

This action was used in the Boulder 4, 8, and 12 flavor studies

(1106.5293, 111.2317, 1404.0984)

It is the action used in the 8 flavor joint project with LSD

(E. Weinberg’s talk)

We understand this action well
Topology evolution

Topology is moving well even with the lightest mass

$m_\chi = 0.010$, 24$^3 \times 48$ volume
Running coupling

\( t^2 \langle E(t) \rangle \) in the chiral limit at various \( m_h \) values

\( g_{GF}^2(t/t_0) \) rescaled by \( t_0 \) at various \( m_h \) values

Rescaling forces the renormalized couplings to agree at \( t_0 \)
Fan-out before and after are due to cut-off lattice artifacts
Improved running coupling

t-shift improved running coupling

\[ \tilde{g}^2_{GF}(\mu = \frac{1}{\sqrt{8t}}) = \frac{1}{N} t^2 \langle E(t + \tau_0) \rangle \]

by adjusting \( \tau_0 \) most cut-off effects can be removed

\[ (1404.0984, 1501.07848) \]
Mixing in the $0^{++}$ channel

There is one major difference between $N_f=4+8$ and 8:
– with non-degenerate masses the $0^{++}$ splits to light and heavy states
– there is mixing the heavy and light species

This is similar to $\eta - \eta'$ mixing in QCD
→ need to diagonalize the correlator matrix

$$C(t) = \begin{pmatrix}
D_{ll}(t) - C_{ll}(t) & \sqrt{2} D_{lh}(t) \\
\sqrt{2} D_{hl}(t) & 2 D_{hh}(t) - C_{hh}(t)
\end{pmatrix}$$

Normalization: even though we describe 4 and 8 flavors, on the lattice they correspond to 1 and 2 staggered species
Mixing in the $0^{++}$ channel

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\end{pmatrix}$$

Diagonalizing $C(t)$ could lead to very large statistical errors.

Fortunately: $D_{\omega\omega} \ll$ diagonal terms for almost all parameter values

Finite difference correlators at $m_{\ell} = 0.05$, $m_{\omega} = 0.005$
Mixing in the $0^{++}$ channel

$$C(t) = \begin{pmatrix} D_{ll}(t) - C_{ll}(t) & \sqrt{2}D_{lh}(t) \\ \sqrt{2}D_{hl}(t) & 2D_{hh}(t) - C_{hh}(t) \end{pmatrix}$$

Diagonalizing $C(t)$ could lead to very large statistical errors.

**Fortunately:** $D_{\text{eff}} \ll$ diagonal terms for almost all parameter values but not always!

Derivative correlators at $m_h = 0.05$, $m_{\text{eff}} = 0.015$
Mixing in the $0^{++}$ channel

$$C(t) = \begin{pmatrix}
D_{ll}(t) - C_{ll}(t) & \sqrt{2}D_{lh}(t) \\
\sqrt{2}D_{hl}(t) & 2D_{hh}(t) - C_{hh}(t)
\end{pmatrix}$$

Diagonalizing $C(t)$ could lead to very large statistical errors.

Fortunately: the lightest excitation in $D_{\chi\chi}$ (and $D_{\chi\gamma}$, $D_{\gamma\gamma}$) is the $0^{++}$

Derivative correlators at $m_h = 0.06$, $m_{\chi} = 0.010$:

$D_{\chi\chi}$ and $D_{\gamma\gamma}$.